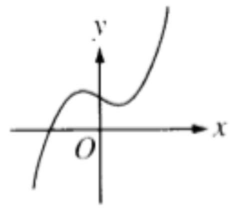
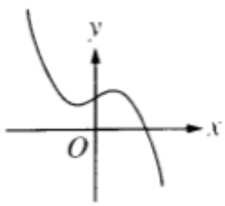
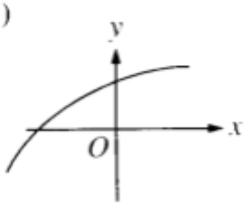


6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

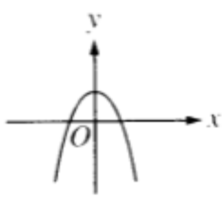
(A)



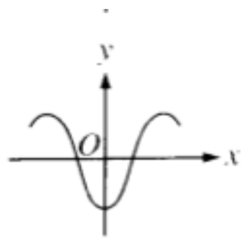
(B)



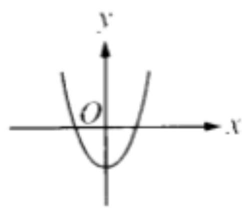
(C)



(D)



(E)



1. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

6. E The graph of h has 2 turning points and one point of inflection. The graph of h' will have 2 x-intercepts and one turning point. Only (C) and (E) are possible answers. Since the first turning point on the graph of h is a relative maximum, the first zero of h' must be a place where the sign changes from positive to negative. This is option (E).

1. C f will be increasing when its derivative is positive.
$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$ $f'(x) = 3(x+3)(x-1) > 0$ for $x < -3$ or $x > 1$.

80. The first derivative of the function f is given by $f'(x)=\frac{\cos^2 x}{x}-\frac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

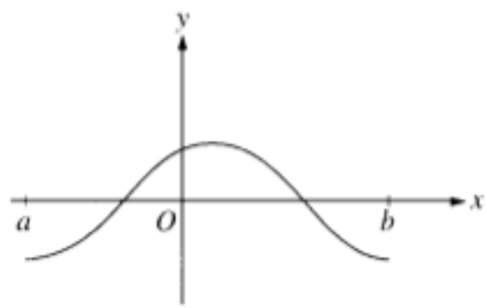
(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

89. If g is a differentiable function such that $g(x)<0$ for all real numbers x and if $f'(x)=\left(x^2-4\right)g(x)$, which of the following is true?

(A) f has a relative maximum at $x=-2$ and a relative minimum at $x=2$.
(B) f has a relative minimum at $x=-2$ and a relative maximum at $x=2$.
(C) f has relative minima at $x=-2$ and at $x=2$.
(D) f has relative maxima at $x=-2$ and at $x=2$.
(E) It cannot be determined if f has any relative extrema.

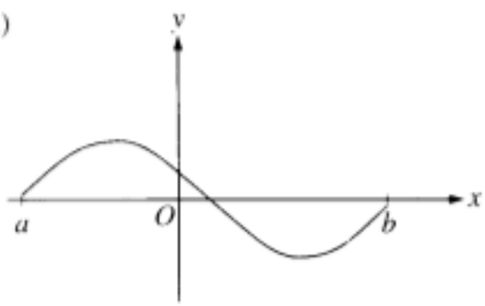
80. B Look at the graph of $f'(x)$ on the interval (0,10) and count the number of x-intercepts in the interval.

89. B The graph of $y = x^2 - 4$ is a parabola that changes from positive to negative at $x = -2$ and from negative to positive at $x = 2$. Since g is always negative, f' changes sign opposite to the way $y = x^2 - 4$ does. Thus f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.

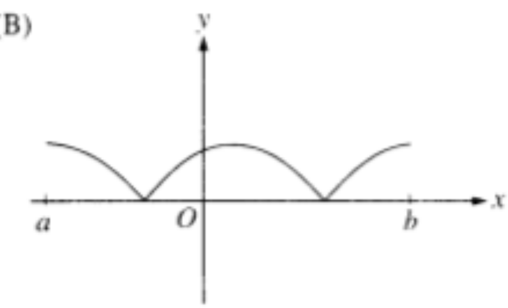


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

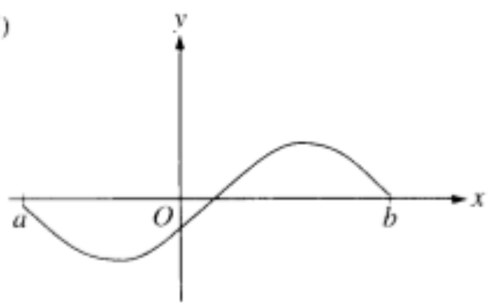
(A)



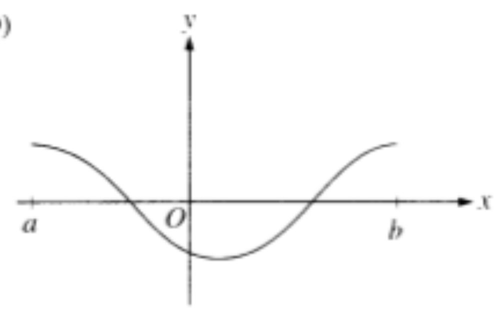
(B)



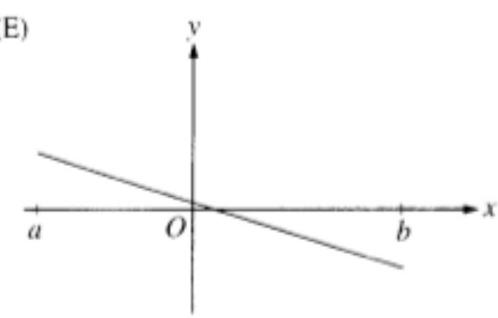
(C)



(D)



(E)



23. A The graph shows that f is increasing on an interval (a,c) and decreasing on the interval (c,b), where $a<c<b$. This means the graph of the derivative of f is positive on the interval (a,c) and negative on the interval (c,b), so the answer is (A) or (E). The derivative is not (E), however, since then the graph of f would be concave down for the entire interval.

