Sec. 12-7 Normal Distributions

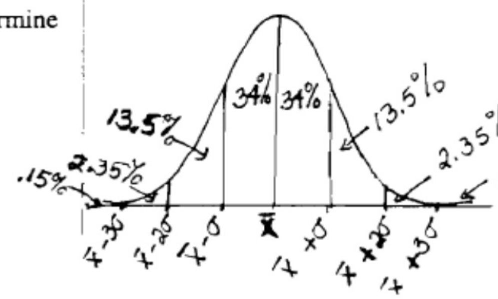
VOCABULARY

\ smooth, symmetrical, bell-shaped curve which approximates a binomial listribution is called a **normal curve.** Areas under this curve represent probabilities from **normal distributions.**

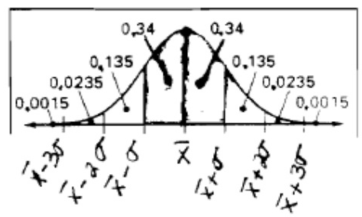
\rea Under a Normal Curve

The mean $\bar{x}$ and standard deviation σ of a normal distribution determine he following areas.

- The total area under the curve is 1.
- 68% of the area lies within 1 standard deviation of the mean.
- 95% of the area lies within 2 standard deviations of the mean.
- 99.7% of the area lies within 3 standard deviations of the mean.



The partial areas can be interpreted as probabilities, as shown.

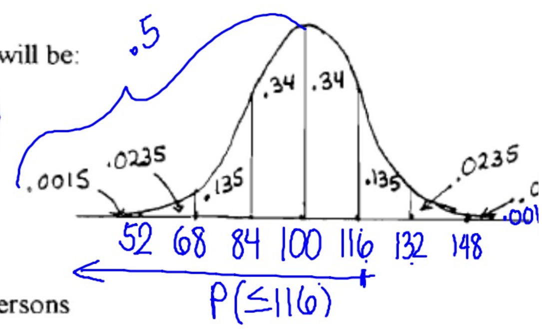


Example 1: A normal distribution of IQ scores has a mean of 100 and a standard deviation of 16. $= \sigma$

Find the probability that a randomly selected person's IQ will be:

a) between 84 and 116 $= .68 \quad (.34 + .34)$

b) at most 116 $= .5 + .34 = .84$



Find the probability that the IQ for 3 randomly selected persons will be at least 132.

$P(\geq 132) = .025$

$\frac{.025}{\text{1st}} \times \text{and} \ \frac{.025}{\text{2nd}} \times \text{and} \ \frac{.025}{\text{3rd}} \approx .00002$

.002%

If n is large, it becomes tedious to compute binomial probabilities using the formula $P(k\ successes) = {}_nC_k\,(p)^k\,(1-p)^{n-k}$. Under certain conditions, you may use a normal distribution to approximate a binomial distribution.

Normal Approximation of a Binomial Distribution

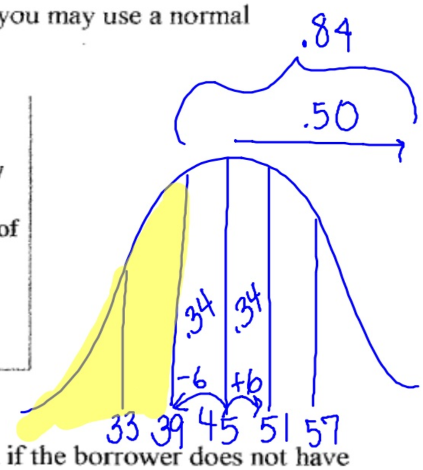
Consider the binomial distribution consisting of n trials with probability p of success on each trial. If $np \geq 5$ and $n(1-p) \geq 5$, then binomial distribution can be approximated by a normal distribution with a mean of

$\bar{x} = np$

mean

and a standard deviation of

$\sigma = \sqrt{np(1-p)}$.



Example 2: A loan officer at a bank may reject a loan application if the borrower does not have enough assets or has too many debts based on total income. At a certain bank, 20% of the loan applications are rejected. Assume there were 225 applications. What is the probability that at most 39 will be rejected?

$1 - .84 = \boxed{.16}$

$\bar{X} = np = 225(.20) = 45$

$\sigma = \sqrt{np(1-p)} = \sqrt{45(.8)} = \boxed{6}$