

Study Guide

for

AP Calculus (AB) 1st Semester Final Exam 2010/2011

Mr. Wissa

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit.

1) $\lim_{x \rightarrow 0} (1 - \cot x)$ 1) _____

- A) ∞ B) 0 C) $-\infty$ D) Does not exist

2) $\lim_{x \rightarrow -2} \frac{1}{x+2}$ 2) _____

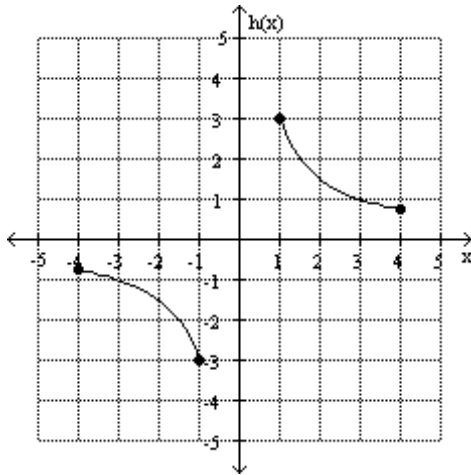
- A) ∞ B) Does not exist C) $1/2$ D) $-\infty$

3) $\lim_{x \rightarrow (-2)^+} \frac{1}{x+2}$ 3) _____

- A) $1/2$ B) $-1/2$ C) ∞ D) $-\infty$

Find the location of the indicated extremum for the function.

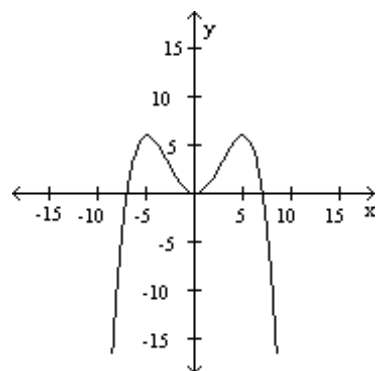
4) Maximum 4) _____



- A) No maximum B) $x = 1$ C) $x = -4$ D) $x = 4$

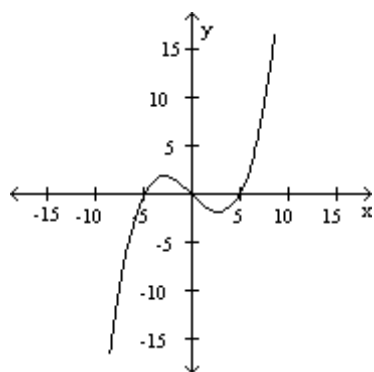
The graph of a function is given. Choose the answer that represents the graph of its derivative.

5)

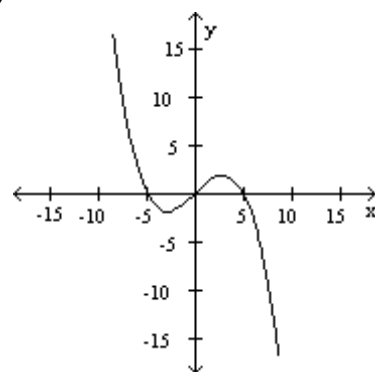


5) _____

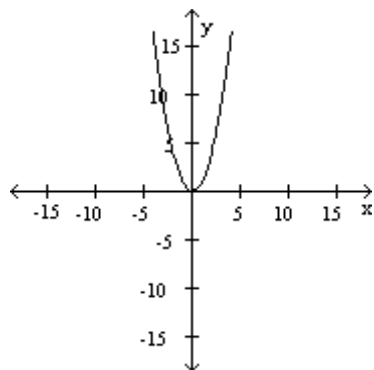
A)



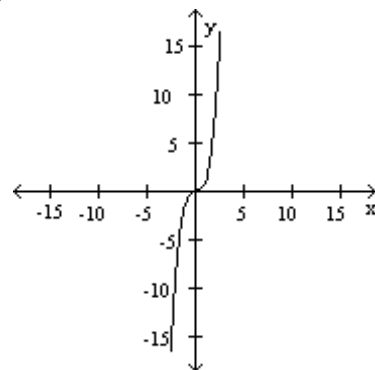
B)



C)



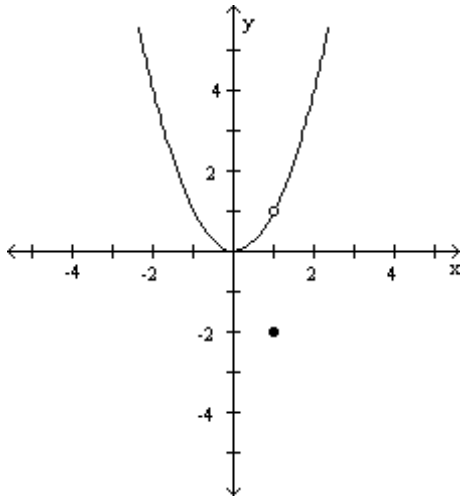
D)



The figure shows the graph of a function. At the given value of x , does the function appear to be differentiable, continuous but not differentiable, or neither continuous nor differentiable?

6) $x = 1$

6) _____



- A) Differentiable
- B) Continuous but not differentiable
- C) Neither continuous nor differentiable

Find all points where the function is discontinuous.

$$7) f(x) = \begin{cases} 0, & x < 0 \\ x^2 - 7x, & 0 \leq x \leq 7 \\ 7, & x > 7 \end{cases}$$

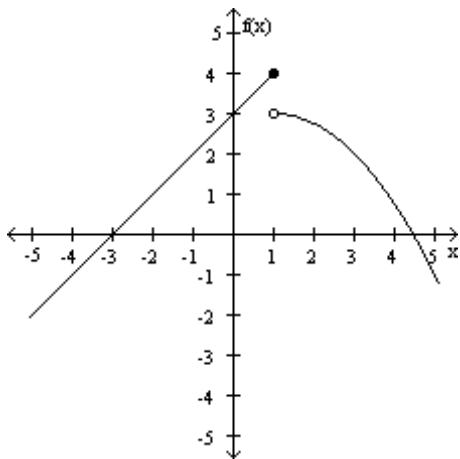
7) _____

- A) $x = 7$
- B) $x = 0$ and $x = 7$
- C) $x = 0$
- D) Nowhere

Determine the limit graphically, if it exists.

