P56

21 $P(x) = 3x^3 - 7x^2 + 2x + 3$

$P(\frac{1}{3}) =$

	3	−7	2	3
$\frac{1}{3}$		1	−2	0
	3	−6	0	(3)

24. $f(2i) = 0$ $\quad f(x) = x^4 + x^2 + a$

$f(2i) = (2i)^4 + (2i)^2 + a$

$0 = 16i^4 + 4i^2 + a$

$0 = 16 + 4(-1) + a$

$0 = 12 + a \rightarrow a = -12$

P61 (12) $x^{20} - 4x^{18} + 3x - 6 = f(x)$

(*) $f(2) = (2)^{20} - 4(2)^{18} + 3(2) - 6 = 0$

so $x-2$ is a factor

(*) $f(-2) = (-2)^{20} - 4(-2)^{18} + 3(-2) - 6 = -12$

so $x+2$ is not a factor

20

	$6x^{(3)}$	11	−4	−4
−2		−12	2	4
	$6x^{(2)}$	−1x	−2	0

$3x \quad -2$

$2x \quad +1$

$(3x-2)(2x+1) = 0$

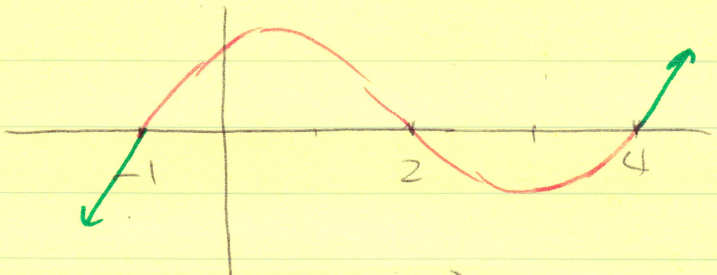
$\frac{2}{3} \quad -\frac{1}{2}$

1. $y = (x+1)(x-2)(x-4)$

Roots: $(x+1) \to -1$
$(x-2) \to 2$
$(x-4) \to 4$
all single roots

Highest exp: x^3 : (3) ; coef +

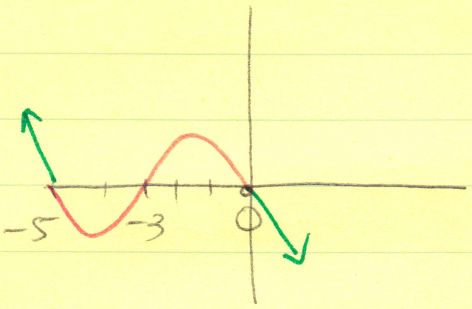
odd pos ⟶



3. $y = -x(x+5)(x+3)$

Roots: $x \to 0$ single root
$(x+5) \to -5$ single root
$(x+3) \to -3$ single root

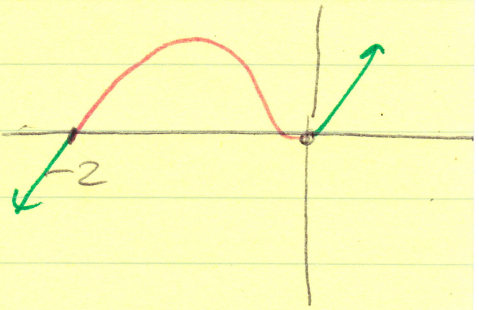
Highest exp: $-x^{(3)}$: odd ; coef : neg



