4.7a

1. (a)

First Number	Second Number	Product
1	22	22
2	21	42
3	20	60
4	19	76
5	18	90
6	17	102
7	16	112
8	15	120
9	14	126
10	13	130
11	12	132

We needn't consider pairs where the first number is larger than the second, since we can just interchange the numbers in such cases. The answer appears to be 11 and 12, but we have considered only integers in the table.

(b) Let the numbers be x any y
Let P be the product we want to maximize

$P = xy$ $\quad\quad x+y=23$
$\quad\quad\quad\quad y=23-x$

$P(x) = x(23-x) = 23x - x^2$ $\quad\quad$ Domain: $x>0$

$P'(x) = 23-2x$

$P'(x)=0$ when $23-2x=0$
$x = \frac{23}{2} = 11.5$

$P'(x)>0$ when $0<x<11.5$
$P'(x)<0$ when $x>11.5$
$\Big\}$ P has an abs max at $P(11.5)=11.5$

the numbers are 11.5 and 11.5

2. Let x and y be the numbers
Let P the product we want to minimize

$P = xy$ $\qquad\qquad x - y = 100$
$\qquad\qquad\qquad\quad x = 100 + y$

$P(y) = (100+y)y = 100y + y^2$ $\qquad$ Domain: $\mathbb{R}$

$P'(y) = 100 + 2y$

$P'(y) = 0$ when $100 + 2y = 0$
$\qquad\qquad\qquad\qquad y = -50$

$\left.\begin{array}{l} P'(y) < 0 \text{ when } y < -50 \\ P'(y) > 0 \text{ when } y > -50 \end{array}\right\}$ P has an abs min at $P(-50) = -2500$

the numbers are -50 and 50

3. let x and y be positive numbers
let N be the sum we are minimizing

$N = x + y \qquad xy = 100$

$y = \frac{100}{x}$

$N(x) = x + \frac{100}{x} = x - 100x^{-1} \qquad \text{Domain: } x > 0$

$N'(x) = 1 - 100x^{-2} = 1 - \frac{100}{x^2} = \frac{x^2 - 100}{x^2}$

$N'(x) = 0$ when $x^2 - 100 = 0$

$x = \pm 10$

but $x > 0$ so $x = 10$

$N'(x)$ is never und since $x > 0$

$$\left.\begin{array}{ll} N'(x) < 0 & \text{when } 0 < x < 10 \\ N'(x) > 0 & \text{when } x > 10 \end{array}\right\} \text{N has an abs min at } N(10) = 20$$

the numbers are 10 and 10

4. Let x be a positive number
Let M be the sum we are minimizing

$M = x + \frac{1}{x}$

$M(x) = x + \frac{1}{x} = x + x^{-1}$ Domain: $x > 0$

$M'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$

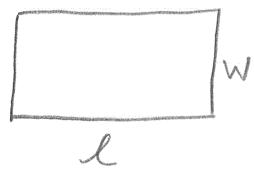
$M'(x) = 0$ when $x^2 - 1 = 0$
$x = \pm 1$
but $x > 0$ so $x = 1$

$M'(x)$ is never und since $x > 0$

$\left.\begin{array}{l} M'(x) < 0 \text{ when } 0 < x < 1 \\ M'(x) > 0 \text{ when } x > 1 \end{array}\right\}$ M has an abs min at $M(1) = 2$

the number is 1

5.



$A \rightarrow$ area

$A = l \cdot w$

$100 = 2w + 2l$

$l = \dfrac{100-2w}{2} = 50 - w$

$A(w) = (50-w)w = 50w - w^2$ Domain: $0 \le w \le 50$

Since A is cont on $[0,50]$ we can use the Closed Interval Method

$A'(w) = 50 - 2w$

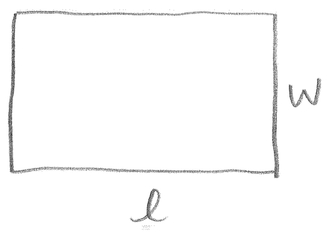
$A'(w) = 0$ when $50 - 2w = 0$

$w = 25$

w	$A(w)$
0	0
25	625 MAX
50	0

Dimensions: 25 m by 25 m

6.



