Station 1

Procedural

Zill

EXERCISES 2.4 *Answers to odd-numbered problems begin on page A-70.*

Fluency ★

In Problems 1–30 find the derivative of the given function.

1. $y = 1/x$
2. $y = (2/x^3)^2$
3. $y = (5x)^{-2}$
4. $y = \pi^4 x^{-4}$
5. $y = 6x^2 + x^{-2}$
6. $y = 5x^4 - 1/2x^5$
7. $y = (x^2 - 7)(x^3 + 4x + 2)$
8. $y = (7x + 1)(x^4 - x^3 - 9x)$
9. $f(x) = \left(4 + \frac{1}{x}\right)\left(2x - \frac{1}{x^2}\right)$
10. $f(x) = \left(x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)$
11. $f(x) = \dfrac{10}{x^2 + 1}$
12. $f(x) = 5(4x - 3)^{-1}$
13. $G(x) = \dfrac{3x + 1}{2x - 5}$
14. $F(x) = \dfrac{2 - 3x}{7 - x}$
15. $y = (6x - 1)^2$
16. $y = (x^4 + 5x)^2$
17. $g(t) = \dfrac{t^2}{2t^2 + t + 1}$
18. $p(y) = \dfrac{y^2 - 10y + 2}{y(y^2 - 1)}$
19. $H(z) = (z + 1)(2z + 1)(3z + 1)$
20. $Q(r) = (r^2 + 1)(r^3 - r)(3r^4 + 2r - 1)$
21. $y = \dfrac{(2x + 1)(x - 5)}{3x + 2}$
22. $y = \dfrac{x^5}{(x^2 + 1)(x^3 + 4)}$
23. $y = \dfrac{2 - 1/x^3}{3 + 1/x^2}$
24. $y = \dfrac{x^{-2}}{x^{-3} + x^{-2} + 1}$
25. $f(u) = \frac{1}{u} + \frac{1}{u^2} + \frac{1}{u^3} + \frac{1}{u^4}$
26. $h(v) = \dfrac{1}{v + v^2 + v^3 + v^4}$
27. $y = \left(\dfrac{x + 1}{x + 3}\right)(x^2 - 2x - 1)$
28. $y = (x + 1)\left(x + 1 - \dfrac{1}{x + 2}\right)$
29. $f(x) = (3x + 1)^{-2}$
30. $g(x) = (x + 1)^3$

In Problems 31 and 32 find dy/dx without the aid of the Quotient Rule.

31. $y = \dfrac{6x^2 - 5x}{x}$
32. $y = \dfrac{x^4 + 2x^3 - 1}{x^2}$

In Problems 33–36 find an equation of the tangent line to the graph of the given function at the indicated value of x.

33. $y = 1/x^2;\ x = \frac{1}{2}$
34. $y = 4x - 1/x;\ x = -1$
35. $y = (2x^2 - 4)(x^3 + 5x + 3);\ x = 0$
36. $y = \dfrac{5x}{x^2 + 1};\ x = 2$

Choose from following sets:

★ #1-10

★ #11-26

★★ #31-32 Explain your reasoning

★★ #33-36

Do #27 alone when you are ready to assess your understanding

53. $V(r) = \frac{4\pi}{3}r^3; \quad S = V'(r) = 4\pi r^2$

54. $v(r) = \frac{P}{4\nu l}(R^2 - r^2) = \frac{PR^2}{4\nu l} - \frac{P}{4\nu l}r^2$. Since $v'(r) = -\frac{P}{2\nu l}r = 0$, r

55. $U(x) = \frac{k}{2}x^2; \quad F = -\frac{dU}{dx} = -\frac{k}{2}2x = -kx$

Given $k = 30$ N/m and $x = \frac{m}{2}$, $F = -(30\text{ N/m})\frac{m}{2} = -15$ N.

Exercises 2.4

Zill key

\#1-4

1. $y = x^{-1}; \quad y' = -x^{-2} = -\frac{1}{x^2}$

2. $y = \frac{4}{x^6} = 4x^{-6}; \quad y' = -24x^{-7} = -\frac{24}{x^7}$

3. $y = \frac{1}{25}x^{-2}; \quad y' = -\frac{2}{25}x^{-3}$

4. $y' = -4\pi^4 x^{-5}$

Exercises 2.4

Zill Key

\# 5 -13

5. $y' = 12x - 2x^{-3}$

6. $y = 5x^4 - \frac{1}{2}x^{-5};\quad y' = 20x^3 + \frac{5}{2}x^{-6} = 20x^3 + \frac{5}{2x^6}$

