26. $\lim_{x \to -3^-} \frac{x+2}{x+3} = \infty$

as $x \to -3^-$, $(x+2) \to -1$
as $x \to -3^-$, $(x+3) \to 0^-$

28 $\lim_{x \to 0} \frac{x-1}{x^2(x+2)} = -\infty$

as $x \to 0$, $(x-1) \to -1$
as $x \to 0$, $x^2 \to 0^+$
as $x \to 0$, $(x+2) \to 2$

30. $\lim_{x \to \pi^-} \cot(x) = \lim_{x \to \pi^-} \frac{\cos(x)}{\sin(x)} = -\infty$

as $x \to \pi^-$, $\cos(x) \to -1$
as $x \to \pi^-$, $\sin(x) \to 0^+$

32. $\lim_{x \to 2^-} \frac{x^2-2x}{x^2-4x+4} = \lim_{x \to 2^-} \frac{x(x-2)}{(x-2)^2}$

$= \lim_{x \to 2^-} \frac{x}{x-2}$

Note: We can cancel $(x-2)$ since $x$ is approaching 2 and not equal to 2

as $x \to 2^-$, $x \to 2$
as $x \to 2^-$, $(x-2) \to 0^-$

$\lim_{x \to 2^-} \frac{x^2-2x}{x^2-4x+4} = -\infty$

34. ⓐ $f(x) = \dfrac{x^2+1}{3x-2x^2} = \dfrac{x^2+1}{x(3-2x)}$

to find VA find where denominator equals 0

$3x - 2x^2 = 0$
$x(3-2x) = 0$
$x = 0 \quad x = \frac{3}{2}$

need limits to justify these are VA

$$\lim_{x \to 0^-} \frac{x^2+1}{x(3-2x)} = -\infty$$

as $x \to 0^-$, $(x^2+1) \to 1$
as $x \to 0^-$, $x \to 0^-$
as $x \to 0^-$, $(3-2x) \to 3$

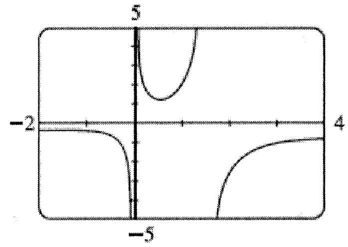
$\therefore$ $x=0$ is a VA since $\lim\limits_{x \to 0^-} f(x) = -\infty$

$$\lim_{x \to \frac{3}{2}^-} \frac{x^2+1}{x(3-2x)} = \infty$$

as $x \to \frac{3}{2}^-$, $(x^2+1) \to \frac{13}{4}$
as $x \to \frac{3}{2}^-$, $x \to \frac{3}{2}$
as $x \to \frac{3}{2}^-$, $(3-2x) \to 0^+$

$\therefore$ $x = \frac{3}{2}$ is a VA since $\lim\limits_{x \to \frac{3}{2}^-} f(x) = \infty$

(b)



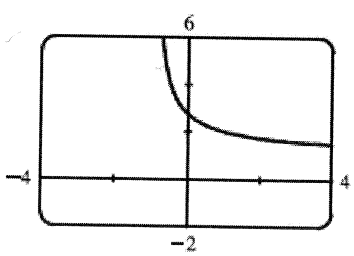

35. ⓐ $\lim_{x \to 0} (1+x)^{\frac{1}{x}}$ let $f(x) = (1+x)^{\frac{1}{x}}$

| $x$ | $f(x)$ |
|---|---|
| 0.1 | $f(0.1) = 2.59374246$ |
| 0.01 | $f(0.01) \approx 2.704813829$ |
| 0.001 | $f(0.001) \approx 2.716923932$ |
| 0.0001 | $f(0.0001) \approx 2.718145927$ |
| 0.00001 | $f(0.00001) \approx 2.718268237$ |
| 0.000001 | $f(0.000001) \approx 2.718280469$ |

| $x$ | $f(x)$ |
|---|---|
| −0.1 | $f(-0.1) \approx 2.867971991$ |
| −0.01 | $f(-0.01) \approx 2.731999026$ |
| −0.001 | $f(-0.001) \approx 2.719642216$ |
| −0.0001 | $f(-0.0001) \approx 2.718417755$ |
| −0.00001 | $f(-0.00001) \approx 2.71829542$ |
| −0.000001 | $f(-0.000001) \approx 2.718283188$ |

$\lim_{x \to 0} (1+x)^{\frac{1}{x}} \approx 2.71828$

ⓑ (b)



