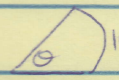
Ch 7 Review (Trig)

1a $\frac{140^\circ}{1} \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{7\pi}{9}$; (1b) $\frac{5\pi}{9} \text{ radians} \cdot \frac{180^\circ}{\pi \text{ rad}} = 100^\circ$

2 

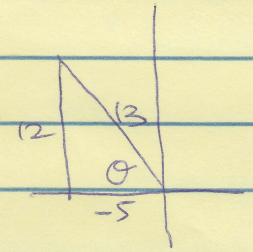
$\theta = \frac{10}{6} = \frac{5}{3}$

3. Arc length : $\frac{48^\circ}{360^\circ} \cdot 2\pi \cdot 9 = \frac{12\pi}{5}$ in

Area : $\frac{48^\circ}{360^\circ} \cdot \pi \cdot 9^2 = \frac{54\pi}{5}$ in^2

4 Arc length : $r\theta = 9(4) = 36$ in

Area = $\frac{1}{2} r^2\theta = \frac{1}{2} \cdot 9^2 \cdot 4 = 162$ in^2

5 

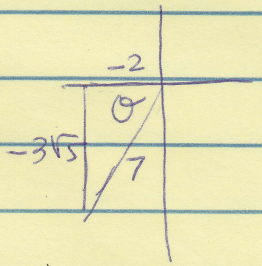
$(-5)^2 + (12)^2 = r^2$

$r = 13$

$\sin\theta = \frac{12}{13}$ $\quad \csc\theta = \frac{13}{12}$

$\cos\theta = \frac{-5}{13}$ $\quad \sec\theta = \frac{-13}{5}$

$\tan\theta = \frac{-12}{5}$ $\quad \cot\theta = \frac{-5}{12}$

6. 

$(-2)^2 + y^2 = 7^2 \rightarrow 4 + y^2 = 49 \rightarrow y^2 = 45$

$y = 3\sqrt{5}$

$\sin\theta = \frac{-3\sqrt{5}}{7}$

7 $\cos 30^\circ = \frac{\sqrt{3}}{2}$ (Q I)

(b) $\csc 45^\circ$: $\sin 45^\circ = \frac{\sqrt{2}}{2}$ (Q I)

$\rightarrow \csc 45^\circ = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

(c) $\tan 225^\circ \rightarrow$ Q III (+)

ref $= 45^\circ \rightarrow \tan 45^\circ = 1$

$\tan 225^\circ = 1$

(d) $\cot \frac{4\pi}{3} \rightarrow$ Q III (+)

ref $= \frac{\pi}{3} \rightarrow \cot \frac{\pi}{3} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cot \frac{4\pi}{3} = \frac{\sqrt{3}}{3}$

(e) $\sin \frac{3\pi}{4} \rightarrow$ Q II (+)

ref $\frac{\pi}{4} \rightarrow \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

(f) $\sec \frac{7\pi}{6} \rightarrow$ Q III (−)

ref $\frac{\pi}{6} \rightarrow \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sec \frac{\pi}{6} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec \frac{7\pi}{6} = \ominus \frac{2\sqrt{3}}{3}$

