Exercises for Section 2.3

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–6, use the Product Rule to differentiate the function.

1. $g(x) = (x^2 + 1)(x^2 - 2x)$
2. $f(x) = (6x + 5)(x^3 - 2)$
3. $h(t) = \sqrt[3]{t}(t^2 + 4)$
4. $g(s) = \sqrt{s}(4 - s^2)$
5. $f(x) = x^3 \cos x$
6. $g(x) = \sqrt{x} \sin x$

In Exercises 7–12, use the Quotient Rule to differentiate the function.

7. $f(x) = \dfrac{x}{x^2 + 1}$
8. $g(t) = \dfrac{t^2 + 2}{2t - 7}$
9. $h(x) = \dfrac{\sqrt[3]{x}}{x^3 + 1}$
10. $h(s) = \dfrac{s}{\sqrt{s} - 1}$
11. $g(x) = \dfrac{\sin x}{x^2}$
12. $f(t) = \dfrac{\cos t}{t^3}$

In Exercises 13–18, find $f'(x)$ and $f'(c)$.

Function	*Value of c*
13. $f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$	$c = 0$
14. $f(x) = (x^2 - 2x + 1)(x^3 - 1)$	$c = 1$
15. $f(x) = \dfrac{x^2 - 4}{x - 3}$	$c = 1$
16. $f(x) = \dfrac{x + 1}{x - 1}$	$c = 2$
17. $f(x) = x \cos x$	$c = \dfrac{\pi}{4}$
18. $f(x) = \dfrac{\sin x}{x}$	$c = \dfrac{\pi}{6}$

In Exercises 19–24, complete the table without using the Quotient Rule.

Function	*Rewrite*	*Differentiate*	*Simplify*
19. $y = \dfrac{x^2 + 2x}{3}$			
20. $y = \dfrac{5x^2 - 3}{4}$			
21. $y = \dfrac{7}{3x^3}$			
22. $y = \dfrac{4}{5x^2}$			
23. $y = \dfrac{4x^{3/2}}{x}$			
24. $y = \dfrac{3x^2 - 5}{7}$			

In Exercises 25–38, find the derivative of the algebraic function.

25. $f(x) = \dfrac{3 - 2x - x^2}{x^2 - 1}$
26. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$
27. $f(x) = x\left(1 - \dfrac{4}{x + 3}\right)$
28. $f(x) = x^4\left(1 - \dfrac{2}{x + 1}\right)$
29. $f(x) = \dfrac{2x + 5}{\sqrt{x}}$
30. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$
31. $h(s) = (s^3 - 2)^2$
32. $h(x) = (x^2 - 1)^2$
33. $f(x) = \dfrac{2 - \frac{1}{x}}{x - 3}$
34. $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x + 1}\right)$
35. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$
36. $f(x) = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$
37. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$, c is a constant
38. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$, c is a constant

In Exercises 39–54, find the derivative of the trigonometric function.

39. $f(t) = t^2 \sin t$
40. $f(\theta) = (\theta + 1) \cos \theta$
41. $f(t) = \dfrac{\cos t}{t}$
42. $f(x) = \dfrac{\sin x}{x}$
43. $f(x) = -x + \tan x$
44. $y = x + \cot x$
45. $g(t) = \sqrt[4]{t} + 8 \sec t$
46. $h(s) = \dfrac{1}{s} - 10 \csc s$
47. $y = \dfrac{3(1 - \sin x)}{2 \cos x}$
48. $y = \dfrac{\sec x}{x}$
49. $y = -\csc x - \sin x$
50. $y = x \sin x + \cos x$
51. $f(x) = x^2 \tan x$
52. $f(x) = \sin x \cos x$
53. $y = 2x \sin x + x^2 \cos x$
54. $h(\theta) = 5\theta \sec \theta + \theta \tan \theta$

In Exercises 55–58, use a computer algebra system to differentiate the function.

55. $g(x) = \left(\dfrac{x + 1}{x + 2}\right)(2x - 5)$
56. $f(x) = \left(\dfrac{x^2 - x - 3}{x^2 + 1}\right)(x^2 + x + 1)$
57. $g(\theta) = \dfrac{\theta}{1 - \sin \theta}$
58. $f(\theta) = \dfrac{\sin \theta}{1 - \cos \theta}$

In Exercises 59–62, evaluate the derivative of the function at the given point. Use a graphing utility to verify your result.

Function	*Point*
59. $y = \dfrac{1 + \csc x}{1 - \csc x}$	$\left(\dfrac{\pi}{6}, -3\right)$
60. $f(x) = \tan x \cot x$	$(1, 1)$
61. $h(t) = \dfrac{\sec t}{t}$	$\left(\pi, -\dfrac{1}{\pi}\right)$
62. $f(x) = \sin x(\sin x + \cos x)$	$\left(\dfrac{\pi}{4}, 1\right)$

In Exercises 63–68, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.