1. (a) $\lim_{x \to 2} (x+8) = 10$

(b) $\lim_{x \to 4} (2x+6) = 14$

(c) as $x - 2$, $x^3 \to 8$

(d) $\lim_{x \to 3} 7 = 7$

2. (a) $\lim_{x \to -5} f(x) = 4$

(b) $\lim_{x \to 2} f(x) = 3$

(c) $\lim_{x \to 0^-} f(x) = 0$ and $\lim_{x \to 0^+} = \infty$

so $\lim_{x \to 0} f(x)$ DNE since $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

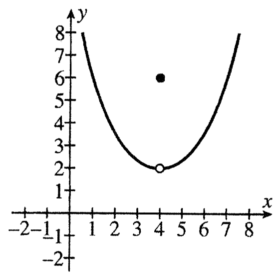
3. $\lim_{x \to 2} |x-5|$ let $f(x) = |x-5|$

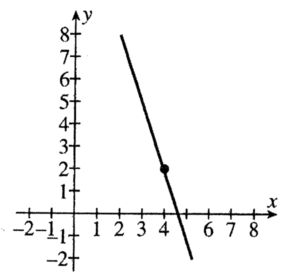
| x | f(x) |
|---|---|
| 1.9 | $f(1.9) = 3.1$ |
| 1.99 | $f(1.99) = 3.01$ |
| 1.999 | $f(1.999) = 3.001$ |

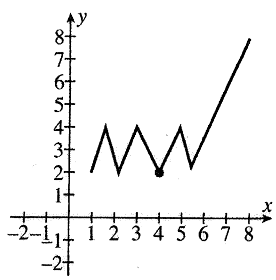
| x | f(x) |
|---|---|
| 2.1 | $f(2.1) = 2.9$ |
| 2.01 | $f(2.01) = 2.99$ |
| 2.001 | $f(2.001) = 2.999$ |

$\lim_{x \to 2} |x-5| = 3$

4.




5. $\lim_{x \to 1} \frac{1}{1-x}$ let $f(x) = \frac{1}{1-x}$

| x | f(x) |
|---|---|
| 0.9 | $f(0.9) = 10$ |
| 0.99 | $f(0.99) = 100$ |
| 0.999 | $f(0.999) = 1000$ |

| x | f(x) |
|---|---|
| 1.1 | $f(1.1) = -10$ |
| 1.01 | $f(1.01) = -100$ |
| 1.001 | $f(1.001) = -1000$ |

$\lim_{x \to 1^-} f(x) = \infty$ and $\lim_{x \to 1^+} f(x) = \infty$

so $\lim_{x \to 1} f(x)$ DNE since $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

6. (a) $\lim_{x \to -2^-} f(x) = 1$

(b) $\lim_{x \to -2^+} f(x) = 0$

(c) $\lim_{x \to -2^-} f(x) = 1$ and $\lim_{x \to -2^+} f(x) = 0$

so $\lim_{x \to -2} f(x)$ DNE since $\lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$

(d) $\lim_{x \to 2^+} f(x) = 3$

(e) $\lim_{x \to 2^-} f(x) = 0$

(f) $\lim_{x \to 2^+} f(x) = 3$ and $\lim_{x \to 2^-} f(x) = 0$

so $\lim_{x \to 2} f(x)$ DNE since $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

(g) $\lim_{x \to 0^-} f(x) = 1$

(h) $\lim_{x \to 0^+} f(x) = 1$

(i) $\lim_{x \to 0} f(x) = 1$

10. ALWAYS

12. (a) $\lim_{x \to -2^+} f(x) = \infty$

(b) $\lim_{x \to -2^-} f(x) = \infty$

(c) $\lim_{x \to -2} f(x) = \infty$

(d) $\lim_{x \to 3^-} f(x) = \infty$

(e) $\lim_{x \to 3^+} f(x) = 0$

(f) $\lim_{x \to 3^-} f(x) = \infty$ and $\lim_{x \to 3^+} f(x) = 0$

so $\lim_{x \to 3} f(x)$ DNE since $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$

(g) $\lim_{x \to 0^+} f(x) = \infty$

(h) $\lim_{x \to 0^-} f(x) = -\infty$

(i) $\lim_{x \to 0^+} f(x) = \infty$ and $\lim_{x \to 0^-} f(x) = -\infty$

so $\lim_{x \to 0} f(x)$ DNE since $\lim_{x \to 0^+} f(x) \neq \lim_{x \to 0^-} f(x)$

14. $\lim_{x \to 0^+} \csc(x) = \lim_{x \to 0^+} \frac{1}{\sin(x)} = \infty$

as $x \to 0^+$, $\sin(x) \to 0^+$