

## BINOMIAL PROBABILITIES AND DISTRIBUTIONS

Section 12.6

In a binomial experiment, there are  $n$  independent trials with each trial having only two possible outcomes: success and failure. The probability of success is the same for each trial.

### Finding a Binomial Probability

For a binomial experiment consisting of  $n$  trials, the probability of exactly  $k$  successes is:

$$P(k \text{ successes}) = {}_n C_k p^k (1-p)^{n-k}, \text{ where } p = \text{probability of success on each trial} \\ \text{and } 1-p = \text{probability of failure.}$$

**Example:** A scientist claims that 40% of mice used in an experiment will become very aggressive after having been administered an experimental drug. Suppose you randomly select 8 mice.

a) What is the probability that exactly 5 of them show aggressive behavior?

$$n=8$$

$$k=5$$

$$p=.4$$

$${}_8 C_5 (.4)^5 (.6)^3 = 56 (.4)^5 (.6)^3 \approx .124$$

$$\frac{8 \cdot 7 \cdot 6}{6} \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

b) What is the probability that at least 7 of them show aggressive behavior?

$$\frac{8!}{1!7!1} \left( \binom{8}{7} (.4)^7 (.6)^1 \right) + \binom{8}{8} (.4)^8 \approx .009$$

or

$$8 (.4)^7 (.6)^1 + 1 (.4)^8$$

A **binomial distribution** shows the probabilities of all possible numbers of successes for an experiment. A **histogram** can be used to show the binomial distribution.

c) Make a histogram for the binomial distribution for this experiment. What is the most likely number of mice to become aggressive?

$$\begin{array}{l}
 1 \quad {}_8C_0 (.4)^0 (.6)^8 \approx .017 \checkmark \\
 8 \quad {}_8C_1 (.4)^1 (.6)^7 \approx .090 \checkmark \\
 28 \quad {}_8C_2 (.4)^2 (.6)^6 \approx .209 \checkmark \\
 56 \quad {}_8C_3 (.4)^3 (.6)^5 \approx .279 \\
 70 \quad {}_8C_4 (.4)^4 (.6)^4 \approx .232 \\
 56 \quad {}_8C_5 (.4)^5 (.6)^3 \approx .124 \checkmark \\
 28 \quad {}_8C_6 (.4)^6 (.6)^2 \approx .041 \checkmark \\
 8 \quad {}_8C_7 (.4)^7 (.6)^1 \approx .008 \checkmark \\
 1 \quad {}_8C_8 (.4)^8 (.6)^0 \approx .001 \checkmark
 \end{array}$$

