

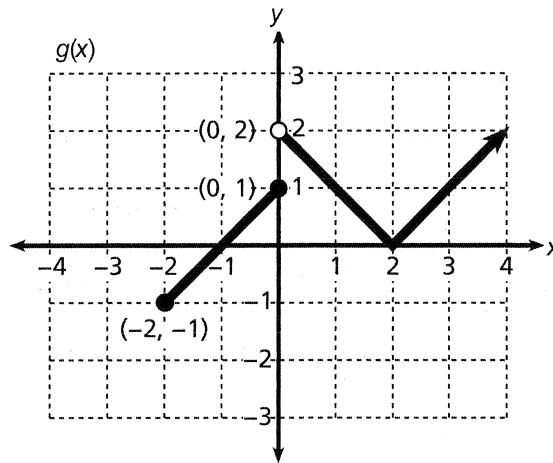
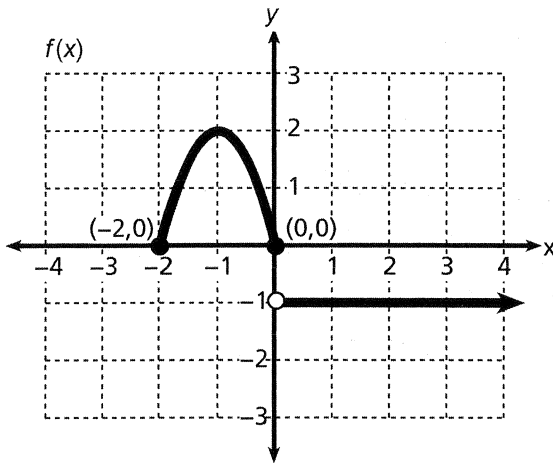
## LIMITS AND THEIR PROPERTIES

Work through the FRQ on a separate sheet of paper and show all your work for the MC questions.

### FREE-RESPONSE QUESTION

A calculator may not be used for this question.

1. Use the graphs of  $f(x)$  and  $g(x)$  given below to answer the following questions:



- Is  $f[g(x)]$  continuous at  $x = 0$ ? Explain why or why not.
- Is  $g[f(x)]$  continuous at  $x = 0$ ? Explain why or why not.
- What is  $\lim_{x \rightarrow \infty} f[g(x)]$ ? Explain your reasoning.
- If  $h(x) = \begin{cases} f(x) + g(x), & -2 \leq x < 0 \\ k \cdot g(x)f(x), & x \geq 0 \end{cases}$ , what is  $k$  so that  $h(x)$  is continuous at  $x = 0$ ?

### MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

1. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{2 - x}$ .

- 5
- 3
- 3
- 5
- The limit does not exist.

2. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 9} \frac{\sqrt{x-5} - 2}{x-9}$ .

- $\frac{1}{4}$
- $-\frac{1}{4}$
- 1
- 0
- The limit does not exist.

3. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$ .

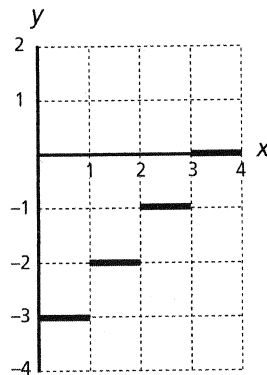
- (A)  $\frac{1}{4}$
- (B)  $-\frac{1}{4}$
- (C) 1
- (D) -1
- (E) The limit does not exist.

4. Evaluate the limit, if it exists:  $\lim_{x \rightarrow 1} \frac{\tan^{-1} x}{\sin^{-1} x + 1}$ .

- (A) 0
- (B)  $\frac{1}{4}$
- (C)  $\frac{1}{2}$
- (D)  $\frac{\pi}{2}$
- (E)  $\frac{\pi}{2\pi + 4}$

5. Estimate the limit, if it exists:  $\lim_{x \rightarrow 3} f(x)$  where  $f(x)$  is represented by the given graph:

- (A) 0
- (B) -1
- (C) 3
- (D) 1
- (E) The limit does not exist.



6. Given the function

$$f(x) = \begin{cases} \sin 2x, & x \leq \pi \\ 2x + k, & x > \pi \end{cases}$$

what value of  $k$  will make this piecewise function continuous?