63. $f(x) = (x^3 - 3x + 1)(x + 2)$, $(1, -3)$

64. $f(x) = (x - 1)(x^2 - 2)$, $(0, 2)$

65. $f(x) = \dfrac{x}{x - 1}$, $(2, 2)$

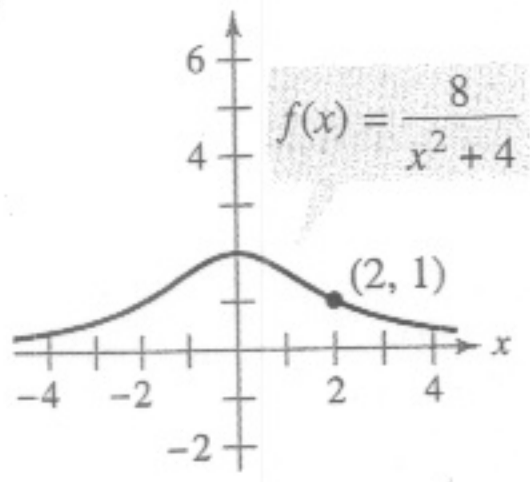
66. $f(x) = \dfrac{(x - 1)}{(x + 1)}$, $\left(2, \dfrac{1}{3}\right)$

67. $f(x) = \tan x$, $\left(\dfrac{\pi}{4}, 1\right)$

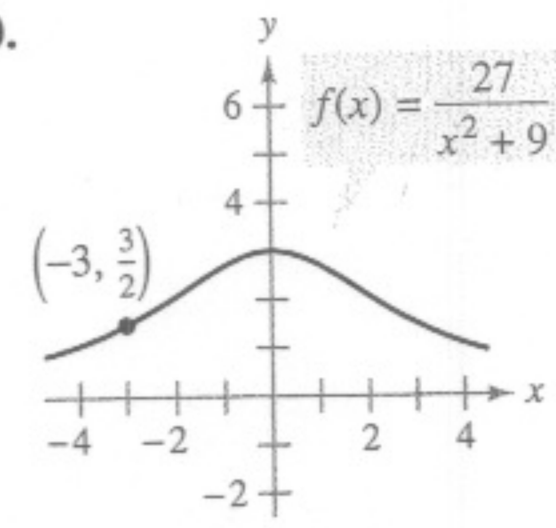
68. $f(x) = \sec x$, $\left(\dfrac{\pi}{3}, 2\right)$

Famous Curves **In Exercises 69–72, find an equation of the tangent line to the graph at the given point. (The graphs in Exercises 69 and 70 are called *witches of Agnesi*. The graphs in Exercises 71 and 72 are called *serpentines*.)**

69.



70.



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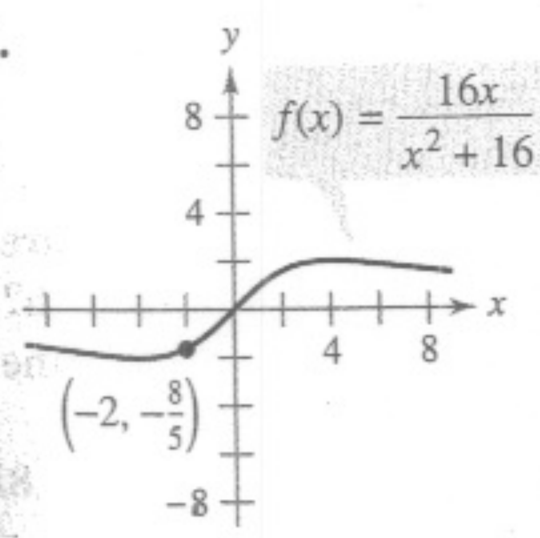
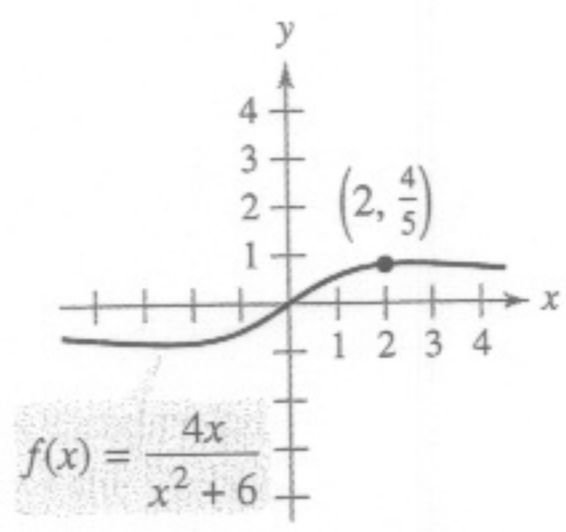


72.