7. $y' = (x^2 - 7)(3x^2 + 4) + (x^3 + 4x + 2)\cdot 2x = 5x^4 - 9x^2 + 4x - 28$

8. $y' = (7x + 1)(4x^3 - 3x^2 - 9) + (x^4 - x^3 - 9x)\cdot 7 = 35x^4 - 24x^3 - 3x^2 - 1$

9. $f(x) = (4 + x^{-1})(2x - x^{-2})$

$f'(x) = (4 + x^{-1})(2 + 2x^{-3}) + (2x - x^{-2})(-x^{-2}) = 8 + 8x^{-3} + 3x^{-4} = 8$

10. $f(x) = x^4 - \frac{1}{x^4} = x^4 - x^{-4};\quad f'(x) = 4x^3 + 4x^{-5} = 4x^3 + \frac{4}{x^5}$

11. $f'(x) = \frac{(x^2+1)\cdot 0 - 10\cdot 2x}{(x^2+1)^2} = \frac{-20x}{(x^2+1)^2}$

12. $f(x) = \frac{5}{(4x-3)};\quad f'(x) = \frac{(4x-3)\cdot 0 - 5\cdot 4}{(4x-3)^2} = -\frac{20}{(4x-3)^2}$

13. $G'(x) = \frac{(2x-5)\cdot 3 - (3x+1)\cdot 2}{(2x-5)^2} = -\frac{17}{(2x-5)^2}$

10. $f(x) = x^3 - \dfrac{\ldots}{x^4}$

11. $f'(x) = \dfrac{(x^2+1)\cdot 0 - 10\cdot 2x}{(x^2+1)^2} = \dfrac{-20x}{(x^2+1)^2}$

12. $f(x) = \dfrac{5}{(4x-3)}$; $f'(x) = \dfrac{(4x-3)\cdot 0 - 5\cdot 4}{(4x-3)^2} = -\dfrac{20}{(4x-3)^2}$

13. $G'(x) = \dfrac{(2x-5)\cdot 3 - (3x+1)\cdot 2}{(2x-5)^2} = -\dfrac{17}{(2x-5)^2}$

Zill key
11 – 18
Sorry for cut off answers

14. $F'(x) = \dfrac{(7-x)(-3) - (2-3x)(-1)}{(7-x)^2} = -\dfrac{19}{(7-x)^2}$

15. $y = (6x-1)(6x-1)$; $y' = (6x-1)\cdot 6 + (6x-1)\cdot 6 = 12(6x-1) = 72x$

16. $y = (x^4+5x)(x^4+5x)$; $y' = (x^4+5x)(4x^3+5) + (x^4+5x)(4x^3+5) = 8$

17. $g'(t) = \dfrac{(2t^2+t+1)\cdot 2t - t^2(4t+1)}{(2t^2+t+1)^2} = \dfrac{t^2+2t}{(2t^2+t+1)^2}$

18. $F(y) = \dfrac{y^2-10y+2}{y^3-y}$

$$F'(y) = \frac{(y^3-y)(2y-10) - (y^2-10y+2)(3y^2-1)}{(y^3-y)^2} = \frac{-y^4+20y^3-7y^2+}{(y^3-y)^2}$$

19. $H'(z) = [(z+1)(2z+1)]\cdot 3 + (3z+1)[(z+1)\cdot 2 + (2z+1)\cdot 1] = 18z^2+2$

20. $Q'(r) = [(r^2+1)(r^3-r)](10\ldots$

14. $f'(z)$ ...

15. $y = (6x-1)(6x-1)$; $y' = (6x-1)$ ...

16. $y = (x^4+5x)(x^4+5x)$; $y' = (x^4+5x)(4x^3+5) + (x^4+5x)(4x^3+5) = 8x^7 + 50x^4 + 50x$ 29. f

Zill Key

\# 17 − 22

17. $$s'(t) = \frac{(2t^2+t+1)\cdot 2t - t^2(4t+1)}{(2t^2+t+1)^2} = \frac{t^2+2t}{(2t^2+t+1)^2}$$ 30.

18. $$f(y) = \frac{y^2-10y+2}{y^3-y}$$ 31.

$$f'(y) = \frac{(y^3-y)(2y-10)-(y^2-10y+2)(3y^2-1)}{(y^3-y)^2} = \frac{-y^4+20y^3-7y^2+2}{(y^3-y)^2}$$ 32. 33.