19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only (C) -1 and 0 only (D) -1 and 2 only (E) -1, 0, and 2 only

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

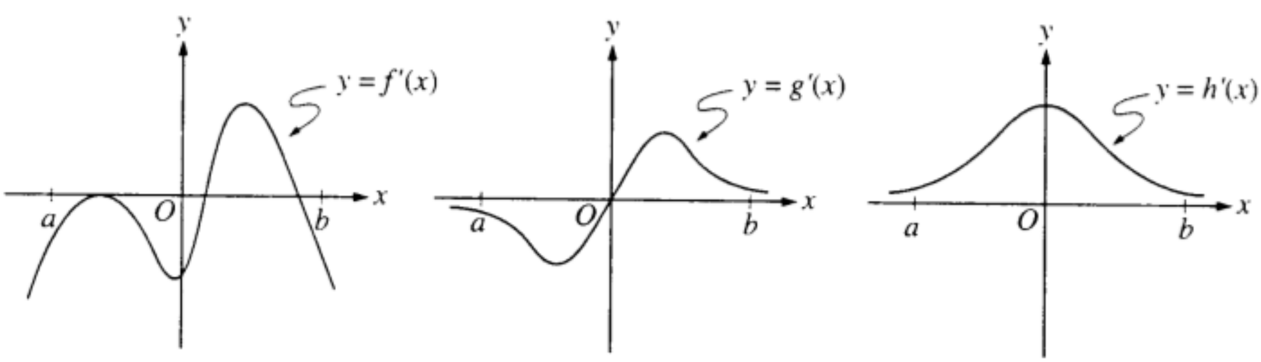
(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

19. C Points of inflection occur where f'' changes sign. This is only at $x=0$ and $x=-1$. There is no sign change at $x=2$.

22. C f is increasing on any interval where $f'(x)>0$. $f'(x)=4x^3+2x=2x(2x^2+1)>0$. Since $(x^2+1)>0$ for all x, $f'(x)>0$ whenever $x>0$.

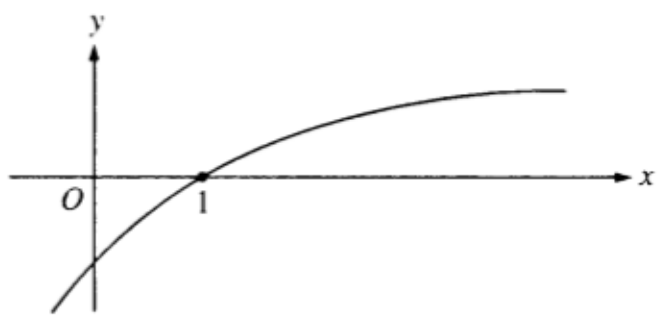
79. A The graph of the derivative would have to change from positive to negative. This is only true for the graph of f'.

80. Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) -5 (E) -10



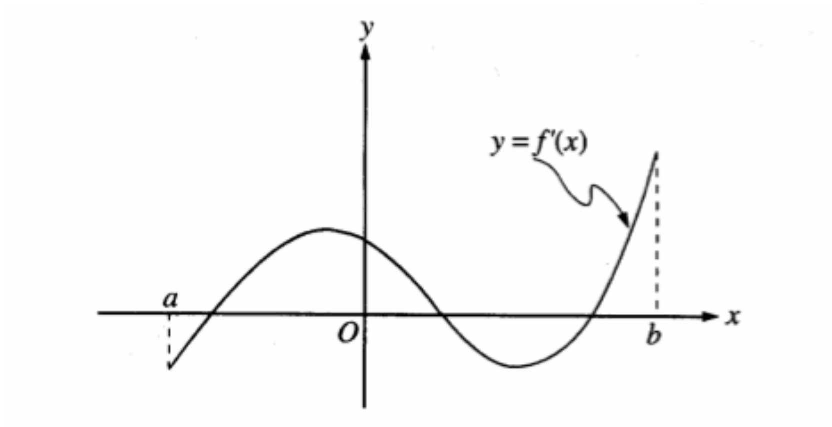
17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

80 B Find the first turning point on the graph of $y = f'(x)$. Occurs at $x = 0.93$.

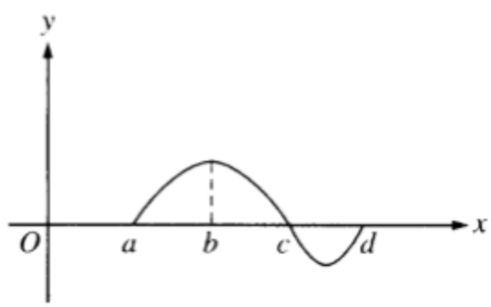
1. D $y' = x^2 + 10x$; $y'' = 2x + 10$; y'' changes sign at $x = -5$

17. D From the graph $f(1) = 0$. Since $f'(1)$ represents the slope of the graph at $x = 1$, $f'(1) > 0$. Also, since $f''(1)$ represents the concavity of the graph at $x = 1$, $f''(1) < 0$.