In Exercises 73–76, determine the point(s) at which the graph of the function has a horizontal tangent line.

73. $f(x) = \dfrac{x^2}{x - 1}$

74. $f(x) = \dfrac{x^2}{x^2 + 1}$

75. $f(x) = \dfrac{4x - 2}{x^2}$

76. $f(x) = \dfrac{x - 4}{x^2 - 7}$

77. ***Tangent Lines*** Find equations of the tangent lines to the graph of $f(x) = \dfrac{x + 1}{x - 1}$ that are parallel to the line $2y + x = 6$. Then graph the function and the tangent lines.

78. ***Tangent Lines*** Find equations of the tangent lines to the graph of $f(x) = \dfrac{x}{x - 1}$ that pass through the point $(-1, 5)$. Then graph the function and the tangent lines.

In Exercises 79 and 80, verify that $f'(x) = g'(x)$, and explain the relationship between f and g.

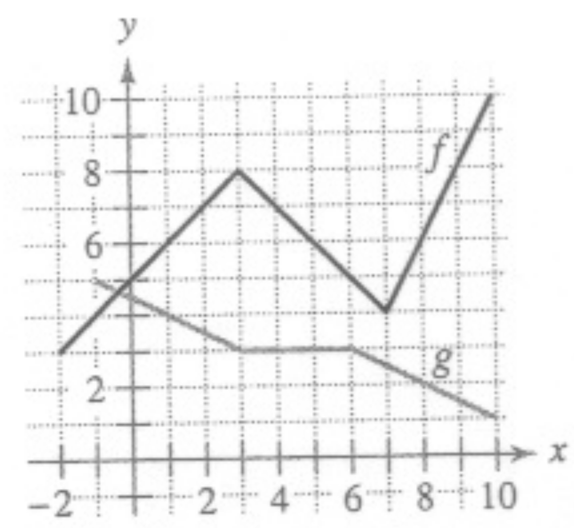
79. $f(x) = \dfrac{3x}{x + 2}$, $g(x) = \dfrac{5x + 4}{x + 2}$

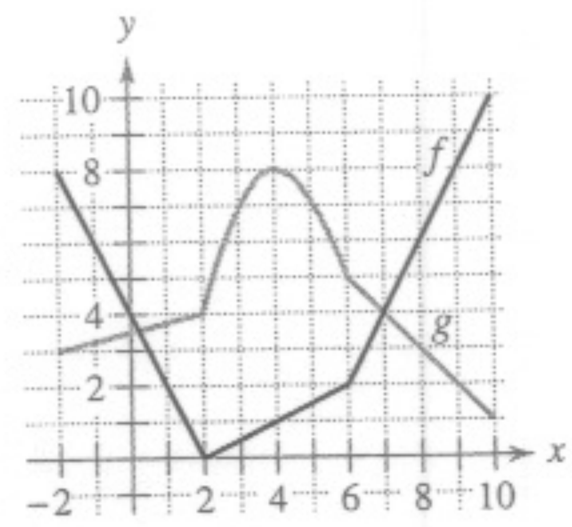
80. $f(x) = \dfrac{\sin x - 3x}{x}$, $g(x) = \dfrac{\sin x + 2x}{x}$

In Exercises 81 and 82, use the graphs of f and g. Let $p(x) = f(x)g(x)$ and $q(x) = \dfrac{f(x)}{g(x)}$.

81. (a) Find $p'(1)$.
(b) Find $q'(4)$.

82. (a) Find $p'(4)$.
(b) Find $q'(7)$.





83. ***Area*** The length of a rectangle is given by $2t + 1$ and its height is $\sqrt{t}$, where t is time in seconds and the dimensions are in centimeters. Find the rate of change of the area with respect to time.

84. ***Volume*** The radius of a right circular cylinder is given by $\sqrt{t + 2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time.

85. ***Inventory Replenishment*** The ordering and transportation cost C for the components used in manufacturing a product is

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \geq 1$$

where C is measured in thousands of dollars and x is the order size in hundreds. Find the rate of change of C with respect to x when (a) $x = 10$, (b) $x = 15$, and (c) $x = 20$. What do these rates of change imply about increasing order size?

86. ***Boyle's Law*** This law states that if the temperature of a gas remains constant, its pressure is inversely proportional to its volume. Use the derivative to show that the rate of change of the pressure is inversely proportional to the square of the volume.

87. ***Population Growth*** A population of 500 bacteria is introduced into a culture and grows in number according to the equation

$$P(t) = 500\left(1 + \frac{4t}{50 + t^2}\right)$$

where t is measured in hours. Find the rate at which the population is growing when $t = 2$.