(8) $\cos^{-1}\left(-\frac{1}{2}\right) =$ Q II

60° in Q II $\rightarrow 120^\circ = \frac{2\pi}{3}$

(b) $\sin^{-1}\left(-\frac{1}{2}\right)$ D : $-\frac{\pi}{2} \le \theta \le 0$

$-30^\circ = -\frac{\pi}{6}$

(c) $\tan^{-1}(\sqrt{3})$ Q I

$60^\circ = \frac{\pi}{3}$

(d) $\cos^{-1}(-1) \rightarrow (-1, 0)$

180° or π

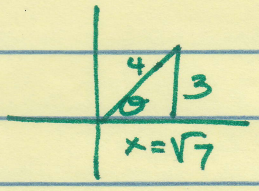
(e) $\sin^{-1}(-1) \rightarrow (0, -1)$

$-90^\circ = -\frac{\pi}{2}$

(f) $\tan^{-1}(-1)$

$-\frac{\pi}{4} = -45^\circ$

9a $\cos\left(\sin^{-1}\frac{3}{4}\right) \rightarrow$ QI



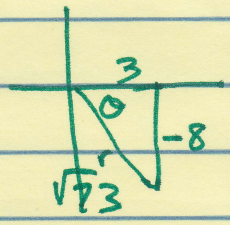
$x^2+3^2=4^2$

$x^2=7$

$x=\sqrt{7}$

$\cos\theta=\frac{\sqrt{7}}{4}$

b $\csc\left(\tan^{-1}\left(-\frac{8}{3}\right)\right) \rightarrow$ QIV



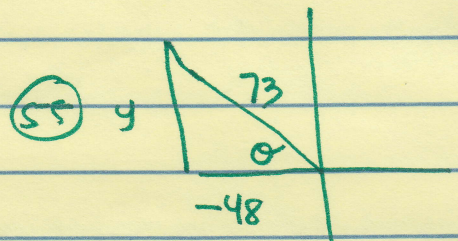
$r^2=3^2+8^2$

$=73$

$r=\sqrt{73}$

$\csc=\frac{hyp}{opp}=\frac{-\sqrt{73}}{8}$

c $\cot\left[\cos^{-1}\left(\frac{-48}{73}\right)\right] \rightarrow$ QII



$(-48)^2+y^2=73^2$

$y^2=3025$

$y=55$

$\cot\theta=\frac{adj}{opp}=\frac{-48}{55}$

10 $-115^\circ+360^\circ=245^\circ$

$-115^\circ-360^\circ=-475^\circ$

11 $-\frac{11\pi}{4}+2\pi=-\frac{3\pi}{4}$

$-\frac{11\pi}{4}+2(2\pi)=\frac{5\pi}{4}$

12 $\sin 4\pi=\sin 0=0$

(b) $\cos\frac{7\pi}{6} \rightarrow$ QII $\rightarrow$ cos is (−)

(c) $\sin\left(-\frac{\pi}{3}\right) \rightarrow$ QIV $\rightarrow$ sin is (−)

13 $\cos -550^\circ = \cos(-550^\circ + 360^\circ + 360^\circ)$

$= \cos 170^\circ$

cos is − in Q II

ref ∠ $180 - 170 = 10^\circ$

$-\cos 10^\circ$

14 $\sin 1400^\circ = \sin 1400^\circ - 360^\circ - 360^\circ - 360^\circ =$

$\sin 320^\circ$

Q IV sin is ⊖

ref ∠ $= 360 - 320^\circ = 40^\circ$

$-\sin 40^\circ$

p284

10. Find one positive and one negative angle that are coterminal with -115°

11. Find one positive and one negative angle that are coterminal with $-\frac{11\pi}{4}$

12. State whether each expression is positive, negative or zero.

a) $\sin 4\pi$ b) $\cos\frac{7\pi}{6}$ c) $\sin\left(-\frac{\pi}{4}\right)$

Write in terms of its reference angle.