12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a,b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima



22. The graph of f is shown in the figure above. If $g(x)=\int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.

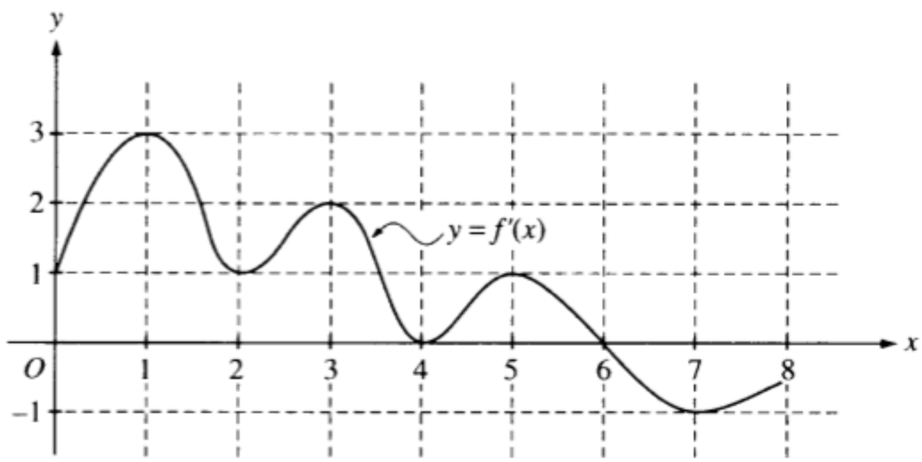
12. A f' changes from positive to negative once and from negative to positive twice. Thus one relative maximum and two relative minimums.

22. C $g'(x) = f(x)$. The only critical value of g on (a,d) is at $x = c$. Since g' changes from positive to negative at $x = c$, the absolute maximum for g occurs at this relative maximum.

3. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) -1 (B) $-\frac{\sqrt{5}}{5}$ (C) 0 (D) $\frac{\sqrt{5}}{5}$ (E) 1

Questions 7-9 refer to the graph and the information below.



The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

8. How many points of inflection does the graph of f have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

9. At what value of x does the absolute minimum of f occur?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

3. A $f(x)=3x^5-4x^3-3x;\ f'(x)=15x^4-12x^2-3=3(5x^2+1)(x^2-1)=3(5x^2+1)(x+1)(x-1)$;
f' changes from positive to negative only at $x=-1$.

8. E Points of inflection occur where f' changes from increasing to decreasing, or from decreasing to increasing. There are six such points.

9. A f increases for $0 \le x \le 6$ and decreases for $6 \le x \le 8$. By comparing areas it is clear that f increases more than it decreases, so the absolute minimum must occur at the left endpoint, $x=0$.

77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

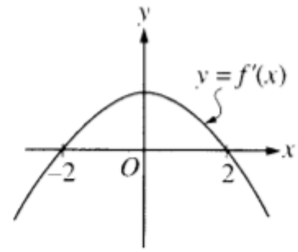
(A) -1.58 (B) -1.63 (C) -1.67 (D) -1.89 (E) -2.33

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) -0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

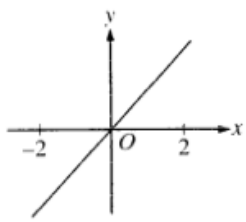
77. D $y = x^3 + 6x^2 + 7x - 2\cos x$. Look at the graph of $y'' = 6x + 12 + 2\cos x$ in the window [–3,–1] since that domain contains all the option values. y'' changes sign at $x = -1.89$.

85. C Look at the graph of f' and locate where the y changes from positive to negative. $x = 0.91$

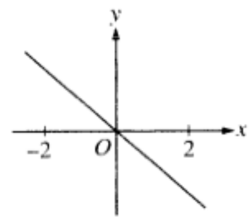


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

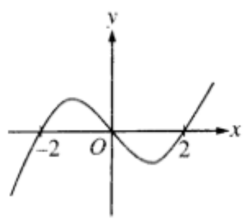
(A)



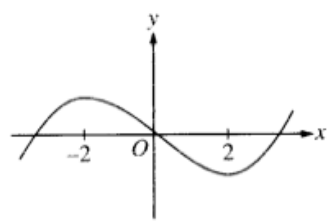
(B)



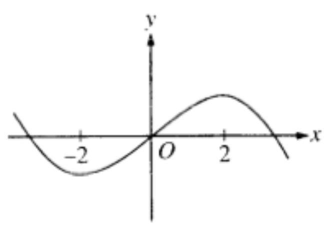
(C)



(D)



(E)



22. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x < -1$ and $x > 3$

(C) $-3 < x < 1$

(D) $-1 < x < 3$

(E) All values of x

11. E Since f' is positive for $-2 < x < 2$ and negative for $x < -2$ and for $x > 2$, we are looking for a graph that is increasing for $-2 < x < 2$ and decreasing otherwise. Only option E.

22. D $f(x) = (x^2 - 3)e^{-x}$; $f'(x) = e^{-x}(-x^2 + 2x + 3) = -e^{-x}(x-3)(x+1)$; $f'(x) > 0$ for $-1 < x < 3$

22. If $f(x) = x^2 e^x$, then the graph of f is decreasing for all x such that

(A) $x < -2$ (B) $-2 < x < 0$ (C) $x > -2$ (D) $x < 0$ (E) $x > 0$

41. Let $f(x) = \int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

22. B $f'(x) = x^2e^x + 2xe^x = xe^x(x+2)$; $f'(x) < 0$ for $-2 < x < 0$

41. C $f'(x) = (2x-3)e^{\left(x^2-3x\right)^2}$; $f' < 0$ for $x < \frac{3}{2}$ and $f' > 0$ for $x > \frac{3}{2}$.
Thus f has its absolute minimum at $x = \frac{3}{2}$.

5. E $y = 3x^4 - 16x^3 + 24x^2 + 48$; $y' = 12x^3 - 48x^2 + 48x$; $y'' = 36x^2 - 96x + 48 = 12(3x-2)(x-2)$
$y'' < 0$ for $\frac{2}{3} < x < 2$, therefore the graph is concave down for $\frac{2}{3} < x < 2$

44. What is the minimum value of $f(x) = x \ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

9. If $f(x) = 1 + x^{\frac{2}{3}}$, which of the following is NOT true?

(A) f is continuous for all real numbers.

(B) f has a minimum at $x = 0$.

(C) f is increasing for $x > 0$.

(D) $f'(x)$ exists for all x.

(E) $f''(x)$ is negative for $x > 0$.