27. $1 - 12/(x+3)^2 = (x^2 + 6x - 3)/(x+3)^2$

29. $\left[2\sqrt{x} - (2x+5)\dfrac{1}{2\sqrt{x}}\right]/x = (2x-5)/2x^{3/2}$

31. $6s^2(s^3 - 2)$ 33. $-(2x^2 - 2x + 3)/[x^2(x-3)^2]$

35. $(3x^3 + 4x)[(x-5)\cdot 1 + (x+1)\cdot 1]$
$+ [(x-5)(x+1)](9x^2 + 4)$
$= 15x^4 - 48x^3 - 33x^2 - 32x - 20$

37. $\dfrac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2} = -\dfrac{4xc^2}{(x^2 - c^2)^2}$

39. $t(t\cos t + 2\sin t)$ 41. $-(t\sin t + \cos t)/t^2$

43. $-1 + \sec^2 x = \tan^2 x$ 45. $\dfrac{1}{4t^{3/4}} + 8\sec t\tan t$

47. $\dfrac{-6\cos^2 x + 6\sin x - 6\sin^2 x}{4\cos^2 x} = \dfrac{3}{2}(-1 + \tan x\sec x - \tan^2 x)$
$= \frac{3}{2}\sec x(\tan x - \sec x)$

49. $\csc x\cot x - \cos x = \cos x\cot^2 x$ 51. $x(x\sec^2 x + 2\tan x)$

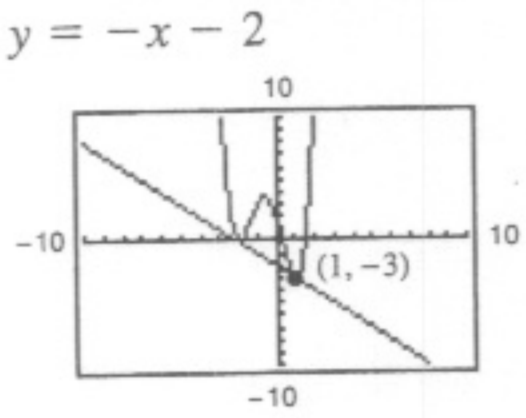
53. $2x\cos x + 2\sin x - x^2\sin x + 2x\cos x$
$= 4x\cos x + (2 - x^2)\sin x$

55. $\left(\dfrac{x+1}{x+2}\right)(2) + (2x-5)\left[\dfrac{(x+2)(1) - (x+1)(1)}{(x+2)^2}\right]$
$= \dfrac{2x^2 + 8x - 1}{(x+2)^2}$

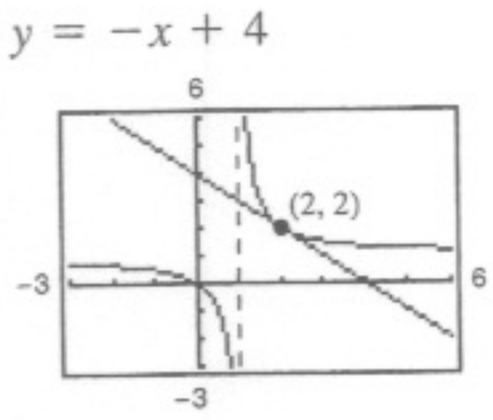
57. $\dfrac{1 - \sin\theta + \theta\cos\theta}{(1 - \sin\theta)^2}$ 59. $y' = \dfrac{-2\csc x\cot x}{(1 - \csc x)^2}$, $-4\sqrt{3}$

61. $h'(t) = \sec t(t\tan t - 1)/t^2$, $1/\pi^2$

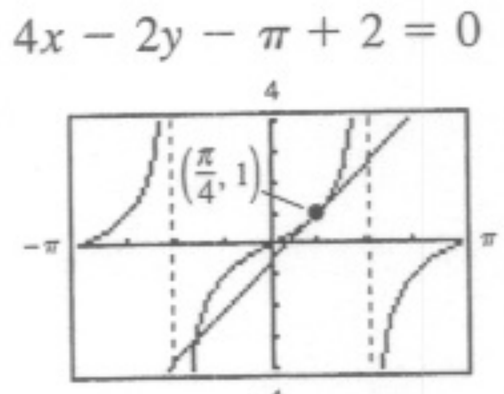
63. (a) $y = -x - 2$
(b)



65. (a) $y = -x + 4$
(b)



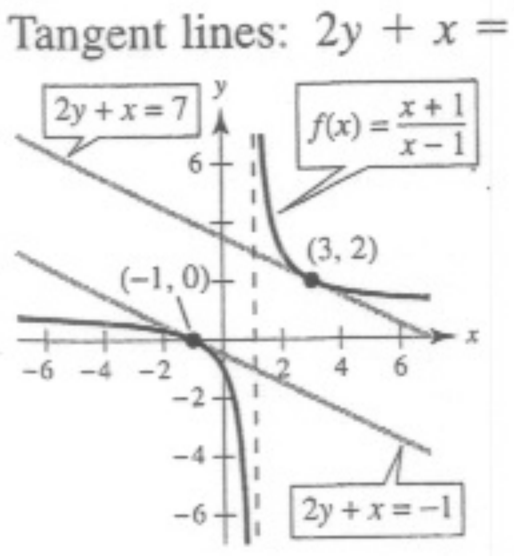
67. (a) $4x - 2y - \pi + 2 = 0$
(b)



