10-3

1. $2\cos^2 10^\circ - 1$ $\qquad \theta = 10^\circ$

$2\cos^2\theta - 1 = \cos 2\theta$

$= \cos 2(10^\circ) = \cos 20^\circ$

3. $\dfrac{4\tan\beta}{1-\tan^2\beta} = 2 \cdot \dfrac{2\tan\beta}{1-\tan^2\beta} = 2(\tan 2\beta)$

5. $2\sin 35^\circ \cos 35^\circ$ $\qquad \theta = 35^\circ$

$2\sin\theta\cos\theta = \sin 2\theta = \sin 2(35^\circ) = \sin 70^\circ$

7. $\dfrac{2\tan 25^\circ}{1-\tan^2 25^\circ}$ $\qquad \theta = 25^\circ \rightarrow \dfrac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$

$= \tan 2(25^\circ) = \tan 50^\circ$

9. $1 - 2\sin^2 \frac{x}{2}$ $\qquad \theta = \frac{x}{2} \rightarrow 1 - 2\sin^2\theta = \cos 2\theta$

$= \cos 2\left(\frac{x}{2}\right) = \cos x$

11. $\sqrt{\dfrac{1-\cos 80^\circ}{2}}$ $\qquad \theta = 80^\circ \rightarrow \sqrt{\dfrac{1-\cos\theta}{2}} = \sin\dfrac{\theta}{2} = \sin\dfrac{80^\circ}{2}$

$= \sin 40^\circ$

13. $2\cos^2\frac{\pi}{8} - 1 \rightarrow \theta = \frac{\pi}{8} \rightarrow 2\cos^2\theta - 1 = \cos 2\theta$

$= \cos 2\left(\frac{\pi}{8}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

15. $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12} \rightarrow \theta = \frac{\pi}{12} \rightarrow \cos^2\theta - \sin^2\theta = \cos 2\theta$

$\rightarrow \cos 2\left(\frac{\pi}{12}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

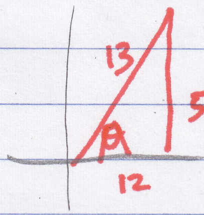
17. $\sin 15^\circ \cos 15^\circ \rightarrow \theta = 15^\circ$

$2 \cdot \sin\theta\cos\theta = y \cdot 2$

$\sin 2\theta = 2y \rightarrow y = \dfrac{\sin 2\theta}{2} = \dfrac{\sin 2(15^\circ)}{2}$

$= \dfrac{\sin 30^\circ}{2} = \frac{1}{2} \div 2 = \frac{1}{4}$

19\. $\angle A$ is acute $\rightarrow$ QI



$\sin A = 5/13$

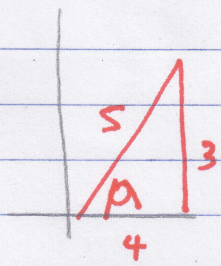
$\cos A = 12/13$

$\sin 2A = 2\sin A \cos A$
$= 2(5/13)(12/13)$
$= \frac{120}{169}$

$\cos 2A = \cos^2 A - \sin^2 A = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144-25}{169} = \frac{119}{169}$

21\.



$\sin A = 3/5$

$\cos A = 4/5$

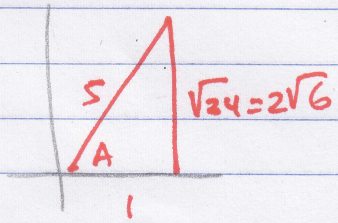
$\sin 2A = 2\sin A \cos A$
$= 2(3/5)(4/5) = 24/25$

$2A = \theta \rightarrow \sin 4A = \sin 2\theta = 2\sin\theta\cos\theta$
$= 2\,\underline{\sin 2A}\,\underline{\cos 2A}$
$= (2\sin A\cos A)(1-2\sin^2 A)$

$= 2\cdot 2(3/5)(4/5)\left(1-2\left(\frac{9}{25}\right)\right)$

$= \frac{48}{25}\left(1-\frac{18}{25}\right) = \frac{48}{25}\cdot\frac{7}{25} = \frac{336}{625}$

23\.



