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1.  $f(x) = 17 - 3x$  polynomial function

zeros:  $17 - 3x = 0$

$$3x = 17$$

$$x = \frac{17}{3} \rightarrow \text{zeros } \left(\frac{17}{3}\right)$$

3.  $h(x) = 9 \rightarrow$  polynomial function

zeros:  $h(x) = 0 \rightarrow 9 = 0 \rightarrow$  not possible

so no zeros

5.  $m(x) = \frac{x^2 - 3x - 4}{x^2 + 1} \rightarrow$  not a polynomial function  
 $\rightarrow$  quadratic expression in the denominator

zeros:  $m(x) = 0$

$$\frac{x^2 - 3x - 4}{x^2 + 1} = 0 \rightarrow \text{a fraction} = 0 \text{ if the numerator is zero}$$

$$0 \div \text{any } \neq = 0!$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$\begin{array}{cc} \downarrow & \downarrow \\ 4 & -1 \end{array}$$

7.  $p(x) = x^3 - 9x \rightarrow$  polynomial function

zeros:  $p(x) = 0 \rightarrow x^3 - 9x = 0$

Factor  $x$ :  $x(x^2 - 9) = 0$

$$x(x + 3)(x - 3) = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 0 & -3 & 3 \end{array}$$

(15)  $h(x) = 2x^2 - 5x + 6$

$$h(-1) \quad \begin{array}{r|rrr} & 2 & -5 & 6 \\ -1 & & -2 & 7 \\ \hline & 2 & -7 & 13 \end{array} \quad h(-1) = 13$$

$$h(2i) \quad \begin{array}{r|rrr} & 2 & -5 & 6 \\ 2i & & 4i & -10i + 8i^2 = -10i - 8 \\ \hline & 2 & -5 + 4i & -2 - 10i \end{array} \quad h(2i) = -2 - 10i$$

$$h(1+i) \quad \begin{array}{r|rrr} & 2 & -5 & 6 \\ 1+i & & 2+2i & -5-i \\ \hline & 2 & -3+2i & 1-i \end{array} \quad h(1+i) = 1-i$$

$(-3+2i)(1+i)$   
 $-3 - 3i + 2i + 2i^2$   
 $-3 - i - 2 \rightarrow -5 - i$

$$h(3a) \quad \begin{array}{r|rrr} & 2 & -5 & 6 \\ 3a & & 6a & -15a + 18a^2 \\ \hline & 2 & -5+6a & 18a^2 - 15a + 6 \end{array}$$

(17)  $f(x) = x^3 - 9x$

$$f\left(-\frac{\sqrt{2}}{3}\right) = \begin{array}{r|rrrr} & 1 & 0 & -9 = -\frac{81}{9} & 0 \\ -\frac{\sqrt{2}}{3} & & -\frac{\sqrt{2}}{3} & \frac{2}{9} & \frac{7a\sqrt{2}}{27} \\ \hline & 1 & -\frac{\sqrt{2}}{3} & -\frac{79}{9} & \frac{7a\sqrt{2}}{27} \end{array}$$

(b)  $f(i\sqrt{3})$

$$\begin{array}{r|rrrr} & 1 & 0 & -9 & 0 \\ i\sqrt{3} & & i\sqrt{3} & 3i^2 = -3 & -12i\sqrt{3} \\ \hline & 1 & i\sqrt{3} & -12 & -12i\sqrt{3} \end{array}$$

(c)  $f\left(\frac{x}{3}\right)$

$$\begin{array}{r|rrrr} & 1 & 0 & -9 = -\frac{81}{9} & 0 \\ \frac{x}{3} & & \frac{x}{3} & \frac{x^2}{9} & \frac{x^3 - 81x}{27} \\ \hline & 1 & \frac{x}{3} & \frac{x^2 - 81}{9} & \frac{x^3 - 81x}{27} = \frac{x^3}{27} - 3x \end{array}$$

$$\begin{array}{r|rrrr}
 1 & 0 & -9 & 0 \\
 x-3 & x-3 & x^2-6x+9 & x^3-9x^2+18x \\
 \hline
 1 & x-3 & x^2-6x & x^3-9x^2+18x
 \end{array}$$

$$\begin{aligned}
 (x^2-6x)(x-3) &= x^3 - 3x^2 - 6x^2 + 18x \\
 &= x^3 - 9x^2 + 18x
 \end{aligned}$$

$$23 \quad f(x) = 3x^3 + kx - 2$$

$$4 \text{ is a zero so } f(4) = 0$$

$$0 = 3(4)^3 + k(4) - 2$$

$$= 3(64) + 4k - 2$$

$$= 192 + 4k - 2 \rightarrow 0 = 190 + 4k$$

$$4k = -190$$

$$k = -190 \div 4 = -47.5$$

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$$1.a \quad (x^5 - 2x^3 + x^2 - 4) \div (x-1)$$

$$1x^5 + 0x^4 - 2x^3 + 1x^2 + 0x - 4$$

$$\begin{aligned}
 x-1 &= 0 \\
 x &= 1
 \end{aligned}$$

$$\begin{array}{r|rrrrrr}
 1 & 1x^5 & 0 & -2 & 1 & 0 & -4 \\
 & 1 & 1 & -1 & 0 & 0 & \\
 \hline
 Q: & \boxed{1x^4 + 1x^3 - 1x^2} & 0x & 0 & \boxed{-4} & \text{remainder}
 \end{array}$$