69. $2y + x - 4 = 0$

71. $25y - 12x + 16 = 0$ 73. $(0, 0), (2, 4)$ 75. $(1, 2)$

77. Tangent lines: $2y + x = 7$; $2y + x = -1$



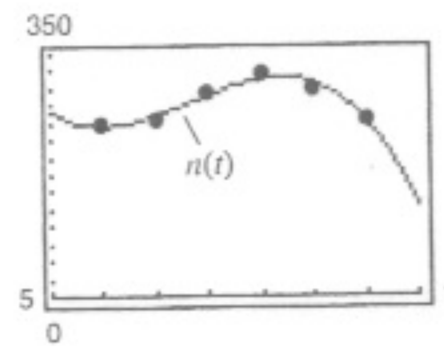
79. $f(x) + 2 = g(x)$ 81. (a) $p'(1) = 1$ (b) $q'(4) = -1/3$

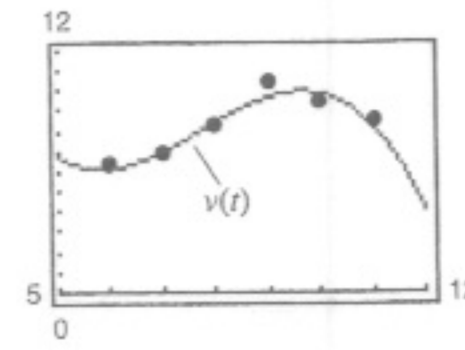
83. $(6t + 1)/(2\sqrt{t})$ cm²/sec

85. (a) −$38.13 (b) −$10.37 (c) −$3.80
The costs decrease with increasing order size.

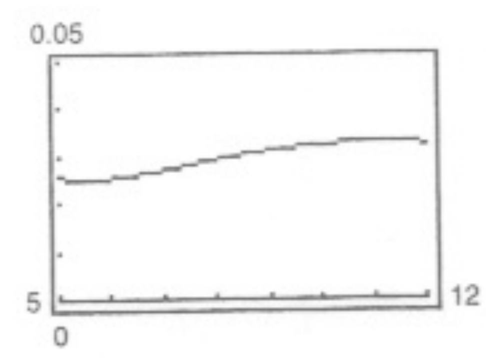
87. 31.55 bacteria/hr 89. Proof

91. (a) $n(t) = -3.5806t^3 + 82.577t^2 - 603.60t + 1667.5$
$v(t) = -0.1361t^3 + 3.165t^2 - 23.02t + 59.8$
(b)





(c) $A = \dfrac{-0.1361t^3 + 3.165t^2 - 23.02t + 59.8}{-3.5806t^3 + 82.577t^2 - 603.60t + 1667.5}$



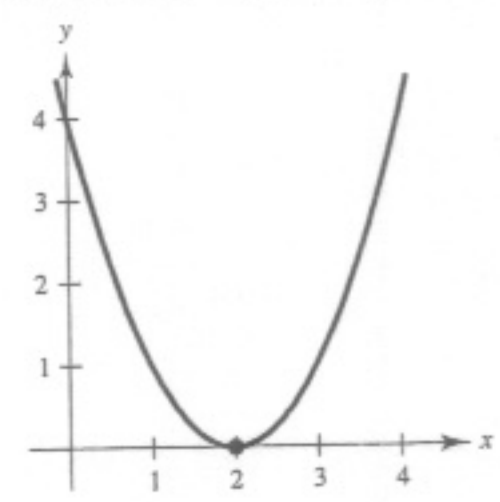
A represents the average retail value (in millions of dollars) per 1000 motor homes.

(d) $A'(t)$ represents the rate of change of the average retail value per 1000 motor homes for the given year.