Let P be perimeter

$P = 2l + 2w$

$1000 = l \cdot w$

$l = \frac{1000}{w}$

$P(w) = 2\left(\frac{1000}{w}\right) + 2w = 2000w^{-1} + 2w$ Domain: $w > 0$

$P'(w) = -2000w^{-2} + 2 = \frac{-2000}{w^2} + 2 = \frac{-2000 + 2w^2}{w^2}$

$P'(w) = 0$ when $-2000 + 2w^2 = 0$

$w^2 = 1000$

$w = \pm\sqrt{1000} = 10\sqrt{10}$

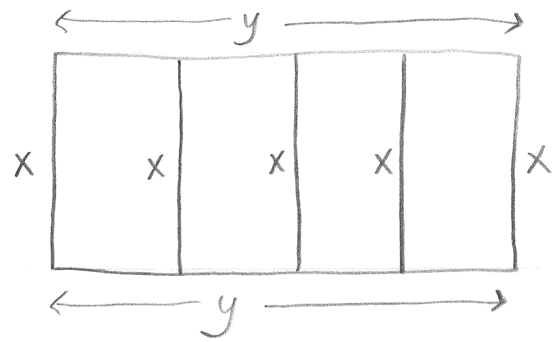
since $w > 0$ $w = 10\sqrt{10}$

$P'(w)$ is never und since $w > 0$

$\left.\begin{array}{l} P'(w) < 0 \text{ when } 0 < w < 10\sqrt{10} \\ P'(w) > 0 \text{ when } w > 10\sqrt{10} \end{array}\right\}$ P has an abs min at $P(10\sqrt{10}) = 40\sqrt{10}$

Dimensions: $10\sqrt{10}$ m and $10\sqrt{10}$ m

9.



Let A be area

$A = xy$

$5x + 2y = 750$

$y = 375 - \frac{5}{2}x$

$A(x) = x(375 - \frac{5}{2}x) = 375x - \frac{5}{2}x^2$ Domain: $0 \le x \le 150$

Since A is cont on $[0, 150]$ we can use the Closed Interval Method

$A'(x) = 375 - 5x$

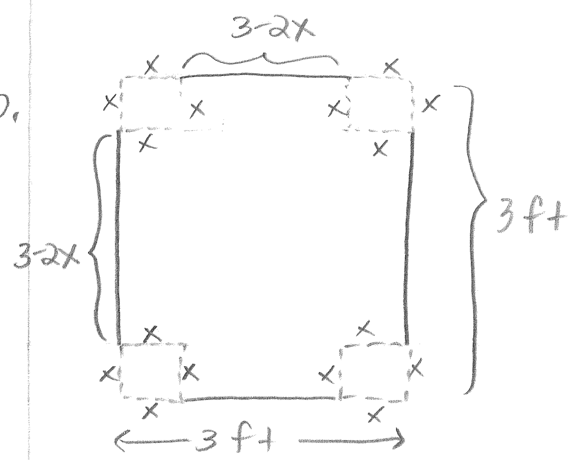
$A'(x) = 0$ when $375 - 5x = 0$

$x = 75$

x	$A(x)$	
0	0	
75	14062.5	MAX
150	0	

The largest possible area is 14062.5 ft^3

10.



Let V be the volume

$V = (3-2x)(3-2x) \cdot x = x(9 - 12x + 4x^2)$

$V(x) = 9x - 12x^2 + 4x^3$ Domain: $0 \leq x \leq 1.5$

Since V is cont on $[0,3]$ we can use the Closed Interval Method

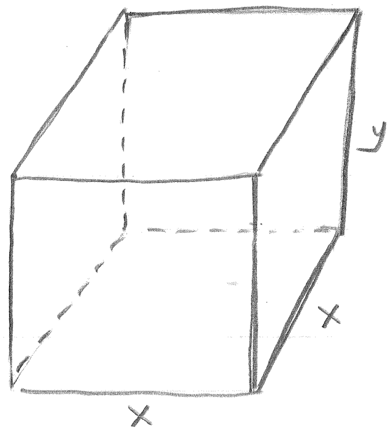
$V'(x) = 9 - 24x + 12x^2 = 3(4x^2 - 8x + 3)$
$= 3(2x - 3)(2x - 1)$

$V'(x) = 0$ when $3(2x-3)(2x-1) = 0$
$x = \frac{3}{2}$ $x = \frac{1}{2}$

x	$V(x)$	
0	0	
$\frac{1}{2}$	2	MAX
$\frac{3}{2}$	0	

the largest volume is $2\ ft^3$

12.



A : amt of material

$A = x^2 + 4xy$ $\qquad x^2 y = 32000$

$y = 32000x^{-2}$

$A(x) = x^2 + 4x(32000x^{-2}) = x^2 + \frac{128000}{x}$ $\qquad$ Domain: $x > 0$

$A'(x) = 2x - 128000x^{-2} = 2x - \frac{128000}{x^2} = \frac{2x^3 - 128000}{x^2}$

$A'(x) = 0$ when $2x^3 - 128000 = 0$

$x = 40$

$A'(x)$ is never und since $x > 0$

$\left.\begin{array}{ll} A'(x) < 0 & \text{when } 0 < x < 40 \\ A'(x) > 0 & \text{when } x > 40 \end{array}\right\}$ A has abs min at $A(40) = 4800$

Dimensions: base 40 cm by 40 cm
height 20 cm