14. The <u>derivative</u> of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

(A) None (B) One (C) Two (D) Three (E) Four

44. C $f'(x) = \ln x + x \cdot \frac{1}{x}$; $f'(x)$ changes sign from negative to positive only at $x = e^{-1}$.

$f(e^{-1}) = -e^{-1} = -\frac{1}{e}$.

9. D $f'(x) = \frac{2}{3} \cdot \frac{1}{x^{1/3}}$. This does not exist at $x = 0$. D is false, all others are true.

14. B The only place that $f'(x)$ changes sign from positive to negative is at $x = -3$.

21. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

23. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

27. The function f given by $f(x) = x^3 + 12x - 24$ is

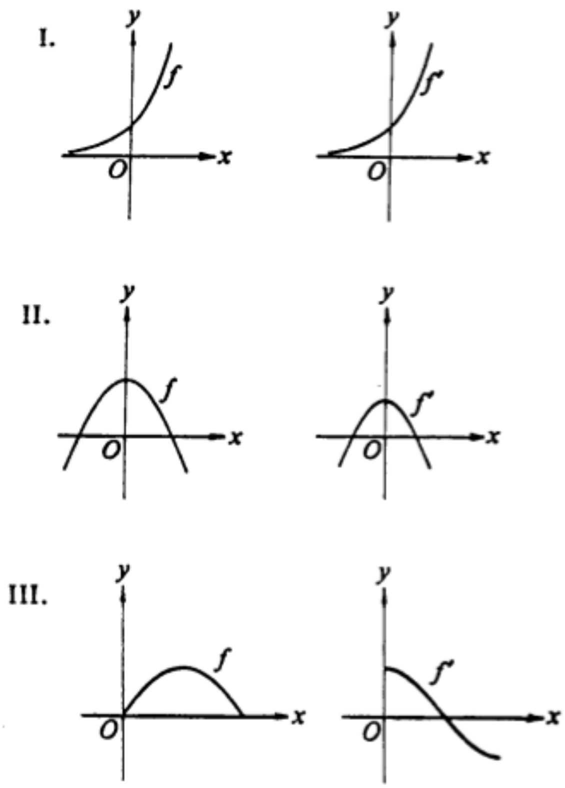
(A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
(B) decreasing for $x < 0$, increasing for $x > 0$
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

21. C $y = x^{-2} - x^{-3}$; $y' = -2x^{-3} + 3x^{-4}$; $y'' = 6x^{-4} - 12x^{-5} = 6x^{-5}(x-2)$. The only domain value at which there is a sign change in y'' is $x = 2$. Inflection point at $x = 2$.

23. C A quick way to do this problem is to use the effect of the multiplicity of the zeros of f on the graph of $y = f(x)$. There is point of inflection and a horizontal tangent at $x = -2$. There is a horizontal tangent and turning point at $x = 3$. There is a horizontal tangent on the interval $(-2, 3)$. Thus, there must be 3 critical points. Also, $f'(x) = (x-3)^3(x+2)^4(9x-7)$.

27. C $f'(x) = 3x^2 + 12 > 0$. Thus f is increasing for all x.

9. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?



(A) I only (B) II only (C) III only (D) I and III (E) II and III

27. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b?

(A) -3 (B) 0 (C) 1 (D) 3

(E) It cannot be determined from the information given.

15. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

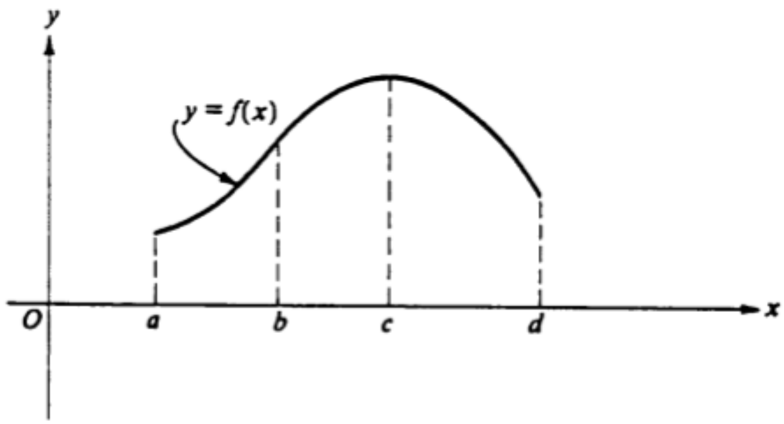
9. D II does not work since the slope of f at $x = 0$ is not equal to $f'(0)$. Both I and III could work. For example, $f(x) = e^x$ in I and $f(x) = \sin x$ in III.

27. B $y'(x) = 3x^2 + 2ax + b$, $y''(x) = 6x + 2a$, $y''(1) = 0 \Rightarrow a = -3$
$y(1) = -6$ so, $-6 = 1 + a + b - 4 \Rightarrow -6 = 1 - 3 + b - 4 \Rightarrow b = 0$

15. D $f'(x) = (x-3)^2 + 2(x-2)(x-3) = (x-3)(3x-7)$; $f'(x)$ changes from positive to negative at $x = \dfrac{7}{3}$.

4. The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$



8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\dfrac{dy}{dx} > 0$ and $\dfrac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

33. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2

4. E Students should know what the graph looks like without a calculator and choose option E.
Or $y=-5(x-2)^{-1}$; $y'=5(x-2)^{-2}$; $y''=-10(x-2)^{-3}$. $y''<0$ for $x>2$.

8. B $\frac{dy}{dx}>0 \Rightarrow y$ is increasing; $\frac{d^2y}{dx^2}<0 \Rightarrow$ graph is concave down. This is only on $b<x<c$.

