

## 14.5 Notes: Transition Matrices

**Transition matrices** are used to analyze, describe, & predict change.

E.g., manufacturers who conduct market analysis to predict consumer trends over set periods of time use transition matrices to determine the percentage of people who switch brands.

ex 1: Crest and Colgate both manufacture toothpaste. (a) Build a transition matrix that shows the percentages of customers who either stay with the brand they are using or switch brands based on the given information:

\* 90% of people who use Crest continue to use Crest while 6% switch to Colgate the next month & 4% switch to some other brand

\* 72% of people who use Colgate continue to use Colgate while 11% switch to Crest the next month & 17% switch to some other brand

\* 60% of people who use some other brand continue to use some other brand while 21% switch to Crest the next month & 19% switch to Colgate

$$\begin{array}{l} \text{From:} \\ \text{Crest} \\ \text{Colgate} \\ \text{other} \end{array} \begin{bmatrix} \text{To: Crest} & \text{Colgate} & \text{other} \\ \text{Crest} & & \\ \text{Colgate} & & \\ \text{other} & & \end{bmatrix} = T \quad \leftarrow \text{transition matrix for consumer buying}$$

(b) Conduct a market survey to collect the current market share in percentages (in this case, we will do a class survey)

$$\begin{array}{c} \text{Crest} \quad \text{Colgate} \quad \text{other} \\ \text{Market Share} \quad \left[ \begin{array}{ccc} & & \end{array} \right] = M_0 \quad \leftarrow \text{initial market profile} \end{array}$$

(c) Suppose toothpaste is bought once per month. Predict market share...

i) one month from now:  $M_1 = M_0 T =$

ii) two months from now:  $M_2 = M_1 T = M_0 T T = M_0 T^2 =$

iii) three months from now:  $M_3 = M_0 T^3 =$

iv) 30 months from now:  $M_{30} = \quad =$

v) 40 months from now:  $M_{40} = \quad =$

\* Notice  $M_{30}$  and  $M_{40}$  are almost the same; this is called a "Steady State"

What market conditions must be true for the 30-month and 40-month predictions to be accurate?

Very little change in  $\quad$ , very little change in  $\quad$ , very little flux in  $\quad$ , no  $\quad$ , et cetera.

(d) Suppose another class was surveyed and their initial market profile was:

$$\begin{array}{c} \text{Crest} \quad \text{Colgate} \quad \text{other} \\ \text{Market Share} \quad [ .75 \quad .15 \quad .10 ] = M_0 \end{array}$$

Find the steady state for this matrix using the same transition matrix.

I.e., we want to find some steady state matrix  $S$  such that  $S = S T$   
 where  $S = M_0 T^?$  ... so multiply by

Idea: Keep multiplying  $M_0 T$  times  $T$  until the decimals level out and don't change much, e.g. calculate  $M_0 T^{50}$ .

$$S = [ \quad ]$$

#### 14.4 HW

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#### Manufacturing

	tables	chairs	desks	amnt. of time available
Carpentry	[[ 120	105	125 ]	Carpentry [[ 20,250 ]
Assembly	[ 40	65	110 ]	Assembly [ 12,070 ]
Finishing	[ 80	90	125 ]]	Finishing [ 17,000 ]]

Want to use all available labor. How many t, c, d, should the manager schedule for production each week? I.e., want to find t, c, d.

$$\begin{array}{c} 3 \times 3 \quad A \quad X = B \quad 3 \times 1 \\ \begin{bmatrix} 120 & 105 & 125 \\ 40 & 65 & 110 \\ 80 & 90 & 125 \end{bmatrix} \begin{bmatrix} t \\ c \\ d \end{bmatrix} = \begin{bmatrix} 20,250 \\ 12,070 \\ 17,000 \end{bmatrix} \end{array}$$

$$X = A^{-1} B \quad \leftarrow \text{Do on T.I.}$$

$$X = \begin{bmatrix} t \\ c \\ d \end{bmatrix} = \begin{bmatrix} 25 \\ 150 \\ 12 \end{bmatrix}$$

**25 tables, 150 chairs, 12 desks**