19. $H'(z) = [(z+1)(2z+1)]\cdot 3 + (3z+1)[(z+1)\cdot 2 + (2z+1)\cdot 1] = 18z^2 + 22z + 6$

20. $$Q'(r) = [(r^2+1)(r^3-r)](12r^3+2) + (3r^4+2r-1)[(r^2+1)(3r^2-1) + (r^3-r)\cdot 2r]$$ 34.

$$= (r^2+1)(r^3-r)(12r^3+2) + (3r^4+2r-1)(r^2+1)(3r^2-1) + 2r(3r^4+2r-1)(r^3 \cdot$$

21. $$y' = \frac{(3x+2)[(2x+1)\cdot 1 + (x-5)\cdot 2] - (2x+1)(x-5)\cdot 3}{(3x+2)^2} = \frac{6x^2+8x-3}{(3x+2)^2}$$ 35.

22. $$y' = \frac{(x^2+1)(x^3+4)\cdot 5x^4 - x^5[(x^2+1)\cdot 3x^2 + (x^3+4)\cdot 2x]}{[(x^2+1)(x^3+4)]^2} = \frac{2x^7+12x^6+20x^4}{(x^2+1)^2(x^3+4)^2}$$

Zill key
23-28

23. $y = \dfrac{2 - 1/x^3}{3 + 1/x^2} \cdot \dfrac{x^3}{x^3} = \dfrac{2x^3 - 1}{3x^3 + x}$; $\quad y' = \dfrac{(3x^3 + x) \cdot 6x^2 - (2x^3 - 1)(9x^2 + 1)}{(3x^3 + x)^2} = \dfrac{4x^3 + 9x^2 + 1}{(3x^3 + x)^2}$

24. $y = \dfrac{x^{-2}}{x^{-3} + x^{-2} + 1} \cdot \dfrac{x^3}{x^3} = \dfrac{x}{1 + x + x^3}$; $\quad y' = \dfrac{(1 + x + x^3) \cdot 1 - x(1 + 3x^2)}{(1 + x + x^3)^2} = \dfrac{1 - 2x^3}{(1 + x + x^3)^2}$

25. $f(u) = u^{-1} + u^{-2} + u^{-3} + u^{-4}$; $\quad f'(u) = -u^{-2} - 2u^{-3} - 3u^{-4} - 4u^{-5} = -\dfrac{1}{u^2} - \dfrac{2}{u^3} - \dfrac{3}{u^4} - \dfrac{4}{u^5}$

26. $h'(v) = \dfrac{(v + v^2 + v^3 + v^4) \cdot 0 - 1 \cdot (1 + 2v + 3v^2 + 4v^3)}{(v + v^2 + v^3 + v^4)^2} = \dfrac{-1 - 2v - 3v^2 - 4v^3}{(v + v^2 + v^3 + v^4)^2}$

27. $y' = \dfrac{x + 1}{x + 3}(2x - 2) + (x^2 - 2x - 1)\dfrac{(x + 3) \cdot 1 - (x + 1) \cdot 1}{(x + 3)^2} = \dfrac{2x^2 - 2}{x + 3} + \dfrac{2x^2 - 4x - 2}{(x + 3)^2}$

$$= \frac{2x^3 + 8x^2 - 6x - 8}{(x + 3)^2}$$

28. $y = (x + 1)^2 - \dfrac{x + 1}{x + 2} = x^2 + 2x + 1 - \dfrac{x + 1}{x + 2}$

$$y' = 2x + 2 - \frac{(x + 2) \cdot 1 - (x + 1) \cdot 1}{(x + 2)^2} = 2x + 2 - \frac{1}{(x + 2)^2} = \frac{2x^3 + 10x^2 + 16x + 7}{(x + 2)^2}$$

29. $f(x) = \dfrac{1}{(3x + 1)^2} = \dfrac{1}{9x^2 + 6x + 1}$

28. $y = (x+1)^2 - \dfrac{x+1}{x+2} = x^2 + 2x + 1 - \dfrac{x+1}{x+2}$

$$y' = 2x + 2 - \frac{(x+2)\cdot 1 - (x+1)\cdot 1}{(x+2)^2} = 2x + 2 - \frac{1}{(x+2)^2} = \frac{2x^3 + 10x^2 + 16x + 7}{(x+2)^2}$$

29. $f(x) = \dfrac{1}{(3x+1)^2} = \dfrac{1}{9x^2 + 6x + 1}$

Zill Key

$$f'(x) = \frac{(9x^2 + 6x + 1)\cdot 0 - 1(18x + 6)}{(3x+1)^4} = -\frac{6(3x+1)}{(3x+1)^4} = -6(3x+1)^{-3}$$

