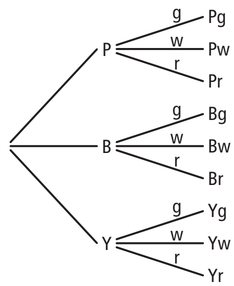
Probability and Statistics
Solutions Key

ARE YOU READY?

1.



2. The entire area is a circle with radius 5 in., so $A = \pi(5)^2 = 25\pi$ in^2. The unshaded area is a square with side x and diagonal 10 in. Using the Pythagorean theorem, $x^2 + x^2 = 100$, so $x^2 = 50$ in^2. The unshaded area is $A = x^2 = 50$ in^2. The shaded area is the entire area minus the unshaded area, so $A = (25\pi - 50)$ in^2. The ratio of the shaded area to the entire area is $\frac{25\pi - 50}{25\pi} = \frac{\pi - 2}{\pi}$.

3. The entire area is a circle with radius 2 cm, so $A = \pi(2)^2 = 4\pi$ in^2. The unshaded area is a circle with radius 1 cm, so the unshaded area is $A = \pi(1)^2$ cm$^2 = \pi$ cm^2. The shaded area is the entire area minus the unshaded area, or $A = 4\pi - \pi = 3\pi$ cm^2. The ratio of the shaded area to the entire area is $\frac{3\pi}{4\pi} = \frac{3}{4}$.

4. $1 - \frac{14}{20} = \frac{20}{20} - \frac{14}{20}$
$= \frac{6}{20} = \frac{3}{10}$

5. $\frac{3}{8} + \frac{5}{6} = \frac{9}{24} + \frac{20}{24}$
$= \frac{29}{24}$

6. $\frac{8}{15} - \frac{2}{5} = \frac{8}{15} - \frac{6}{15}$
$= \frac{2}{15}$

7. $\frac{1}{12} + \frac{1}{10} = \frac{5}{60} + \frac{6}{60}$
$= \frac{11}{60}$

8. $\frac{1}{2} \cdot \frac{3}{7} = \frac{3}{14}$

9. $2\frac{1}{3} \cdot \frac{1}{4} = \frac{7}{3} \cdot \frac{1}{4}$
$= \frac{7}{12}$

10. $\frac{4}{5} \div \frac{1}{2} = \frac{4}{5} \cdot \frac{2}{1}$
$= \frac{8}{5}$

11. $5\frac{1}{3} \div \frac{1}{4} = \frac{16}{3} \div \frac{1}{4}$
$= \frac{16}{3} \cdot \frac{4}{1} = \frac{64}{3}$

12. 7% of 150 = x
$0.07(150) = x$
$x = 10.5$

13. 90% of $x = 45$
$0.9x = 45$
$x = 50$

14. Price increased by 12% of 24 = 0.12(24) = $2.88

15. The amount of water to be changed is 20% of 65 = 0.20(65) = 13 gal.

7-1 PERMUTATIONS AND COMBINATIONS

CHECK IT OUT!

1a. start plot end
6 × 4 × 5 = 120
There are 120 adventures.

b. letter letter letter letter digit
52 × 52 × 52 × 52 × 10 = 73,116,160
There are 73,116,160 possible passwords.

2a. $_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!}$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
$= 8 \cdot 7 \cdot 6 = 336$
There are 336 ways to award the costumes.

b. $_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!}$
$= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$
$= 5 \cdot 4 = 20$
There are 20 ways a 2-digit number can be formed.

3. The order does not matter. It is a combination.
$_8C_2 = \frac{8!}{2!(8-2)!} = \frac{8!}{2!6!}$
$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}$
$= \frac{8 \cdot 7}{2 \cdot 1} = 28$
There are 28 ways to select 2 swimmers from 8.

THINK AND DISCUSS

1. Possible answer: selecting a 9-player batting order from 20 players; selecting 3 magazine subscriptions from a list of 20
2. 1; possible answer: there is only 1 way to choose the entire group from a group.
3. Possible answer: the number of ways to select 4 items from 3; you can't select more than the total number of items.
4.

	Fundamental Counting Principle	Permutation	Combination
Formula	$m_1 \cdot m_2 \cdot \ldots \cdot m_n$	$\frac{n!}{(n-r)!}$	$\frac{n!}{r!(n-r)!}$
Examples	5 shirts × 3 skirts × 4 pairs of shoes = 5 × 3 × 4 outfits	Permutations of 8 items taken 3 at a time $\frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$	3 items chosen from 8 $\frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = 56$

EXERCISES

GUIDED PRACTICE

1. important; permutation
2. blouse jacket skirt
3 × 3 × 2 = 18
There are 18 different outfits.