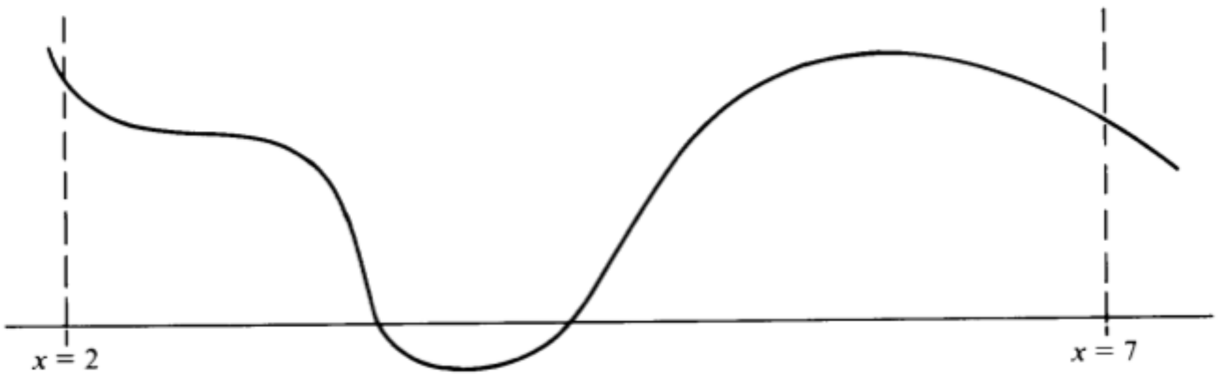
33. A Check the critical points and the endpoints.
$f'(x)=3x^2-6x=3x(x-2)$ so the critical points are 0 and 2.

x	-2	0	2	4
$f(x)$	-8	12	8	28

Absolute maximum is at $x=4$.

2. At what values of x does $f(x)=3x^5-5x^3+15$ have a relative maximum?

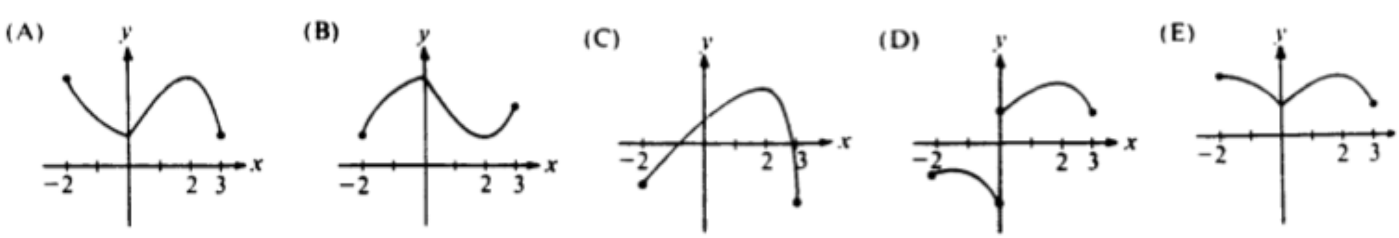
(A) -1 only (B) 0 only (C) 1 only (D) -1 and 1 only (E) -1, 0 and 1



20. The graph of $y=f(x)$ on the closed interval $[2,7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two (C) Three (D) Four (E) Five

43. Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x)<0$ for all x except $x=0$. Which of the following could be the graph of f?



2. A $f'(x) = 15x^4 - 15x^2 = 15x^2(x^2 - 1) = 15x^2(x-1)(x+1)$, changes sign from positive to negative only at $x = -1$. So f has a relative maximum at $x = -1$ only.

20. C Look for concavity changes, there are 3.

43. E Graphs A and B contradict $f'' < 0$. Graph C contradicts $f'(0)$ does not exist. Graph D contradicts continuity on the interval $[-2,3]$. Graph E meets all given conditions.

36. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.
II. The maximum value of $|f(x)|$ is 7.
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

39. If $f(x) = \dfrac{\ln x}{x}$, for all $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

43. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$

(D) $y = 3x - 1$ (E) $y = 4x + 1$

36. B II is true since $|-7|=7$ will be the maximum value of $|f(x)|$. To see why I and III do not have to be true, consider the following: $f(x)=\begin{cases} 5 & \text{if} \quad x\le -5 \\ -x & \text{if} \quad -5<x<7 \\ -7 & \text{if} \quad x\ge 7 \end{cases}$

For $f(|x|)$, the maximum is 0 and the minimum is –7.

39. E $f'(x)=\frac{1}{x}\cdot\frac{1}{x}-\frac{1}{x^2}\ln x=\frac{1}{x^2}(1-\ln x)<0$ for $x>e$. Hence f is decreasing. for $x>e$.

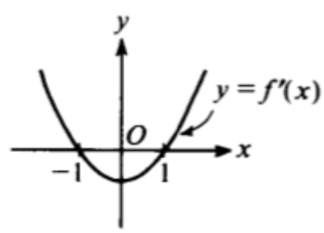
43. B $y'=3x^2+6x$, $y''=6x+6=0$ for $x=-1$. $y'(-1)=-3$. Only option B has a slope of –3.

27. If $f(x)=\frac{1}{3}x^3-4x^2+12x-5$ and the domain is the set of all x such that $0\le x\le 9$, then the absolute maximum value of the function f occurs when x is

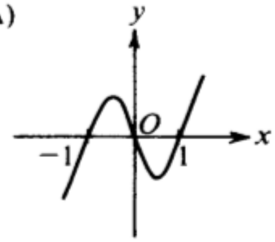
(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

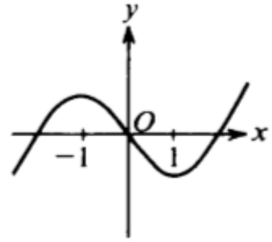
16. The function defined by $f(x)=x^3-3x^2$ for all real numbers x has a relative maximum at $x=$