93. $3/\sqrt{x}$ 95. $2/(x-1)^3$ 97. $-3\sin x$

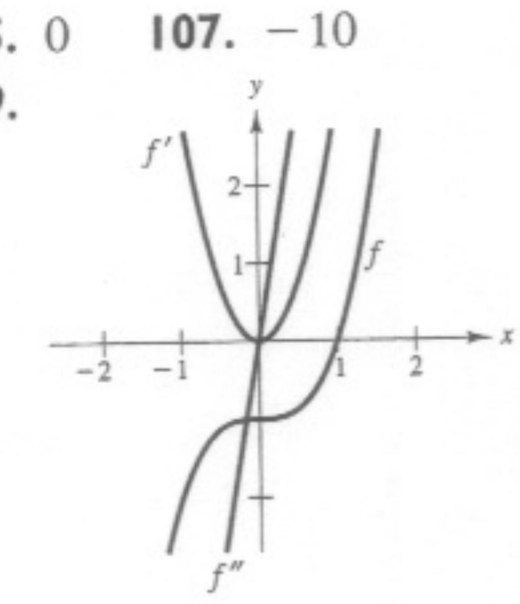
99. $2x$ 101. $1/\sqrt{x}$

103. Answers will vary. For example: $(x-2)^2$

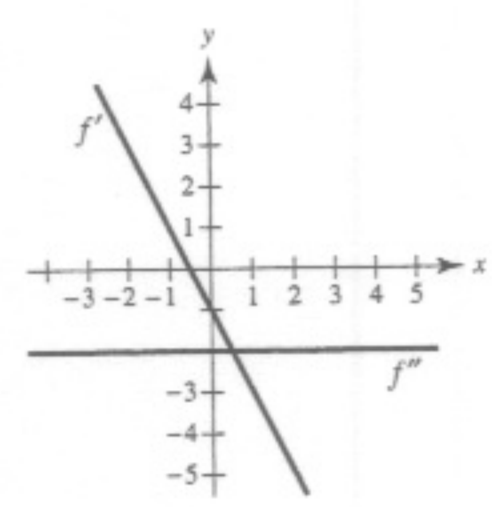


105. 0 107. −10

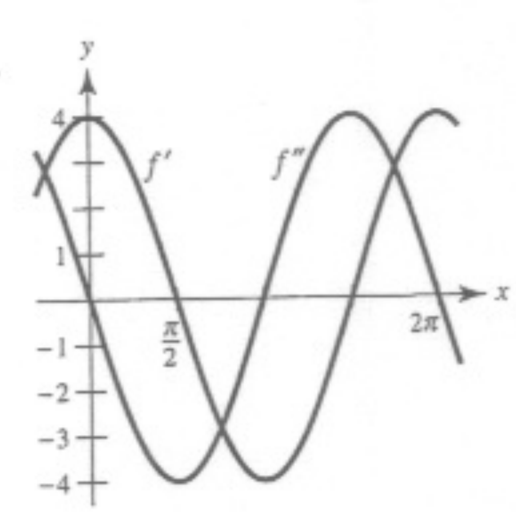
109.



111.



113.



115. $v(3) = 27$ m/sec
$a(3) = -6$ m/sec²
The speed of the object is decreasing, but the rate of that decrease is increasing.

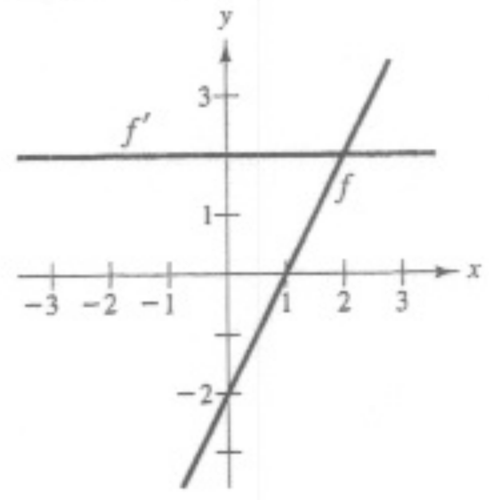
117.

t	0	1	2	3	4
$s(t)$	0	57.75	99	123.75	132
$v(t)$	66	49.5	33	16.5	0
$a(t)$	−16.5	−16.5	−16.5	−16.5	−16.5

The average velocity on [0, 1] is 57.75, on [1, 2] is 41.25, on [2, 3] is 24.75, and on [3, 4] is 8.25.

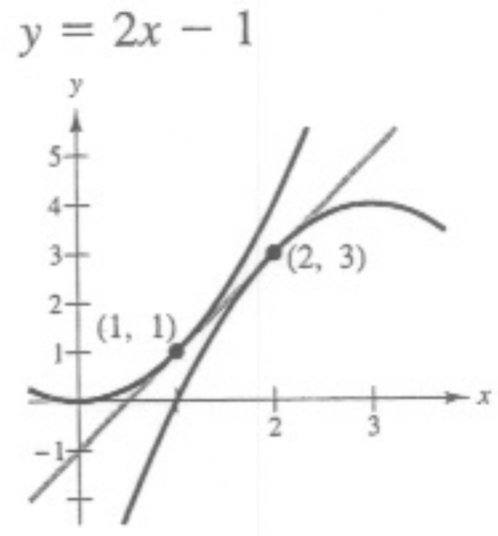
69. $g'(x) = f'(x)$

71.

