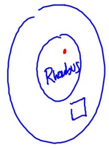
WARMUP

Classify each conditional as true or false.

1. If $j > k$, then $k < j$. True
2. If $a > b$ and $b = c$, then $a > c$. True
3. If $r > t$ and $s > t$, then $r > s$. False
4. If $\angle BCD$ is an exterior angle of $\triangle ABC$, then $m\angle BCD > m\angle A + m\angle B$. False



Use **the conditional: If $\triangle ABC$ is acute, then $m\angle C \neq 90$.**

5. Write the inverse of the statement. Is it true or false?
6. Write the contrapositive of the statement. Is it true or false?

5. If $\triangle ABC$ is not acute, then $m\angle C = 90$. False
6. If $m\angle C = 90$, then $\triangle ABC$ is not acute. True

7. Write the letter paired with the statement that is logically equivalent to "If Dan can't go, then Valerie can go." C
   A. If Valerie can go, then Dan can't go.
   B. If Dan can go, then Valerie can't go.
   C. If Valerie can't go, then Dan can go.
   D. If Dan can go, then Valerie can go.

8. Given: All rhombuses are parallelograms.
   What can you conclude from each additional statement? If no conclusion is possible, write *no conclusion*.
   a. *ABCD* is not a parallelogram. a. *ABCD* is not a rhombus.
   b. *QRST* is not a rhombus. No conclusion
   c. *MNOP* is a parallelogram. No concl.
   d. *GHIJ* is a rhombus. *GHIJ* is a ▱.

SECTION 6.3: INDIRECT PROOFS

Standards:
2.0: Students write geometric proofs, including proofs by contradiction.

DIRECT PROOFS

What we have been doing

if p then q

GIVEN PROVE

if $\boxed{\text{not } q}$ then not p

INDIRECT PROOF

Real-life example of talking to your neighbor:
You say that my dog, Rex, dug a hole in your yard on July 15. Temporarily assume that Rex did dig the hole. Then he would have been in your yard on July 15. But I have bills that show Rex was in the veterinarian's kennel from July 14 to July 17. Rex couldn't have been in two places at once. Therefore, Rex is not the dog that dug the hole in your yard.

HOW TO START AN INDIRECT PROOF

Begin by assuming temporarily that what you want to prove is NOT true.

Assume the opposite of what you want to prove

EXAMPLE 1

Write the first sentence of an indirect proof of each conditional shown.

(a) If $n^2 > 6n$, then $n \neq 4$.
Assume temporarily that n = 4.

(b) If $m(\angle 1) = m(\angle 2)$, then $\overline{XY} \parallel \overline{CD}$.
Assume temporarily that $\overline{XY} \nparallel \overline{CD}$

(c) If $\overline{XY} \parallel \overline{CD}$, then $m(\angle 1) = m(\angle 2)$.
Assume temporarily that $m(\angle 1) \neq m(\angle 2)$

(d) If $\overline{AB} \nparallel \overline{CD}$, then *ABCD* is not a parallelogram.
Assume temporarily that ABCD is a parallelogram.

(e) If *M* is the midpoint of $\overline{AB}$, then $AM = MB$.
Assume temporarily that $AM \neq MB$

HOW TO WRITE AN INDIRECT PROOF

1) Assume temporarily that the conclusion is NOT true. (opp of what you are proving)

2) Reason logically until you reach a contradiction of the given.

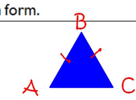
3) Point out that the temporary assumption must be false, and that the conclusion must then be true.

EXAMPLE 2

Write an indirect proof in paragraph form.

(a) Given: $\triangle ABC$; $AB \cong BC$

Prove: $m(\angle A) \neq 90$



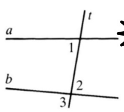
Assume temporarily that $m(\angle A) = 90$.
Since $\overline{AB}$ is congruent to $\overline{BC}$ we know $\angle A$ is congruent to $\angle C$ so $m(\angle C) = 90$. (If 2 sides of a triangle are congruent then the angles opposite those sides are congruent.)
Now $m(\angle A) + m(\angle B) + m(\angle C) = 90 + m(\angle B) + 90 > 180$.
But this contradicts the fact that the sum of the measures of the angles of a triangle is 180.
Therefore, the temporary assumption must be false
So $m(\angle A) \neq 90$

EXAMPLE 2

Write an indirect proof in paragraph form.

(b) Given: Transversal t cuts lines a and b; $m(\angle 1) \neq m(\angle 2)$

Prove: $m(\angle 1) \neq m(\angle 3)$



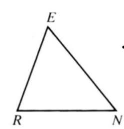
Assume temporarily that $m(\angle 1) = m(\angle 3)$.
Then a//b (if 2 lines are cut by a transversal and corr. angles are congruent then the lines are //.)
Then $m(\angle 1) = m(\angle 2)$. (If 2 // lines are cut by a transversal then alt. int. angles are congruent.)
But this contradicts the fact that $m(\angle 1) \neq m(\angle 2)$
Therefore the temporarty assumption must be false, so $m(\angle 1) \neq m(\angle 3)$

EXAMPLE 2

Write an indirect proof in paragraph form.

(c) Given: Scalene $\triangle REN$

Prove: $\angle R$ is not congruent $\angle N$



Assume temporarily that $\angle R$ is congruent to $\angle N$.
Then $\overline{RE}$ is comgruent to $\overline{EN}$ (if 2 angles of a triangle are congruent then the sides opp those angles are congruent). So triangle REN is isosceles.
But this contradicts the fact that triangle REN is scalene. Therefore the temporary assumption must be false so $\angle R$ is congruent $\angle N$.

EXAMPLE 2

Write an indirect proof in paragraph form.

(d) If $n^2 > 6n$, then $n \neq 4$.

Given: $n^2 > 6n$
Prove: $n \neq 4$

Assume temporarily that $n = 4$.
Then $n^2 = 16$ and $6n = 24$
But this contradicts the given fact that $n^2 > 6n$ since $16 \not> 24$.
Therefore the temporary assumption must be false, so $n \neq 4$.

HOMEWORK

Assignment #6.3

- Page 216-217

  # 1-5, 7-10, 11, 14, 15, & challenge