- (A)  $-2\pi$
- (B)  $-\pi$
- (C) 0
- (D)  $\pi$
- (E)  $2\pi$

7. Find the limit, if it exists:  $\lim_{x \rightarrow 0} x \left( e^x + \frac{1}{x} \right)$ .

- (A) 0
- (B) 1
- (C) 2
- (D) The limit does not exist.
- (E) None of these

8. Identify the vertical asymptotes for  $f(x) = \frac{x^2 + 3x - 4}{x^2 + x - 2}$ .

- (A)  $x = -2, x = 1$
- (B)  $x = -2$
- (C)  $x = 1$
- (D)  $y = -2, y = 1$
- (E)  $y = -2$

9. If  $p(x)$  is a continuous function on the closed interval  $[1, 3]$ , with  $p(1) \leq K \leq p(3)$  and  $c$  is in the closed interval  $[1, 3]$ , then which of the following statements must be true?

(A)  $p(c) = \frac{p(3) + p(1)}{2}$

(B)  $p(c) = \frac{p(3) - p(1)}{2}$

(C) There is at least one value  $c$  such that  $p(c) = K$ .

(D) There is only one value  $c$  such that  $p(c) = K$ .

(E)  $c = 2$

10. How many vertical asymptotes exist for the function

$$f(x) = \frac{1}{2 \sin^2 x - \sin x - 1} \text{ in the open interval } 0 < x < 2\pi?$$

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

$x$	1	2	3	4
$f(x)$	4	2	3	1
$g(x)$	2	3	1	4

11. Selected values for continuous functions  $f(x)$  and  $g(x)$  are given in

the table above.  $\lim_{x \rightarrow 3} \frac{f(g(x))}{g(f(x))} =$

(A)  $\frac{1}{4}$

(B)  $\frac{1}{3}$

(C) 1

(D) 3

(E) 4

12.  $\lim_{x \rightarrow \infty} \frac{\sin x}{e^x + \cos x} =$

(A)  $-1$

(B) 0

(C)  $\frac{1}{e}$

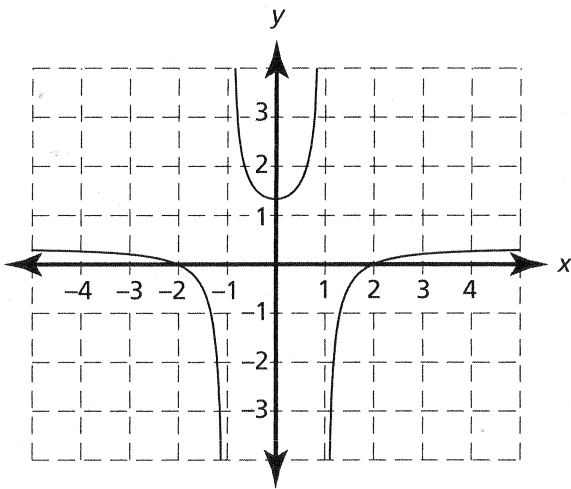
(D) 1

(E) The limit does not exist.

13. For what value of  $k$  is the function  $f(x) = \begin{cases} \frac{2x^2 + 5x - 3}{x^2 - 9}, & x \neq -3 \\ k, & x = -3 \end{cases}$

continuous at  $x = -3$ ?

- (A)  $-\frac{7}{6}$   
 (B)  $-\frac{5}{6}$   
 (C) 0  
 (D)  $\frac{5}{6}$   
 (E)  $\frac{7}{6}$



14. The function  $g(x)$  is shown in the graph above and is of the form

$g(x) = \frac{x^2 + a}{bx^2 - 3}$ . Which of the following could be the values of the constants  $a$  and  $b$ ?

- (A)  $a = -2, b = -1$   
 (B)  $a = -2, b = -3$   
 (C)  $a = -4, b = 3$   
 (D)  $a = -4, b = -3$   
 (E)  $a = 4, b = 3$
15. Which of the following statements is true?
- (A)  $\lim_{x \rightarrow 3} \log_3 x = 2$   
 (B)  $\lim_{x \rightarrow 0^+} \log_3 x$  does not exist.  
 (C)  $\lim_{x \rightarrow -\infty} e^x$  does not exist.  
 (D)  $\lim_{x \rightarrow -\frac{\pi}{2}} \csc x = 1$   
 (E)  $\lim_{x \rightarrow 1} e^{x-1} = 0$