13. $\cos -550^\circ$ 14. $\sin 1400^\circ$

10. 245° ; -475°

11. $-\frac{3\pi}{4}$; $\frac{5\pi}{4}$

12. 0 ; Neg ; Neg

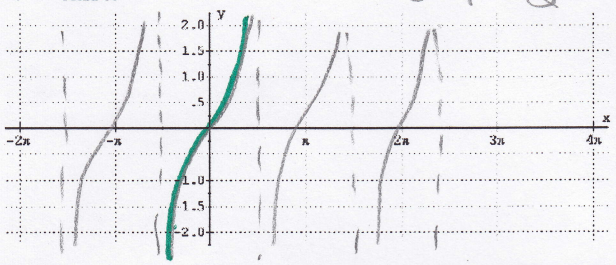
13. $-\cos 10^\circ$

14. $-\sin 40^\circ$

15. Graph over $-2\pi < x < 2\pi$.

a) $y = \tan x$

0 π/4 π/2 3π/4 π 5π/4 3π/2 7π/4 2π
0 1 Ø -1 0 1 Ø -1 0

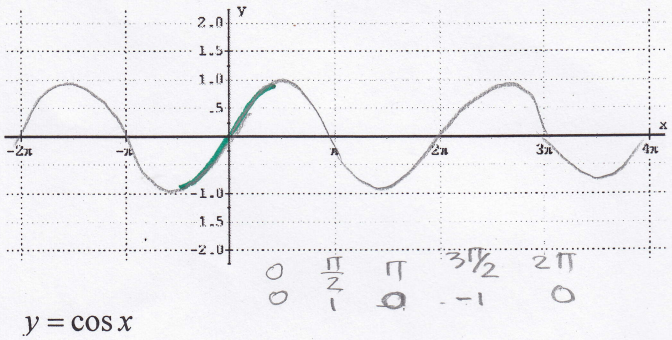


Domain: $x \neq \frac{\pi}{2} + n\pi$ Range: all real #s not 1 -1

Does this graph have an inverse? Why or why not?
Darken a portion of the graph that is one-to-one and thus have an inverse.
Domain of $y = \text{Tan } x$: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Graph over $-2\pi < x < 2\pi$ and then darken the portion that has an inverse.

16. $y = \sin x$

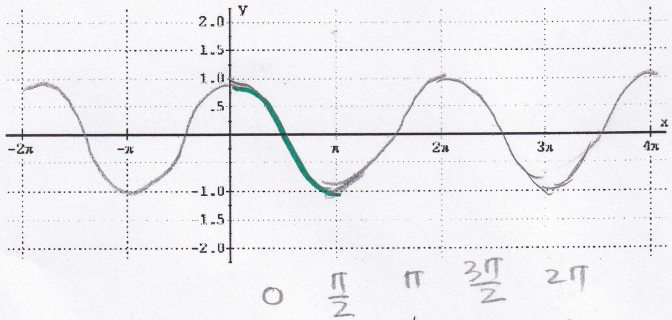


0 π/2 π 3π/2 2π
0 1 0 -1 0

Domain: all real #s Range: $-1 \leq x \leq 1$ not 1-1

Does this graph have an inverse? Why or why not?
Darken a portion of the graph that is one-to-one and thus have an inverse.
Domain of $y = \text{Sin } x$: $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

17. $y = \cos x$



0 π/2 π 3π/2 2π
1 0 -1 0 1

Domain: all real #s Range: $-1 \leq y \leq 1$

Does this graph have an inverse? Why or why not?
Darken a portion of the graph that is one-to-one and thus have an inverse.
Domain of $y = \text{Cos } x$: $0 \leq x \leq \pi$

[0, π]

p293

1 $\frac{270^\circ}{1} \cdot \frac{\pi}{180^\circ} = \frac{3\pi}{2}$

b. $\frac{192^\circ}{1} \cdot \frac{\pi}{180^\circ} = 3.35$

2a. $\frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$

b $\frac{2.5}{1} \cdot \frac{180^\circ}{\pi} = 143.2^\circ$

3. $-200^\circ + 360^\circ = 160^\circ$

$-200^\circ - 360^\circ = -560^\circ$

(b) $313.2^\circ + 360^\circ = 673.2^\circ$

$313.2^\circ - 360^\circ = -46.8^\circ$

(c) $\frac{5\pi}{6} + \frac{2\pi}{1} \cdot \frac{6}{6} = \frac{17\pi}{6}$

$\frac{5\pi}{6} - \frac{2\pi}{1} \cdot \frac{6}{6} = \frac{-7\pi}{6}$

(d) $142^\circ 10' + 360^\circ = 502^\circ 10'$

$-360^\circ + 142^\circ 10'$

$-359^\circ 60'$

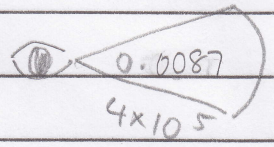
$+ 142^\circ 10'$

$217^\circ 50'$

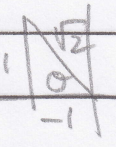
(4) $r = 5\text{cm} \quad \theta = 48^\circ$

$s = \frac{48^\circ}{360^\circ} \cdot 2\pi(5) \approx 4.2\text{cm}$

$A = \frac{48^\circ}{360^\circ} \cdot \pi \cdot 5^2 \approx 10.5\text{cm}^2$

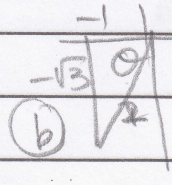
5. 