28-35

30. $g(x) = [(x+1)(x+1)](x+1)$

$g'(x) = [(x+1)(x+1)]\cdot 1 + (x+1)[(x+1)\cdot 1 + (x+1)\cdot 1] = (x+1)^2 + 2(x+1)^2 = 3(x+1)^2$

31. $y = 6x - 5,\ x \neq 0; \quad y' = 6,\ x \neq 0$

32. $y = x^2 + 2x - x^{-2}; \quad y' = 2x + 2 + 2x^{-3}$

33. $y = x^{-2}; \quad y' = -2x^{-3}$

When $x = 1/2$, the slope of the tangent line is $-2(1/2)^{-3}$ or -16. The point of tangency is $(1/2, y(1/2))$ or $(1/2, 4)$. Hence, an equation of the tangent line is $y - 4 = -16(x - 1/2)$ or $y = -16x + 12$.

34. $y = 4x - x^{-1}; \quad y' = 4 + x^{-2}$

When $x = -1$, the slope of the tangent line is $4 + (-1)^{-2}$ or 5. The point of tangency is $(-1, y(-1))$ or $(-1, -3)$. Hence, an equation of the tangent line is $y + 3 = 5(x+1)$ or $y = 5x + 2$.

35. $y' = (2x^2 - 4)(3x^2 + 5) + (x^3 + 5x + 3)\cdot 4x$

When $x = 0$, the slope of the tangent line is $-4(5) + 3(0)$ or -20. The point of tangency is $(0, y(0))$ or $(0, -12)$. Hence, an equation of the tangent line is $y + 12 = -20(x - 0)$ or $y = -20x - 12$.

Exercises 2.4

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#36 – 39

36. $y' = \dfrac{(x^2+1)\cdot 5 - (5x)\cdot 2x}{(x^2+1)^2} = \dfrac{-5x^2+5}{(x^2+1)^2}$

When $x = 2$, the slope of the tangent line is $\dfrac{-5(2)^2+5}{(2^2+1)^2}$ or $-\dfrac{3}{5}$. The point of tangency is $(2, y(2))$ or $(2, 2)$. Hence, an equation of the tangent line is $y - 2 = -\frac{3}{5}(x-2)$ or $y = -\frac{3}{5}x + \frac{16}{5}$.

37. $y' = (x^2-4)\cdot 2x + (x^2-6)\cdot 2x = 4x^3 - 20x$.

The tangent line is horizontal when $4x^3 - 20x = 4x(x^2-5) = 0$, or $x = 0, \pm\sqrt{5}$. Given $y = (x^2-4)(x^2-6)$ we see that for $x = 0$, $y = 24$, and for $x = \pm\sqrt{5}$, $y = -1$. Thus, the points on the graph are $(0, 24)$, $(\sqrt{5}, -1)$, and $(-\sqrt{5}, -1)$.

38. $y = x(x^2 - 2x + 1) = x^3 - 2x^2 + x$; $\quad y' = 3x^2 - 4x + 1 = (x-1)(3x-1)$

The tangent is horizontal when $(x-1)(3x-1) = 0$, or $x = 1, 1/3$. Given $y = x(x-1)^2$ we see that for $x = 1$, $y = 0$, and for $x = 1/3$, $y = 4/27$. Thus, the points on the graph are $(1, 0)$ and $(1/3, 4/27)$.

39. $y' = \dfrac{(x^4+1)\cdot 2x - x^2(4x^3)}{(x^4+1)^2} = \dfrac{2x - 2x^5}{(x^4+1)^2}$

The tangent is horizontal when $y' = 0$ or $2x - 2x^5 = 0$. Then $2x(1-x^4) = 0$ and $x = 0, \pm 1$. Given $y = \dfrac{x^2}{x^4+1}$, we see that for $x = 0$, $y = 0$, and for $x = \pm 1$, $y = 1/2$. Thus, the points on the graph are $(0, 0)$, $(1, 1/2)$, and $(-1, 1/2)$.

40. $y' = \dfrac{(x^2-6x)\cdot 0 - 1\cdot(2x-6)}{(x^2-6x)^2} = \dfrac{-2x+6}{(x^2-6x)^2}$

The tangent is horizontal when $y' = 0$, so $-2x + 6 = 0$ and $x = 3$. Given $y = 1$ ... for $x = 3$, $y = -1/9$. Thus, the point on the graph is $(3, -1/9)$.

... Since the slope of the tangent line i...