$$3.b \quad (1x^3 - 3x^2 + 0x + 5) \div (x+2)$$

$$\begin{array}{r|rrrr}
 1x^3 & -3 & 0 & 5 \\
 -2 & -2 & 10 & -20
 \end{array}$$

$$Q: \boxed{1x^2 - 5x + 10} \quad \boxed{-15} \text{ remainder}$$

$$5. \quad (1x^4 - 2x^3 + 5x + 2) \div (x+1)$$

$$\begin{array}{r|rrrrr}
 1x^4 & -2 & 0x^2 & 5 & 2 \\
 -1 & -1 & 3 & -3 & -2
 \end{array}$$

$$Q: \boxed{1x^3 - 3x^2 + 3x + 2} \quad \boxed{0} \text{ remainder}$$

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7  $(1x^5 + x^3 + x) \div (x-3)$

|   |        |          |           |         |        |                 |
|---|--------|----------|-----------|---------|--------|-----------------|
|   | $1x^5$ | $0x^4$   | $1$       | $0x^2$  | $1$    | $0$             |
| 3 |        | 3        | 9         | 30      | 90     | 273             |
|   | $1x^4$ | $+ 3x^3$ | $+ 10x^2$ | $+ 30x$ | $+ 91$ | $273$ Remainder |

9 Must use long division  $\rightarrow$  divisor not linear

Q:  $3x^2 - 8x + 21$

|            |                |               |                 |              |                 |
|------------|----------------|---------------|-----------------|--------------|-----------------|
| $x^2 + 2x$ | $3x^4$         | $- 2x^3$      | $+ 5x^2$        | $+ x$        | $+ 1$           |
|            | $\ominus 3x^4$ | $\oplus 6x^3$ |                 |              |                 |
|            |                | $- 8x^3$      | $+ 5x^2$        |              |                 |
|            |                | $\oplus 8x^3$ | $\oplus 16x^2$  |              |                 |
|            |                |               | $21x^2$         | $+ x$        |                 |
|            |                |               | $\ominus 21x^2$ | $\oplus 42x$ |                 |
|            |                |               |                 | $- 41x$      | $+ 1$ remainder |

11.  $f(x) = x^{100} - 4x^{99} + 3$

$f(1) = 1^{100} - 4(1)^{99} + 3 = 1 - 4 + 3 = 0 \rightarrow (x-1)$  is a factor of  $f(x)$

$f(-1) = (-1)^{100} - 4(-1)^{99} + 3 = 1 + 4 + 3 = 8 \neq 0 \rightarrow (x+1)$  is not a factor of  $f(x)$

13b  $P(x) = x^3 - 5x^2 + 3x + 9$

|                         |    |     |      |      |                   |                       |
|-------------------------|----|-----|------|------|-------------------|-----------------------|
| $(x+3)$                 |    | $1$ | $-5$ | $3$  | $9$               | $P(-3) \neq 0$        |
|                         | -3 |     | $-3$ | $24$ | $-81$             | $\text{so } (x+3)$ is |
|                         |    | $1$ | $-8$ | $27$ | $\ominus 72$      | not a factor          |
| $\textcircled{a} (x-3)$ |    | $1$ | $-5$ | $3$  | $9$               | $P(3) = 0$ so         |
|                         | 3  |     | $3$  | $-6$ | $-9$              | $(x-3)$ is a factor   |
|                         |    | $1$ | $-2$ | $-3$ | $\textcircled{0}$ | of $P(x)$             |

$$17 \quad \begin{array}{c} x^2 - x + 4 \\ \hline 2x + 1 \mid P(x) \\ \hline R3 \end{array}$$

$$\begin{aligned} P(x) &= (x^2 - x + 4)(2x + 1) + 3 \\ &= 2x^3 + x^2 - 2x^2 - x + 8x + 4 + 3 \\ &= 2x^3 - x^2 + 7x + 7 \end{aligned}$$

$$19 \quad \begin{array}{c} 2x^3 - 5x^2 - 4x + 3 \\ \hline 3 \mid \phantom{2x^3} 6x^2 + 3x - 3 \\ \hline 2x^2 + 1x - 1 \quad \boxed{0} \end{array}$$

$$\begin{array}{r} 2x^2 + x - 1 \\ 2x \quad -1 \\ \hline x \quad +1 \\ \hline (2x-1)(x+1) = 0 \\ \downarrow \quad \downarrow \\ \boxed{\frac{1}{2}} \quad ; \quad \boxed{-1} \end{array}$$

$$21. \quad \begin{array}{c} 2x^4 - 9x^3 + 2x^2 + 9x - 4 \\ \hline -1 \mid \phantom{2x^4} -2x^3 + 11x^2 - 13x + 4 \\ \hline 2x^3 - 11x^2 + 13x - 4 \quad \boxed{0} \\ \hline 2x^2 - 9x + 4 \quad \boxed{0} \end{array}$$

$$\begin{array}{r} 2x^2 - 9x + 4 \\ 2x \quad -1 \\ \hline x \quad -4 \\ \hline (2x-1)(x-4) = 0 \\ \downarrow \quad \downarrow \\ \boxed{\frac{1}{2}} \quad \boxed{4} \end{array}$$

$$23 \quad \begin{array}{c} 1x^4 + 3x^3 - 3x^2 + 3x - 4 \\ \hline -4 \mid \phantom{1x^4} -4x^3 + 4x^2 - 4x + 4 \\ \hline 1x^3 - 1x^2 + 1x - 1 \quad \boxed{0} \\ \hline 1x^2 + 0x + 1 \quad \boxed{0} \end{array}$$

$$\begin{aligned} x^2 + 1 &= 0 \\ x^2 &= -1 \\ x &= \pm \sqrt{-1} \\ x &= \pm i \end{aligned}$$