3. digit letter

$9 \times 25 = 225$

There are 225 different codes.

4. $_7P_2 = \frac{7!}{(7-2)!} = \frac{7!}{5!}$

$= 7 \cdot 6 = 42$

There are 42 ways to schedule the 2 activities.

5. $_{12}P_3 = \frac{12!}{(12-3)!} = \frac{12!}{9!}$

$= 12 \cdot 11 \cdot 10 = 1320$

There are 1320 ways to listen to 3 songs.

6. $_6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!}$

$= 6 \cdot 5 \cdot 4 = 120$

There are 120 ways the prizes can be awarded.

7. $_{21}C_4 = \frac{21!}{4!(21-4)!} = \frac{21!}{4!17!}$

$= \frac{21 \cdot 20 \cdot 19 \cdot 18}{4 \cdot 3 \cdot 2 \cdot 1} = 5985$

There are 5985 ways to send 4 students to the library.

8. $_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!}$

$= \frac{5 \cdot 4}{2 \cdot 1} = 10$

There are 10 ways to choose 3 boxes of cereal.

PRACTICE AND PROBLEM SOLVING

9. lake cabin

$4 \times 3 = 12$

There are 12 routes from the lake to the cabins.

10. posters markers

$3 \times 4 = 12$

There are 12 different posters.

11. $_9P_2 = \frac{9!}{(9-2)!} = \frac{9!}{7!}$

$= 9 \cdot 8 = 72$

There are 72 ways to choose a manager and an assistant.

12. $_{26}P_3 = \frac{26!}{(26-3)!} = \frac{26!}{23!}$

$= 26 \cdot 25 \cdot 24 = 15{,}600$

There are 15,600 possible identification codes.

13. $_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!}$

$= 5 \cdot 4 = 20$

There are 20 ways to assign 2 planes to the runways.

14. $_6C_3 = \frac{6!}{3!(6-3)!} = \frac{6!}{3!3!}$

$= \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$

There are 20 choices of 3 hamburger toppings.

15. $_{49}C_7 - {}_{49}C_6$

$= \frac{49!}{7!(49-7)!} - \frac{49!}{6!(49-6)!}$

$= \frac{49!}{7!42!} - \frac{49!}{6!43!}$

$= \frac{49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$- \frac{49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$= 85{,}900{,}584 - 13{,}983{,}816$

$= 71{,}916{,}768$

There are 71,916,768 more ways to select the numbers.

16. $_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!}$

$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

17. $_5C_5 = \frac{5!}{5!(5-5)!} = \frac{5!}{5!0!} = 1$

18. $_9P_1 = \frac{9!}{(9-1)!} = \frac{9!}{8!} = 9$

19. $_6C_1 = \frac{6!}{1!(6-1)!} = \frac{6!}{1!5!} = 6$

20. $\frac{2!}{6!} = \frac{1}{6 \cdot 5 \cdot 4 \cdot 3} = \frac{1}{360}$

21. $\frac{4!3!}{2!} = \frac{(4 \cdot 3 \cdot 2 \cdot 1)(3 \cdot 2 \cdot 1)}{2 \cdot 1}$

$= 4 \cdot 3 \cdot 2 \cdot 1(3) = 72$

22. $\frac{9!}{7!} = 9 \cdot 8 = 72$

23. $\frac{8! - 5!}{(8-5)!} = \frac{8! - 5!}{3!} = \frac{8!}{3!} - \frac{5!}{3!}$

$= (8 \cdot 7 \cdot 6 \cdot 5 \cdot 4) - (5 \cdot 4)$

$= 6720 - 20 = 6700$

24. $_6C_2 = \frac{6!}{2!(6-2)!} = \frac{6!}{2!4!}$

$= \frac{6 \cdot 5}{2 \cdot 1} = 15$

25. $_7C_4 = \frac{7!}{4!(7-4)!} = \frac{7!}{4!3!}$

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$

26. $_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!}$

$= 7 \cdot 6 \cdot 5 = 210$

$_7C_4 = \frac{7!}{4!(7-4)!} = \frac{7!}{4!3!}$

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$

Therefore, $_7P_3 > {}_7C_4$.

27. $_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!}$

$= 7 \cdot 6 \cdot 5 \cdot 4 = 840$

$_7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!}$

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$

Therefore, $_7P_4 > {}_7C_3$.