$\cos 2A = 2\cos^2 A - 1$
$= 2\left(\frac{1}{5}\right)^2 - 1 = 2\left(\frac{1}{25}\right) - 1$
$= \frac{2}{25} - \frac{1 \cdot 25}{1 \cdot 25} = \frac{-23}{25}$

$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}} = \sqrt{\frac{1+1/5}{2}} = \sqrt{\frac{\frac{5}{5}+\frac{1}{5}}{2}}$

$= \sqrt{\frac{6}{5}\cdot\frac{1}{2}} = \sqrt{\frac{3}{5}\cdot\frac{5}{5}} = \frac{\sqrt{15}}{5}$

A is in QI $\rightarrow$ $0 < A < \frac{\pi}{2}$

$\frac{A}{2}$ $\rightarrow$ $0 < \frac{A}{2} < \frac{\pi}{4} \rightarrow$ QI

so $\cos\frac{A}{2}$ is $\oplus$ $\frac{\sqrt{15}}{5}$

10-3 #4

25. $\cos 105^\circ = \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

b $\cos 105^\circ = \cos \frac{210^\circ}{2} = \sqrt{\frac{1 + \cos 210^\circ}{2}}$

$= \sqrt{\frac{1 + -\sqrt{3}/2}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{} \div \frac{2}{1}}$

$= \sqrt{\frac{2-\sqrt{3}}{2} \cdot \frac{1}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

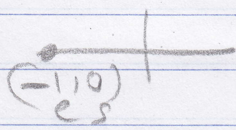
105° is in Q II so $\cos 105^\circ$ is negative

$\cos 105^\circ = - \sqrt{\frac{2-\sqrt{3}}{2}}$

p373

4. $\sin \frac{4\pi}{3} \cos \frac{\pi}{3} - \cos \frac{4\pi}{3} \sin \frac{\pi}{3} = \sin\left(\frac{4\pi}{3} - \frac{\pi}{3}\right)$

$= \sin \frac{3\pi}{3} = \sin \pi$



$= 0$

24 $\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right)$

$\sin \frac{\pi}{4} \cos x + \cancel{\cos \frac{\pi}{4} \sin x} + \sin \frac{\pi}{4} \cos x - \cancel{\cos \frac{\pi}{4} \sin x}$

$\frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \cos x = \frac{2\sqrt{2}}{2} \cos x = \sqrt{2} \cos x$

p 377

2. $\tan\alpha = 2 \qquad \tan\beta = -\frac{1}{3}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{2+(-\frac{1}{3})}{1-(2)(-\frac{1}{3})}$$

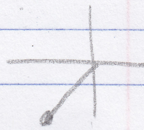
$$= \frac{5/3}{1+2/3} = \frac{5/3}{5/3} = 1$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{2-(-\frac{1}{3})}{1+(2)(-\frac{1}{3})}$$

$$= \frac{7/3}{1-2/3} = \frac{7}{3} \div \frac{1}{3} = \frac{7}{3}\cdot\frac{3}{1} = 7$$

4. $$\frac{\tan\frac{4\pi}{3} - \tan\frac{\pi}{12}}{1+\tan\frac{4\pi}{3}\tan\frac{\pi}{12}} = \tan\left(\frac{4\pi^{4}}{3_{4}} - \frac{\pi}{12}\right) =$$

$$\tan\frac{15\pi}{12} = \tan\frac{5\pi}{4} = 1$$



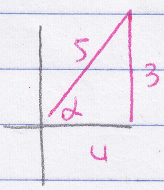
8. $$\tan\left(\frac{3\pi}{4} - \theta\right) = \frac{\tan 3\pi/4 - \tan\theta}{1+\tan\frac{3\pi}{4}\tan\theta}$$

$\tan\theta = \frac{1}{3}$

$$= \frac{-1-\frac{1}{3}}{1+(-1)(\frac{1}{3})} = \frac{-4/3}{1-1/3}$$

$$= -\frac{4}{3} \div \frac{2}{3} = -\frac{4}{3}\cdot\frac{3}{2} = -2$$

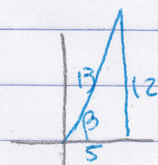
25



$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\tan\alpha = \frac{3}{4}$



$\sin\beta = \frac{12}{13}$

$\cos\beta = \frac{5}{13}$

$\tan\beta = \frac{12}{5}$

a $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{3}{5}\frac{5}{13} + \frac{4}{5}\frac{12}{13} = \frac{63}{65}$

b $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{4}{5}\frac{5}{13} - \frac{3}{5}\frac{12}{13} = \frac{-16}{65}$

c $$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{\frac{5}{5}\frac{3}{4} + \frac{12}{5}\frac{4}{4}}{1-\frac{3}{4}\cdot\frac{12}{5}} = \frac{\frac{63}{20}}{\frac{-16}{20}} = \frac{63}{20}\cdot\frac{-20}{16} = \frac{-63}{16}$$