3.4c

1. $f(x) = 2\cos(x) - 3\sin(x)$

$f'(x) = 2[-\sin(x)] - 3[\cos(x)]$
$= -2\sin(x) - 3\cos(x)$

2. $f(x) = \sin(x)\cos(x)$ PR

$f'(x) = \sin(x)[-\sin(x)] + \cos(x)[\cos(x)]$
$= -\sin^2(x) + \cos^2(x)$

3. $f(x) = \dfrac{\sin(x)}{x}$ QR

$f'(x) = \dfrac{x \cdot \cos(x) - \sin(x) \cdot 1}{x^2} = \dfrac{x\cos(x) - \sin(x)}{x^2}$

4. $f(x) = x^2\cos(x)$ PR

$f'(x) = x^2[-\sin(x)] + \cos(x)[2x]$
$= -x^2\sin(x) + 2x\cos(x)$
$= x[-x\sin(x) + 2\cos(x)]$

5. $f(x) = \underbrace{x^3\sin(x)}_{PR} - 5\cos(x)$

$f'(x) = \left[x^3 \cdot \cos(x) + \sin(x) \cdot 3x^2\right] - 5[-\sin(x)]$

$= x^3\cos(x) + 3x^2\sin(x) + 5\sin(x)$

6. $f(x) = \dfrac{\cos(x)}{x\sin(x)}$ QR

$$f'(x) = \frac{x\sin(x)\cdot\frac{d}{dx}\cos(x) - \cos(x)\cdot\overbrace{\frac{d}{dx}[x\sin(x)]}^{PR}}{[x\sin(x)]^2}$$

$$= \frac{x\sin(x)[-\sin(x)] - \cos(x)[x\cdot\cos(x) + \sin(x)\cdot 1]}{x^2\sin^2(x)}$$

$$= \frac{-x\sin^2(x) - x\cos^2(x) - \cos(x)\sin(x)}{x^2\sin^2(x)}$$

/ 7. $f(x) = \sec(x) - \sqrt{2}\tan(x)$

$$f'(x) = \sec(x)\tan(x) - \sqrt{2}\sec^2(x)$$

$$= \sec(x)[\tan(x) - \sqrt{2}\sec(x)]$$

8. $f(x) = (x^2+1)\sec(x)$ PR

$$f'(x) = (x^2+1)\cdot\sec(x)\tan(x) + \sec(x)\cdot 2x$$

$$= x^2\sec(x)\tan(x) + \sec(x)\tan(x) + 2x\sec(x)$$

$$= \sec(x)[x^2\tan(x) + \tan(x) + 2x]$$

/ 9. $f(x) = \sec(x)\tan(x)$ PR

$$f'(x) = \sec(x)\cdot\sec^2(x) + \tan(x)\cdot\sec(x)\tan(x)$$

$$= \sec(x)[\sec^2(x) + \tan^2(x)]$$

10. $f(x) = \dfrac{\sec(x)}{1+\tan(x)}$ QR

$$f'(x) = \frac{[1+\tan(x)]\cdot \sec(x)\tan(x) - \sec(x)\cdot[0+\sec^2(x)]}{[1+\tan(x)]^2}$$

$$= \frac{\sec(x)\tan(x) + \sec(x)\tan^2(x) - \sec^3(x)}{[1+\tan(x)]^2}$$

$$= \frac{\sec(x)[\tan(x) + \tan^2(x) - \sec^2(x)]}{[1+\tan(x)]^2}$$

$$= \frac{\sec(x)[\tan(x) - 1]}{[1+\tan(x)]^2}$$

Note: $\tan^2(x)+1=\sec^2(x)$

11. $f(x) = \csc(x)\cot(x)$ PR

$$f'(x) = \csc(x)[-\csc^2(x)] + \cot(x)[-\csc(x)\cot(x)]$$

$$= -\csc^3(x) - \csc(x)\cot^2(x)$$

$$= -\csc(x)[\csc^2(x) + \cot^2(x)]$$

12. $f(x) = x - 4\csc(x) + 2\cot(x)$

$$f'(x) = 1 - 4[-\csc(x)\cot(x)] + 2[-\csc^2(x)]$$

$$= 1 + 4\csc(x)\cot(x) - 2\csc^2(x)$$

/ 13 $f(x) = \dfrac{\cot(x)}{1+\csc(x)}$ QR

$$f'(x) = \frac{[1+\csc(x)]\cdot[-\csc^2(x)] - \cot(x)[0-\csc(x)\cot(x)]}{[1+\csc(x)]^2}$$