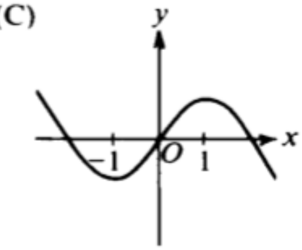
(A) –2 (B) 0 (C) 1 (D) 2 (E) 4

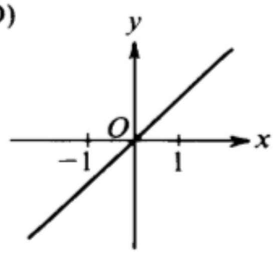


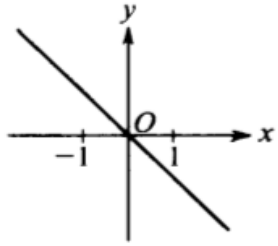
33. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

(A)


(B)


(C)


(D)


(E)


27. E $f'(x) = x^2 - 8x + 12 = (x-2)(x-6)$; the candidates are: $x = 0, 2, 6, 9$

x	0	2	6	9
$f(x)$	-5	$17/3$	-5	22

the maximum is at $x = 9$

16. B $f'(x) = 3x^2 - 6x = 3x(x-2)$ changes sign from positive to negative only at $x = 0$.

33. B f' changes sign from positive to negative at $x = -1$ and therefore f changes from increasing to decreasing at $x = -1$.

Or f' changes sign from positive to negative at $x = -1$ and from negative to positive at $x = 1$. Therefore f has a local maximum at $x = -1$ and a local minimum at $x = 1$.

22. Given the function defined by $f(x)=3x^5-20x^3$, find all values of x for which the graph of f is concave up.

(A) $x>0$

(B) $-\sqrt{2}<x<0$ or $x>\sqrt{2}$

(C) $-2<x<0$ or $x>2$

(D) $x>\sqrt{2}$

(E) $-2<x<2$

3. If $f(x)=x+\dfrac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $[-1,1]$ (C) $(-\infty,\infty)$

(D) $(0,\infty)$ (E) $(-\infty,0)\cup(0,\infty)$

26. Which of the following is true about the graph of $y=\ln\left|x^2-1\right|$ in the interval $(-1,1)$?

(A) It is increasing.

(B) It attains a relative minimum at $(0,0)$.

(C) It has a range of all real numbers.

(D) It is concave down.

(E) It has an asymptote of $x=0$.

22. B $f(x) = 3x^5 - 20x^3$; $f'(x) = 15x^4 - 60x^2$; $f''(x) = 60x^3 - 120x = 60x\left(x^2 - 2\right)$

The graph of f is concave up where $f'' > 0$: $f'' > 0$ for $x > \sqrt{2}$ and for $-\sqrt{2} < x < 0$.

3. A $f'(x) = 1 - \dfrac{1}{x^2} = \dfrac{(x+1)(x-1)}{x^2}$. $f'(x) > 0$ for $x < -1$ and for $x > 1$.

f is increasing for $x \le -1$ and for $x \ge 1$.

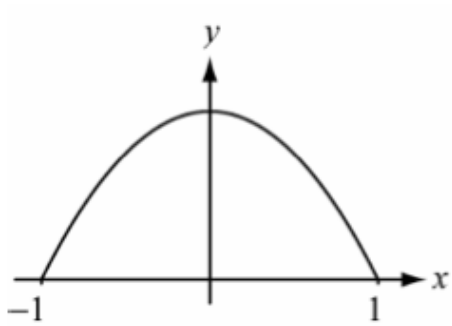
26. D For x in the interval $(-1, 1)$, $g(x) = \left|x^2 - 1\right| = -(x^2 - 1)$ and so $y = \ln g(x) = \ln(-(x^2 - 1))$.

Therefore

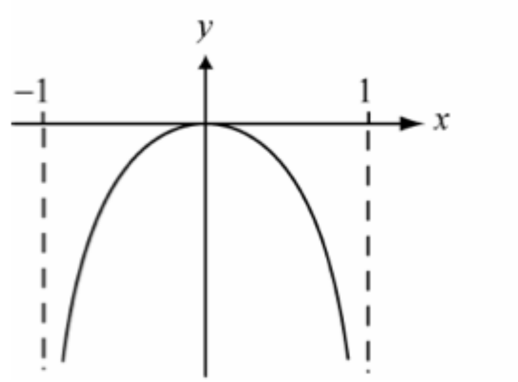
$$y' = \frac{2x}{x^2 - 1},\quad y'' = \frac{\left(x^2 - 1\right)(2) - (2x)(2x)}{\left(x^2 - 1\right)^2} = \frac{-2x^2 - 2}{\left(x^2 - 1\right)^2} < 0$$

Alternative graphical solution: Consider the graphs of $g(x) = \left|x^2 - 1\right|$ and $\ln g(x)$.

concave down



$g(x) = \left|x^2 - 1\right|$



$\ln\left|x^2 - 1\right|$

2. What are the coordinates of the inflection point on the graph of $y=(x+1)\arctan x$?

(A) $(-1,0)$ (B) $(0,0)$ (C) $(0,1)$ (D) $\left(1,\frac{\pi}{4}\right)$ (E) $\left(1,\frac{\pi}{2}\right)$

7. For what value of *k* will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

10. The *derivative* of $f(x)=\frac{x^4}{3}-\frac{x^5}{5}$ attains its maximum value at $x=$

(A) –1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

2. E $y = (x+1)\tan^{-1} x,\ y' = \dfrac{x+1}{1+x^2} + \tan^{-1} x$

$$y'' = \frac{\left(1+x^2\right)(1) - (x+1)(2x)}{\left(1+x^2\right)^2} + \frac{1}{1+x^2} = \frac{2-2x}{\left(1+x^2\right)^2}$$