5. $y = x^2(x+2)$

Roots: $x^2 \to 0$ dbl root ∪∩
$x+2 \to -2$ single root

Highest exp: $x^3 \to$ odd : coef +



7. $y = x(1-x)(1+x)(2+x)$

$= x(-x+1)(x+1)(x+2)$

Roots: $x \to 0$

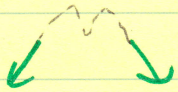
$-x+1 \to 1$

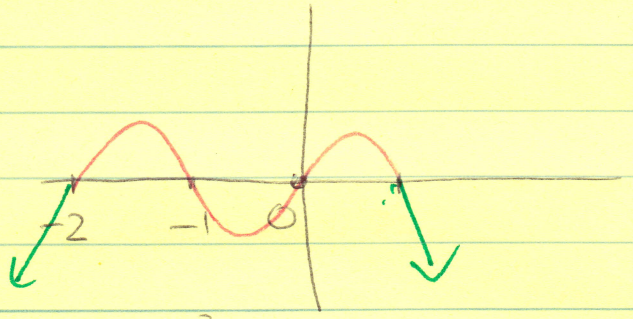
$x+1 \to -1$

$x+2 \to -2$

} single roots

Highest exp: $-x^4$: even ; neg exp



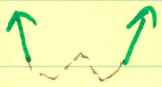


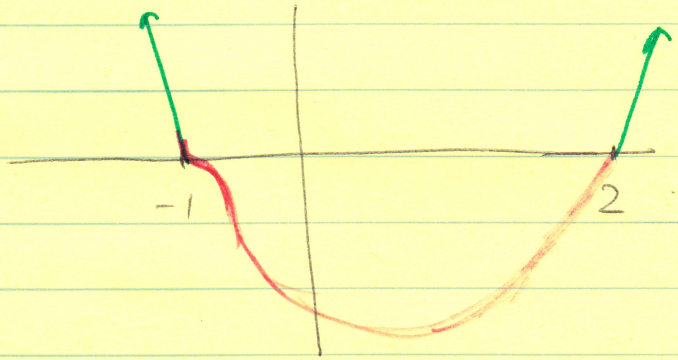
9. $y = (x+1)^3(x-2)$

Roots: $(x+1)^3 \to -1$; triple root

$x-2 \to 2$; single root

Highest exp: x^4 : even ; coef pos





11. $y = x^2(x+2)(x-1)(x+1)$

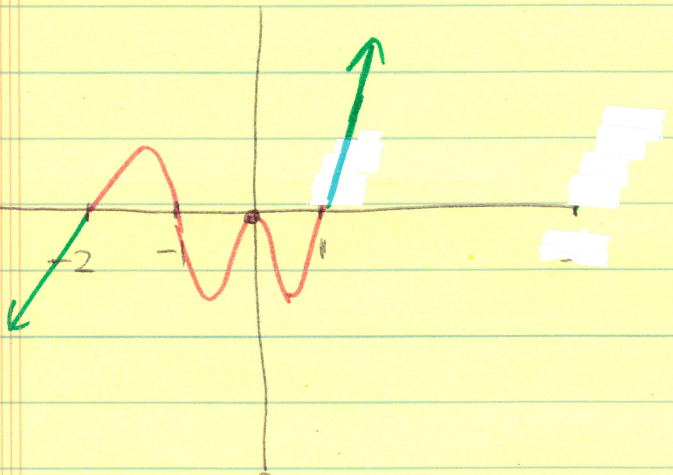
Zeros: $x^2 \to 0$: double root ∪ or ∩

$(x+2) \to -2$: single root

$(x-1) \to 1$: single root

$(x+1) \to -1$ single root

Highest exp: 5 odd ; coef: +.



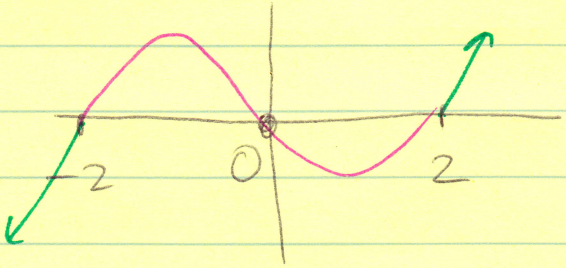
13. $f(x) = x^3 - 4x = x(x^2-4) = x(x+2)(x-2)$

Roots: $x \to 0$

$x+2 \to -2$ } all single roots

$x-2 \to 2$

Highest exp x^3: Odd & pos coef



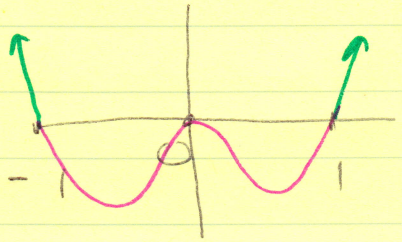
15 $f(x) = x^4 - x^2 = x^2(x^2-1) = x^2(x+1)(x-1)$

Roots: $x^2 \to 0$ dbl root ∪ or ∩

$x+1 \to -1$ single root

$x-1 \to 1$ single root

x^4: even ; pos coef ↑ ↑



17. $f(x) = x^4 - 2x^3 + 2x - 1$

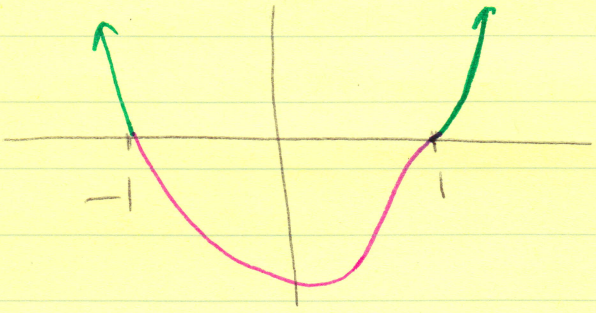
1 is a triple root : ~

-1 is a single root

exp: even

pos coeff

↑ ↑

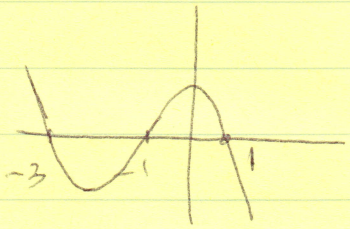


	$1x^4$	-2	0	2	-1
1		1	-1	-1	1
	$1x^3$	-1	-1	1	0
1		1	0	-1	
	$1x^2$	0	-1	0	

$x^2 - 1 \to (x+1)(x-1)$

so the 4th root is -1

21.