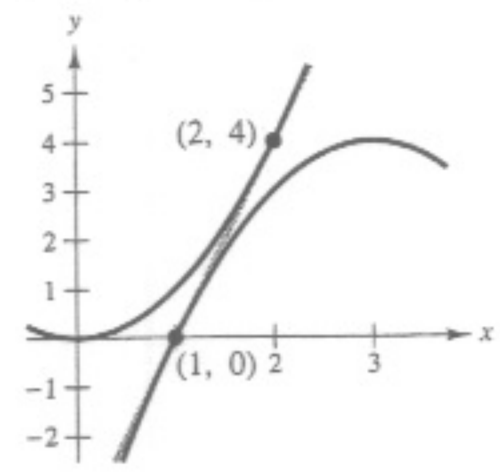


The rate of change of f is constant and therefore f' is a constant function.

73. $y = 2x - 1$

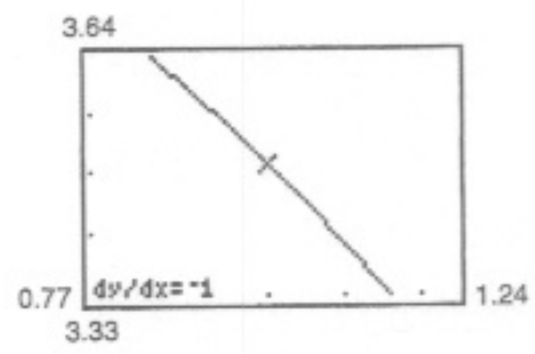


$y = 4x - 4$



75. $f'(x) = 3 + \cos x \neq 0$ for all x. **77.** $x - 4y + 4 = 0$

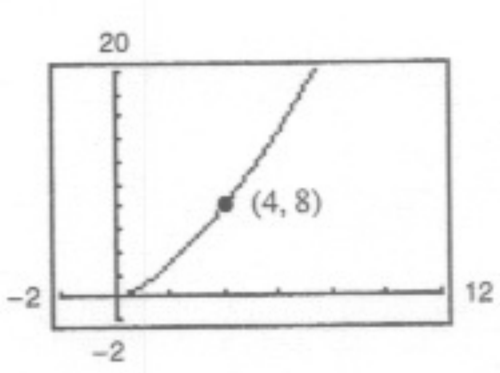
79.



$f'(1)$ appears to be close to -1.

$f'(1) = -1$

81. (a)

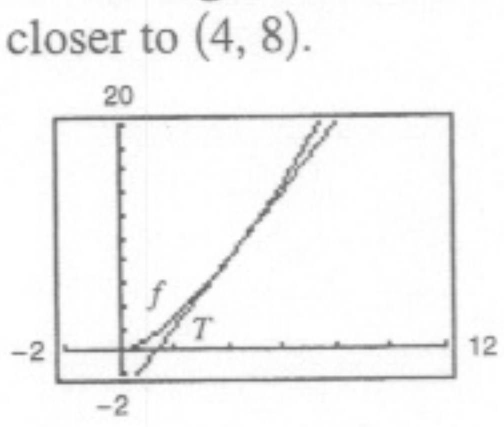


$(3.9, 7.7019)$, $S(x) = 2.981x - 3.924$

(b) $T(x) = 3(x - 4) + 8 = 3x - 4$

The slope (and equation) of the secant line approaches that of the tangent line at $(4, 8)$ as you choose points closer and closer to $(4, 8)$.

(c)



The approximation becomes less accurate.

(d)

Δx	-3	-2	-1	-0.5	-0.1	0
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8
$T(4 + \Delta x)$	-1	2	5	6.5	7.7	8

Δx	0.1	0.5	1	2	3
$f(4 + \Delta x)$	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	8.3	9.5	11	14	17

83. False. Let $f(x) = x$ and $g(x) = x + 1$.

85. False. $dy/dx = 0$ **87.** True

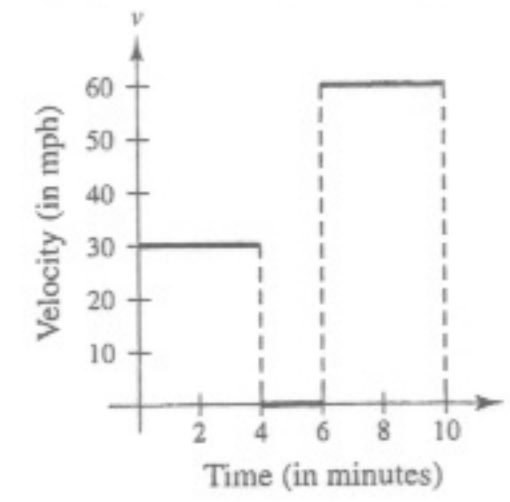
89. Average rate: 2
Instantaneous rates:
$f'(1) = 2$; $f'(2) = 2$