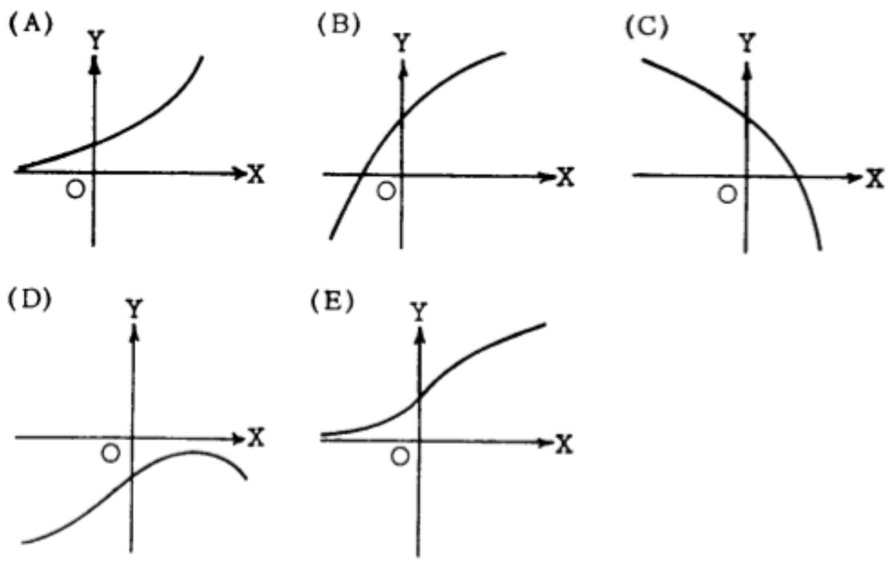
y'' changes sign at $x = 1$ only. The point of inflection is $\left(1,\ \pi/2\right)$

7. D With $f(x) = x + \dfrac{k}{x}$, we need $0 = f'(-2) = 1 - \dfrac{k}{4}$ and so $k = 4$. Since $f''(-2) < 0$ for $k = 4$, f does have a relative maximum at $x = -2$.

10. C $f(x) = \dfrac{x^4}{3} - \dfrac{x^5}{5};\ f'(x) = \dfrac{4x^3}{3} - x^4;\ f''(x) = 4x^2 - 4x^3 = 4x^2(1-x)$

$f'' > 0$ for $x < 1$ and $f'' < 0$ for $x > 1 \Rightarrow f'$ has its maximum at $x = 1$.

16. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only
(B) $(3,162)$ only
(C) $(4,256)$ only
(D) $(0,0)$ and $(3,162)$
(E) $(0,0)$ and $(4,256)$

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

16. B $y' > 0 \Rightarrow y$ is increasing; $y'' < 0 \Rightarrow$ the graph is concave down. Only B meets these conditions.

17. B $y' = 20x^3 - 5x^4$, $y'' = 60x^2 - 20x^3 = 20x^2(3 - x)$. The only sign change in y'' is at $x = 3$. The only point of inflection is $(3,162)$.

21. B $f'(x) = 2x - 2e^{-2x}$, $f'(0) = -2$, so f is decreasing

No Calculators

12. The difference between the absolute maximum value and absolute mimimum value of the function defined by $f(x) = \cos x + \sin x$ is

(A) $\frac{\pi}{2}$ (B) 2 (C) $\sqrt{2}$ (D) $2\sqrt{2}$ (E) $\sqrt{6} - 1$

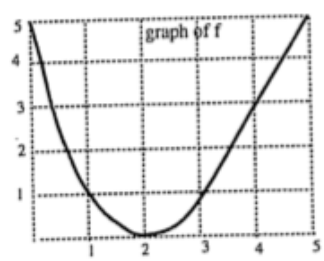
Calculators Active

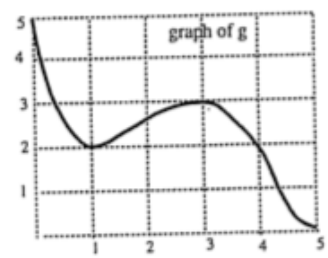
5. The graphs of functions f and g are shown below. If $h(x) = f[g(x)]$, which of the following statements are true about the function h?

I. $h(3) = 5$

II. h is increasing at $x = 2$

III. the graph of h has a horizontal tangent at $x = 1$





(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

12. D p. 5

With $f(x) = \cos x + \sin x$, we have $f'(x) = -\sin x + \cos x$.
The critical numbers occur when $f'(x) = 0$. That is when $\sin x = \cos x$.

These critical numbers are at $x = \frac{\pi}{4} + n\pi$.

The maximum value of the function is $f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$.

The minimum value of the function is $f(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$.

The difference between these two values is $2\sqrt{2}$.

5. D p. 11

I. $h(3) = f(g(3)) = f(3) \approx 1$ — False

II. $h'(x) = f'(g(x)) \cdot g'(x)$

Thus $h'(2) = f'(g(2)) \cdot g'(2)$

$\approx f'(2.6) \cdot g'(2) \approx \frac{1}{2} \cdot \frac{1}{2} > 0$ — True

III. $h'(1) = f'(g(1)) \cdot g'(1) = f'(2) \cdot g'(1) = 0$ — True

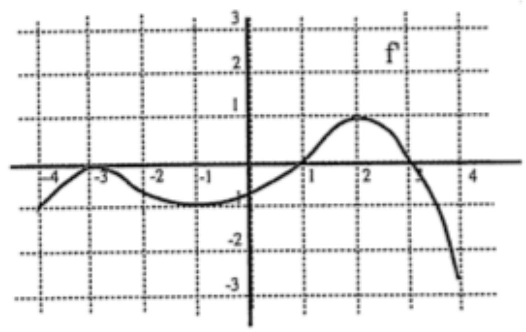
Calculators Active

8. The number of inflection points for the graph of $y = 2x + \cos(x^2)$ in the interval $0 \le x \le 5$ is

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Calculators Active

13. The figure shows the graph of f', the *derivative* of a function f. The domain of f is the interval $-4 \le x \le 4$. Which of the following are true about the graph of f?



graph of the derivative of f

I. At the points where $x = -3$ and $x = 2$ there are horizontal tangents.
II. At the point where $x = 1$ there is a relative minimum point.
III. At the point where $x = -3$ there is an inflection point.

