Algebra 2H Notes Section 10-4 May 14, 2014

Inverses of Trigonometric Functions

The trigonometric functions have inverse relations.

Trigonometric Function	Inverse Relation
$\sin\theta = a$	$\sin^{-1}a = \theta$
$\cos\theta = a$	$\cos^{-1}a = \theta$
$\tan\theta = a$	$\tan^{-1}a = \theta$

Read "the inverse sine of *a*."

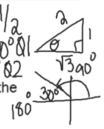
Read "the inverse cosine of *a*."

Read "the inverse tangent of *a*."

How do we evaluate the inverse of a trigonometric function? $\sin^{-1}\frac{1}{2} = \theta$ $\quad \sin\theta = \frac{1}{2}$

$\theta = 30°$ Q1

$= 150°$ Q2



Step 1 Each inverse trigonometric relation has multiple values.
Think, "What reference angle has the given trigonometric value?"
Find the two angles between 0 and 2π radians that have the given value for the trigonometric function.

$\theta = \pi/6,\ 5\pi/6$

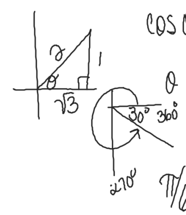
Step 2 Add $(2\pi)n$ to each angle to represent all the coterminal angles, where *n* is an integer and represents the number of revolutions.

$\theta = \pi/6 + (2\pi)n,\ 5\pi/6 + (2\pi)n$

Example 1:

Find all possible values of $\cos^{-1}\frac{\sqrt{3}}{2}$.

$\cos\theta = \frac{\sqrt{3}}{2}$



$\theta = 30° = \pi/6$ Q1

$330° = \frac{11\pi}{6}$ Q4

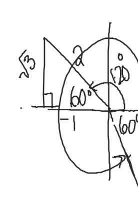
$\pi/6 + (2\pi)n$

$11\pi/6 + (2\pi)n$

Example 2:

Find all possible values of $\tan^{-1}(-\sqrt{3})$.

$\tan\theta = -\sqrt{3}$



$\theta = 120° = 2\pi/3$ Q2

$= 300° = 5\pi/3$ Q4

$2\pi/3 + (2\pi)n$

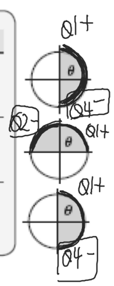
$5\pi/3 + (2\pi)n$

Because more than one value of θ produces the same output value for a given trigonometric function, it is necessary to restrict the domain of each trigonometric function in order to define the inverse trigonometric functions.

Trigonometric functions with restricted domains are indicated with a capital letter. For each value of *a* in the domain of the inverse trigonometric functions, there is only one value of θ.

Inverse Trigonometric Functions

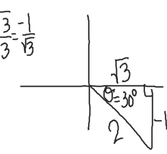
WORDS	SYMBOL	DOMAIN	RANGE
The inverse sine function is $\text{Sin}^{-1}a = \theta$, where $\text{Sin}\,\theta = a$.	$\text{Sin}^{-1}a$	$\{a \mid -1 \le a \le 1\}$	$\left\{\theta \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right\}$ $\{\theta \mid -90° \le \theta \le 90°\}$ Q4, Q1
The inverse cosine function is $\text{Cos}^{-1}a = \theta$, where $\text{Cos}\,\theta = a$.	$\text{Cos}^{-1}a$	$\{a \mid -1 \le a \le 1\}$	$\{\theta \mid 0 \le \theta \le \pi\}$ $\{\theta \mid 0° \le \theta \le 180°\}$ Q1, Q2
The inverse tangent function is $\text{Tan}^{-1}a = \theta$, where $\text{Tan}\,\theta = a$.	$\text{Tan}^{-1}a$	$\{a \mid -\infty < a < \infty\}$	$\left\{\theta \mid -\frac{\pi}{2} < \theta < \frac{\pi}{2}\right\}$ $\{\theta \mid -90° < \theta < 90°\}$ Q1, Q4



How do we evaluate $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ without a calculator? $\text{Tan}\,\theta = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$

Step 1 Decide which quadrant θ lies in. Q4

Draw and label the reference triangle in that quadrant.



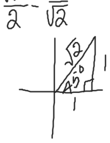
Step 2 Evaluate the function by finding the value of θ for which the trig function has stated value.

$-30° = -\pi/6$

Example 3: **Evaluate. Give angles in both radians and degrees.**

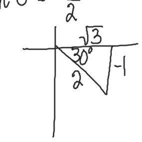
a. $\text{Cos}^{-1}\frac{\sqrt{2}}{2} = 45° = \pi/4$

$\text{Cos}\,\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



b. $\text{Sin}^{-1}\left(-\frac{1}{2}\right) = -30° = -\pi/6$

$\text{Sin}\,\theta = -\frac{1}{2}$



c. $\sec\left(\text{Tan}^{-1}(1)\right)$ $\quad \text{Tan}\,\theta = 1$

$= \sec\ \theta = \sqrt{2}$