28. $_7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!}$

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$

$_7C_4 = \frac{7!}{4!(7-4)!} = \frac{7!}{4!3!}$

$= \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$

Therefore, $_7C_3 = {_7C_4}$.

29. $_{10}C_{10} = \frac{10!}{10!(10-10)!} = \frac{10!}{10!0!} = 1$

$_{10}P_{10} = \frac{10!}{(10-10)!} = \frac{10!}{0!} = 3{,}628{,}800$

Therefore, $_{10}C_{10} < {_{10}P_{10}}$.

30.

***n*!**	4!	3!	2!	1
***n*(*n* − 1)!**	4(3!) = 24	3(2)! = 6	2(1)! = 2	1(0)! = 1

$n(n-1)! = n!$
$(1)(1-1)! = 1!$
$1(0)! = 1$
$0! = 1$

31. The Es in GEESE are identical. The order of the Es is not important.

32. Number of sequences in a peal is 8! = 40,320. It would take 0.25(40,320) = 10,080 s = 2.8 h to ring a complete peal.

33a.

President	A	A	A	A	A	A	A	A	A	A	A	A
Vice President	B	B	B	C	C	C	D	D	D	E	E	E
Secretary	C	D	E	B	D	E	B	C	E	B	C	D

b.

President	B	B	B	B	B	B	B	B	B	B	B	B
Vice President	A	A	A	C	C	C	D	D	D	E	E	E
Secretary	C	D	E	A	D	E	A	C	E	A	C	D

A president, a vice president, and a secretary can be chosen in 60 ways.

c. $\frac{5!}{(5-3)!} = 60$

d. $\frac{5!}{3!(5-3)!} = 10$

The answer 60 is a number of permutations, and the answer 10 is a number of combinations.

34. Possible answer:

$$\frac{_nP_r}{_nC_r} = \frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{n!}{(n-r)!} \times \frac{r!(n-r)!}{n!} = r!;$$

$$\frac{_6P_3}{_6C_3} = 3! = 6;\ {_6C_3} = \frac{_6P_3}{3!};$$

the number of combinations of n items taken r at a time, is the number of permutations of the items divided by the number of ways to order the r items.

35. $_9C_2 = \frac{9!}{2!(9-2)!} = \frac{9!}{2!7!}$

$= \frac{9 \cdot 8}{2 \cdot 1} = 36;$

$_9C_7 = \frac{9!}{7!(9-7)!} = \frac{9!}{7!2!}$

$= \frac{9 \cdot 8}{2 \cdot 1} = 36;$

$_{10}C_6 = \frac{10!}{6!(10-6)!} = \frac{10!}{6!4!}$

$= \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210;$

$_{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10!}{4!6!}$

$= \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210;$

$\frac{n!}{r!(n-r)!}$ is the same as $\frac{n!}{(n-r)!r!}$.

36a. Jen can arrange the dice in 5! = 120 ways.

b. $_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!}$

$= \frac{5 \cdot 4}{2 \cdot 1} = 10$

37. A; order is important.

38. Choosing 3 times from 9 digits: there are 9 possible choices the first time, 8 the second, and 7 the third. The total number of permutations is $9 \times 8 \times 7 = 504$.

TEST PREP

39. D

$_{14}C_5 = \frac{14!}{5!(14-5)!} = \frac{14!}{5!9!}$

40. J

$_9C_4 = \frac{9!}{4!(9-4)!} = \frac{9!}{4!5!}$

$_9C_5 = \frac{9!}{5!(9-5)!} = \frac{9!}{5!4!}$

41. $_{15}C_4 = \frac{15!}{4!(15-4)!} = \frac{15!}{4!11!}$

$= \frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1} = 1365$

There are 1365 ways Rene can choose her electives.

CHALLENGE AND EXTEND

42a. 4 points: $\frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{4 \cdot 3}{2 \cdot 1} = 6$

5 points: $\frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \cdot 4}{2 \cdot 1} = 10$

6 points: $\frac{6!}{2!(6-2)!} = \frac{6!}{2!4!} = \frac{6 \cdot 5}{2 \cdot 1} = 15$

n points: $_nC_2$

b. 20 points: $_{20}C_2 = \frac{20!}{2!(20-2)!} = \frac{20!}{2!18!}$

$= \frac{20 \cdot 19}{2 \cdot 1} = 190$

43. Select 12 jurors out of 30 potential jurors, $_{30}C_{12}$. Select 2 alternate jurors out of the remaining 18 potential jurors, $_{18}C_2$. Use the Fundamental Counting Principle to combine the number of ways the jurors can be selected, $(_{30}C_{12})(_{18}C_2)$.