(A) None (B) II only (C) III only (D) II and III only (E) I, II, III

8. C p. 12

$y = 2x + \cos(x^2)$
$y' = 2 - 2x\sin(x^2)$
$y'' = -2\sin(x^2) - 4x^2\cos(x^2) = 0$

This second derivative, graphed on the interval $[0,5]$, has eight zeros at which there are sign changes. Each corresponds to a point of inflection of the original curve.

13. D p. 36

I. The function f has a horizontal tangent at each x-coordinate that $f'(x) = 0$. This is the case at $x = -3, 1,$ and 3, but **not** at $x = 2$. False

II. Since $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$, the function f is decreasing to the left of $x = 1$ and increasing to the right. True

III. Since $f'(x)$ is increasing to the left of $x = -3$ and decreasing to the right of $x = -3$, the concavity if the graph of f will change at $x = -3$. True

No Calculators

23. Let f be defined by $f(x) = x^{2/3}(5 - 2x)$. f is increasing on the interval

(A) $x < -\frac{5}{2}$ (B) $x > 0$ (C) $x < 1$ (D) $0 < x < \frac{5}{8}$ (E) $0 < x < 1$

3. If $y = 2xe^{-x}$, then y has a point of inflection at $x =$

(A) 0

(B) 1

(C) 2

(D) –2

(E) 4

23. E p. 52

$$f(x) = x^{2/3}(5-2x)$$

$$f'(x) = \frac{2}{3}x^{-1/3}(5-2x) - 2x^{2/3}$$

$$= \frac{2}{3}x^{-1/3}[(5-2x) - 3x]$$

$$= \frac{2}{3}x^{-1/3}[5-5x]$$

$$= \frac{10}{3}x^{-1/3}(1-x)$$

This is positive when $0 < x < 1$.

3. To find the point of inflection, we set $y'' = 0$ and check the concavity at that point (making sure that it switches from concave up to concave down or vice versa).

$$y' = (2x)(-e^{-x}) + 2e^{-x} = -2xe^{-x} + 2e^{-x} = -2e^{-x}(x-1)$$

$$y'' = (-2e^{-x}) + (x-1)(2e^{-x}) = 2xe^{-x} - 4e^{-x} = 2e^{-x}(x-2)$$

$y'' = 0$ only when $x = 2$ (e^{-x} is never equal to zero).

	$x < 2$	$x > 2$
y''	$-$	$+$

The only point of inflection is at $x = 2$.

The correct choice is (C).

Calculators Active

6. Let $f(x) = ax + \frac{b}{x}$ where a and b are positive constants.

(a) Find in terms of a and b, the intervals on which f is increasing.

(b) Find the coordinates of all local maximum and minimum points.

(c) On what interval(s) is the graph concave up?

(d) Find any inflection points. Explain briefly.

6. p. 44

(a) $f(x) = ax + \frac{b}{x}$, where a and b are positive.

$f'(x) = a - \frac{b}{x^2}$

$f'(x) > 0 \quad \Rightarrow \quad ax^2 - b > 0$

$\Rightarrow \quad x^2 > \frac{b}{a}$

$\Rightarrow \quad x < -\sqrt{\frac{b}{a}} \quad \text{or} \quad x > \sqrt{\frac{b}{a}}$

(b) There is a relative maximum at $x = -\sqrt{\frac{b}{a}}$; $y = a\left[-\sqrt{\frac{b}{a}}\right] + \frac{b}{-\sqrt{\frac{b}{a}}}$

$= -2\sqrt{ab}$.

There is a relative minimum at $x = \sqrt{\frac{b}{a}}$; $y = a\left[\sqrt{\frac{b}{a}}\right] + \frac{b}{\sqrt{\frac{b}{a}}}$

$= 2\sqrt{ab}$.

(c) $f''(x) = \frac{2b}{x^3} > 0$ when $x > 0$

(d) $f''(x)$ is never 0. Although the curve is concave down for $x < 0$ and the curve is concave up for $x > 0$, there is not a point of inflection at $x = 0$ since there is no point on the curve there.

Calculators Active

1. Let $f(x) = x^3 + px^2 + qx$.

(a) Find the values of p and q so that $f(-1) = -8$ and $f'(-1) = 12$.

(b) Find the value of p so that the graph of f changes concavity at $x = 2$.

(c) Under what conditions on p and q will the graph of f be increasing everywhere.

1. p. 62

(a) $f(x) = x^3 + px^2 + qx \quad \Rightarrow \quad f'(x) = 3x^2 + 2px + q \quad \Rightarrow \quad f''(x) = 6x + 2p$

$$\begin{Bmatrix} f(-1) = -8 \\ f'(-1) = 12 \end{Bmatrix} \quad \Rightarrow \quad \begin{cases} -8 = -1 + p - q \\ 12 = 3 - 2p + q \end{cases}$$

When we add these two equations, we obtain $4 = 2 - p$.
Thus $\boxed{p = -2.}$ Substituting this value into one of the equations, we find that $\boxed{q = 5.}$

(b) If the graph of f is to have a change in concavity at $x = 2$, then $f''(2) = 0$ and $f''(x)$ changes its sign at $x = 2$.

$f''(2) = 12 + 2p = 0 \quad \Rightarrow \quad p = -6$

Then $f''(x) = 6x - 12 = 6(x - 2)$. This <u>does</u> have a sign change at $x = 2$.

(c) For f to be increasing everywhere, we must have $f'(x) > 0$ for all x.
Then $\boxed{3x^2 + 2px + q > 0}$ for all x.
$f'(x)$ is a quadratic function, opening upward.
$f'(x)$ will be positive valued $\Leftrightarrow$ f' has no zeros
$\Leftrightarrow$ the discriminant of f' is less than 0
$\Leftrightarrow 4p^2 - 12q < 0$
$\Leftrightarrow p^2 < 3q.$