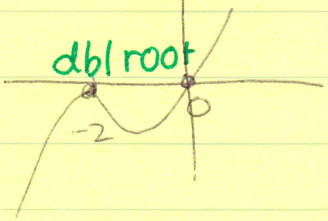
Roots: -3 ; -1 ; 1

$(x+3)(x+1)(x-1)$

↑ ↓ → neg leading coef.

$y = -(x+3)(x+1)(x-1)$

22

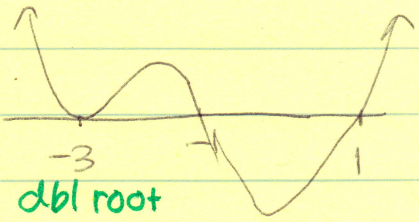


Roots: -2 dbl root ; 0

$(x+2)^2 \quad x$

↙ ↑ → pos leading coef

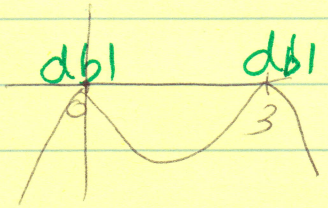
$y = x(x+2)^2$

23. 

dbl
ROOTS: -3 ; -1 1
$(x+3)^2$ $(x+1)$ $(x-1)$

↑ ↑ pos leading coef

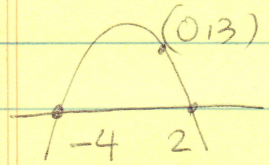
$y = (x+3)^2(x+1)(x-1)$

24 

ROOTS: dbl root 0 dbl root 3
x^2 $(x-3)^2$

↙ ↘ neg leading coef

$y = \ominus x^2(x-3)^2$

29 

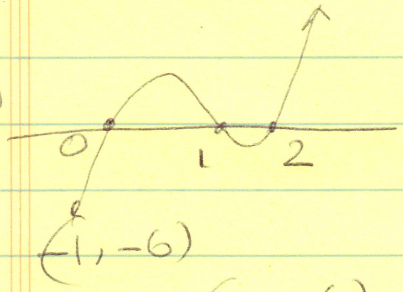
ROOTS: -4 ; 2
$(x+4)$ $(x-2)$

$y = a(x+4)(x-2)$

$(0,3) \rightarrow 3 = a(0+4)(0-2)$

$3 = -8a \rightarrow a = -3/8$

$y = -\frac{3}{8}(x+4)(x-2)$

30 

ROOTS: 0 1 2
x $(x-1)$ $(x-2)$

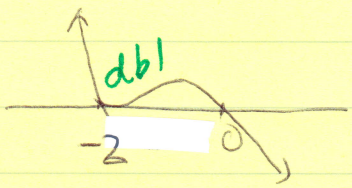
$y = a(x)(x-1)(x-2)$

$(-1,-6) \rightarrow -6 = a(-1)(-1-1)(-1-2)$

$-6 = a(-6) \rightarrow a = 1$

$y = x(x-1)(x-2)$

31



Roots: -2 (dbl root) $\quad 0$

$(x+2)^2 \quad x$

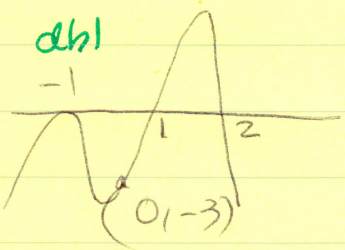
$y = a(x)(x+2)^2$

$(1,-3) \rightarrow -3 = a(1)(1+2)^2$

$-3 = 9a \rightarrow a = -\frac{1}{3}$

$y = -\frac{1}{3}x(x+2)^2$

32



Roots: -1 (dbl) $\quad 1 \quad 2$

$(x+1)^2 \quad (x-1) \quad (x-2)$

$y = a(x+1)^2(x-1)(x-2)$

$(0,-3) \rightarrow -3 = a(0+1)^2(0-1)(0-2)$

$-3 = 2a \rightarrow a = -3/2$

$y = -\frac{3}{2}(x+1)^2(x-1)(x-2)$