$$= \frac{-\csc^2(x) - \csc^3(x) + \csc(x)\cot^2(x)}{[1+\csc(x)]^2}$$

$$= \frac{\csc(x)[-\csc(x) - \csc^2(x) + \cot^2(x)]}{[1+\csc(x)]^2}$$

$$= \frac{\csc(x)[-\csc(x) - 1]}{[1+\csc(x)]^2} = \frac{-\csc(x)}{1+\csc(x)}$$

Note: $1+\cot^2(x) = \csc^2(x)$

14. $f(x) = \dfrac{\csc(x)}{\tan(x)}$ QR

$$f'(x) = \frac{\tan(x)[-\csc(x)\cot(x)] - \csc(x)\cdot\sec^2(x)}{\tan^2(x)}$$

$$= \frac{-\csc(x) - \csc(x)\sec^2(x)}{\tan^2(x)}$$

Note: $\tan(x)\cot(x) = 1$

$$= \frac{-\csc(x)[1+\sec^2(x)]}{\tan^2(x)}$$

✓ 15. $f(x) = \sin^2(x) + \cos^2(x) = 1$

$f'(x) = 0$

16. $f(x) = \dfrac{1}{\cot(x)} = \tan(x)$

$f'(x) = \sec^2(x)$

✓ 19. $y = x\cos(x)$ PR

$y' = x[-\sin(x)] + \cos(x)\cdot 1 = \underbrace{-x\sin(x)}_{PR} + \cos(x)$

$y'' = -[x\cdot\cos(x) + \sin(x)\cdot 1] - \sin(x)$

$= -x\cos(x) - \sin(x) - \sin(x)$

$= -x\cos(x) - 2\sin(x)$

25. $f(x) = \tan(x)$

$f'(x) = \sec^2(x) = \dfrac{1}{\cos^2(x)}$

ⓐ $f(0) = \tan(0) = 0$ pt $(0,0)$

$f'(0) = \dfrac{1}{\cos^2(1)} = \dfrac{1}{1} = 1$ $f'(0) = 1$

$y - 0 = 1(x - 0)$

$y = x$

ⓑ $f(\frac{\pi}{4}) = \tan(\frac{\pi}{4}) = 1$ pt $(\frac{\pi}{4}, 1)$

$f'(\frac{\pi}{4}) = \dfrac{1}{\cos^2(\frac{\pi}{4})} = \dfrac{1}{(\frac{\sqrt{2}}{2})^2} = \dfrac{1}{\frac{1}{2}} = 2$ $f'(\frac{\pi}{4}) = 2$

$y - 1 = 2(x - \frac{\pi}{4})$

1 25 cont'd ⓒ $f(\frac{-\pi}{4}) = \tan(\frac{-\pi}{4}) = -1$ pt $(\frac{-\pi}{4}, -1)$

$f'(\frac{-\pi}{4}) = \frac{1}{\cos^2(\frac{-\pi}{4})} = \frac{1}{(\frac{\sqrt{2}}{2})^2} = \frac{1}{\frac{1}{2}} = 2$ $f'(\frac{-\pi}{4}) = 2$

$y + 1 = 2(x + \frac{\pi}{4})$

1 29. $[-2\pi, 2\pi]$

ⓐ $f(x) = \sin(x)$

$f'(x) = \cos(x)$

need to find where $f'(x) = 0$

$\cos(x) = 0$

$x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}$

ⓑ $f(x) = x + \cos(x)$

$f'(x) = 1 - \sin(x)$

need to find where $f'(x) = 0$

$1 - \sin(x) = 0$

$\sin(x) = 1$

$x = \frac{-3\pi}{2}, \frac{\pi}{2}$

ⓒ $f(x) = \tan(x)$

$f'(x) = \sec^2(x)$

need to find where $f'(x) = 0$

$\sec^2(x) \neq 0$ ∴ no horizontal tangent

ⓓ $f(x) = \sec(x)$

$f'(x) = \sec(x)\tan(x)$

need to find where $f'(x) = 0$

$\frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)} = 0$

$\sin(x) = 0$

$x = \pm 2\pi, \pm \pi, 0$