$s = r\theta = 4 \times 10^5 \times 0.0087 = 3480\text{km}$

(6) 

$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

(b) 

$r = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$

$\sin\theta = \frac{-\sqrt{3}}{2}$

$\cos\theta = \frac{-1}{2}$

6c (0,1) C S

$\cos\theta = 0$

$\sin\theta = 1$

8. $\sin\frac{5\pi}{4} \to$ Q III $\to \sin(-)$

$= -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

C S

(b) $\cos -90^\circ = \cos 270^\circ$

$(0,-1)$ C $= 0$

(c) $\sin 150^\circ \to$ Q II $\to (+)$

$180^\circ - 150^\circ = 30^\circ$

$\sin 30^\circ = \frac{1}{2}$

S

(d) $\cos\frac{11\pi}{6} \to$ Q IV $\to +$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

cos

9. $\tan 135^\circ \to$ Q II $\to (-)$

$180^\circ - 135^\circ = 45^\circ$

$\to +1$

(b) $\cot\frac{2\pi}{3} \to$ Q II $\to (-)$

C S $= \frac{1}{\sqrt{3}} \quad \frac{-\sqrt{3}}{3}$

(c) $\sec(-60^\circ) \to$ Q IV $\to (+)$

$\cos 60^\circ = \frac{1}{2}$

$\sec 60^\circ = 2$

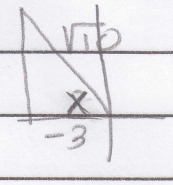
(d) $\csc -\pi = \csc -\pi + 2\pi = \csc\pi$

$(-1,0)$ C C

$\sin\pi = 0 \to \csc\pi = \frac{1}{0}$

undef.

10. $\tan x = -\frac{1}{3}$; $\frac{\pi}{2} < x < \pi$ Q II



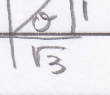
$\sin x = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \to \csc x = \sqrt{10}$

$\cos x = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10} \to \sec x = -\frac{\sqrt{10}}{3}$

$\cot x = -3$

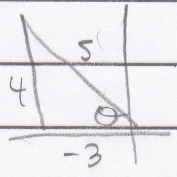
11. $\text{Tan}^{-1}\sqrt{3} \to$ QI $\to \frac{\pi}{3}$

b $\cot\left(\sin^{-1}\frac{1}{2}\right) \to$ QI



$\cot\theta = \sqrt{3}$

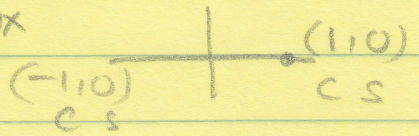
c $\sec\left(\text{Cos}^{-1}\left(-\frac{3}{5}\right)\right) \to$ QII



$\sec\theta = \frac{5}{-3} = -\frac{5}{3}$

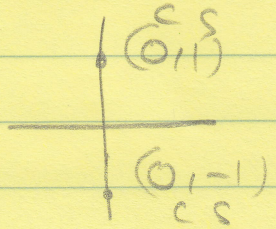
p285

8 $\cot x = \frac{\cos x}{\sin x}$ → undefined when $\sin x = 0$



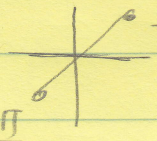
→ $0 + n\pi$

b $\cot x = 0$ → when $\cos x = 0$



→ $\frac{\pi}{2} + n\pi$

c $\cot x = 1$ → when $\cos x = \sin x$



$\cos x = \frac{\sqrt{2}}{2}$, $\sin x = \frac{\sqrt{2}}{2}$

$\cos x = -\frac{\sqrt{2}}{2}$

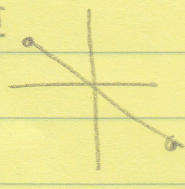
$\sin x = -\frac{\sqrt{2}}{2}$

→ $\frac{\pi}{4} + n\pi$

d. $\cot x = -1$ → $\cos x = -\sin x$

$\cos x = -\frac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2}$



$\cos x = \frac{\sqrt{2}}{2}$ $\sin x = -\frac{\sqrt{2}}{2}$

→ $\frac{3\pi}{4} + n\pi$