91. Average rate: $\frac{1}{2}$
Instantaneous rates:
$f'(1) = 1$; $f'(2) = \frac{1}{4}$

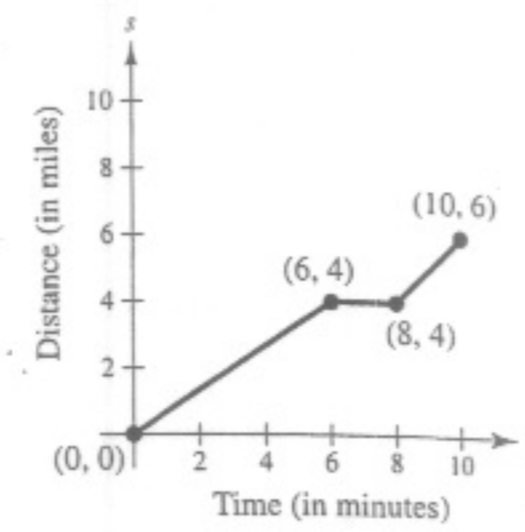
93. (a) $s(t) = -16t^2 + 1362$; $v(t) = -32t$ (b) -48 ft/sec
(c) $s'(1) = -32$ ft/sec; $s'(2) = -64$ ft/sec
(d) $t = \frac{\sqrt{1362}}{4} \approx 9.226$ sec (e) -295.242 ft/sec

95. $v(5) = 71$ m/sec; $v(10) = 22$ m/sec

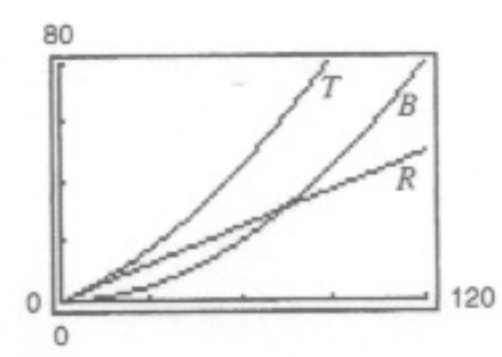
97.



99.



101. (a) $R(v) = 0.417v - 0.02$
(b) $B(v) = 0.0056v^2 + 0.001v + 0.04$
(c) $T(v) = 0.0056v^2 + 0.418v + 0.02$
(d)



(e) $T'(v) = 0.0112v + 0.418$
$T'(40) = 0.866$
$T'(80) = 1.314$
$T'(100) = 1.538$

(f) Stopping distance increases at an increasing rate.

103. $V'(4) = 48$ cm^2 **105.** Proof

107. (a) The rate of change of the number of gallons of gasoline sold when the price is \$1.479
(b) In general, the rate of change when $p = 1.479$ should be negative. As prices go up, sales go down.

109. $y = 2x^2 - 3x + 1$ **111.** $y = -9x$, $y = -\frac{9}{4}x - \frac{27}{4}$

113. $a = \frac{1}{3}$, $b = -\frac{4}{3}$

115. $f_1(x) = |\sin x|$ is differentiable for all $x \neq n\pi$, n an integer.
$f_2(x) = \sin|x|$ is differentiable for all $x \neq 0$.

Section 2.3 (page 126)

1. $2(2x^3 - 3x^2 + x - 1)$ **3.** $(7t^2 + 4)/(3t^{2/3})$

5. $x^2(3\cos x - x\sin x)$ **7.** $(1 - x^2)/(x^2 + 1)^2$

9. $(1 - 8x^3)/[3x^{2/3}(x^3 + 1)^2]$ **11.** $(x\cos x - 2\sin x)/x^3$

13. $f'(x) = (x^3 - 3x)(4x + 3) + (2x^2 + 3x + 5)(3x^2 - 3)$
$= 10x^4 + 12x^3 - 3x^2 - 18x - 15$
$f'(0) = -15$

15. $f'(x) = \frac{x^2 - 6x + 4}{(x - 3)^2}$
$f'(1) = -\frac{1}{4}$

17. $f'(x) = \cos x - x\sin x$
$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{8}(4 - \pi)$

	Function	*Rewrite*	*Differentiate*	*Simplify*
19.	$y = \frac{x^2 + 2x}{3}$	$y = \frac{1}{3}(x^2 + 2x)$	$y' = \frac{1}{3}(2x + 2)$	$y' = \frac{2(x + 1)}{3}$
21.	$y = \frac{7}{3x^3}$	$y = \frac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\frac{7}{x^4}$
23.	$y = \frac{4x^{3/2}}{x}$	$y = 4x^{1/2}$, $x > 0$	$y' = 2x^{-1/2}$	$y' = \frac{2}{\sqrt{x}}$, $x > 0$

25. $\frac{(x^2 - 1)(-2 - 2x) - (3 - 2x - x^2)(2x)}{(x^2 - 1)^2} = \frac{2}{(x + 1)^2}$, $x \neq 1$