8) $\lim_{x \rightarrow 1^+} f(x)$

8) _____



- A) $3\frac{1}{2}$
- B) Does not exist
- C) 4
- D) 3

Find a value for a so that the function f(x) is continuous.

$$9) f(x) = \begin{cases} x^2 - 1, & x < 5 \\ 3ax, & x \geq 5 \end{cases}$$

9) _____

A) $a = \frac{5}{3}$

B) $a = 6$

C) $a = \frac{8}{5}$

D) $a = 24$

Find all possible functions with the given derivative.

10) $f'(x) = 6 \cos 6x$

10) _____

A) $\sin 6x + C$

B) $\sin x + C$

C) $\cos 6x + C$

D) $\cos x + C$

Find dy/dx.

11) $f(x) = e^{6x}$

11) _____

A) e^{6x}

B) $\frac{1}{6}e^{6x}$

C) $6e^x$

D) $6e^{6x}$

12) $y = 8\cos x$

12) _____

A) $8\cos x \ln 8$

B) $8\cos x \ln 8 \sin x$

C) $-8\cos x \ln 8 \sin x$

D) $8\cos x$

Find the derivative at each critical point and determine the local extreme values.

13) $y = \begin{cases} 3 - x, & x < 0 \\ 3 + 2x - x^2, & x \geq 0 \end{cases}$

13) _____

A)

Critical Pt.	Derivative	Extremum	Value
$x = 0$	undefined	local min	-3
$x = 1$	0	local max	2

B)

Critical Pt.	Derivative	Extremum	Value
$x = 3$	undefined	local min	3
$x = 0$	0	local max	4

C)

Critical Pt.	Derivative	Extremum	Value
$x = 0$	undefined	local min	3
$x = 1$	0	local max	4

D)

Critical Pt.	Derivative	Extremum	Value
$x = 0$	undefined	local min	3
$x = 2$	0	local max	7

Find the extreme values of the function on the interval and where they occur.

14) $y = 8 - 7x^2$ on $[-2, 3]$

14) _____

A) Maximum at (0, 56); minimum at (-2, -20)

B) Maximum at (0, 16); minimum at (3, -20)

C) Maximum at (0, 8); minimum at (3, -55)

D) Maximum at (0, 7); minimum at (3, -71)

Find the indicated limit.

15) $\lim_{x \rightarrow -\infty} \frac{\sin x}{x}$

15) _____

A) ∞

B) 0

C) 1

D) Does not exist

Find the limit of $f(x)$ as (a) $x \rightarrow -\infty$, (b) $x \rightarrow \infty$, (c) $x \rightarrow 0^-$, and (d) $x \rightarrow 0^+$.

$$16) f(x) = \begin{cases} \frac{5}{x} & x < 0 \\ -5, & x \geq 0 \end{cases} \quad 16) \underline{\hspace{2cm}}$$

- A) (a) -5
(b) 0
(c) -5
(d) $-\infty$

- B) (a) 0
(b) 5
(c) ∞
(d) 5

- C) (a) 0
(b) 0
(c) -5
(d) -5

- D) (a) 0
(b) -5
(c) $-\infty$
(d) -5

Find the limit, if it exists.

$$17) \lim_{x \rightarrow -\infty} \frac{4x^3 + 2x^2}{x - 6x^2} \quad 17) \underline{\hspace{2cm}}$$

- A) ∞

- B) 4

- C) $-\infty$

- D) $-\frac{1}{3}$

If the function is not differentiable at the given value of x , tell whether the problem is a corner, cusp, vertical tangent, or a discontinuity.

$$18) y = 4|x| + 6, \text{ at } x = 0 \quad 18) \underline{\hspace{2cm}}$$

- A) cusp

- B) vertical tangent

- C) corner

- D) function is differentiable at $x = 0$

Find the derivative of the given function.

$$19) y = 2 \sin^{-1}(4x^4) \quad 19) \underline{\hspace{2cm}}$$

- A) $\frac{32x^3}{\sqrt{1-16x^4}}$

- B) $\frac{32x^3}{\sqrt{1-16x^8}}$

- C) $\frac{32x^3}{1-16x^8}$

- D) $\frac{2}{\sqrt{1-16x^8}}$

Find the equation of the normal line to the indicated curve at the given point.

$$20) y = 5x^2 \text{ at } (-3, 45) \quad 20) \underline{\hspace{2cm}}$$

- A) $x - 30y + 1353 = 0$

- B) $x - 30y + 1347 = 0$

- C) $x + 30y - 1347 = 0$

- D) $x - 30y - 1347 = 0$

Find the function with the given derivative whose graph passes through the point P.

$$21) f'(x) = x^2 + 4, \text{ P}(3, 40) \quad 21) \underline{\hspace{2cm}}$$

- A) $f(x) = \frac{x^3}{3} + 4x + 19$

- B) $f(x) = x^3 + 4x + 1$

- C) $f(x) = x^3 + 4x^2 + 19$

- D) $f(x) = \frac{x^3}{3} + 4x$

Find the intervals on which the function is continuous.

$$22) y = \sqrt{5x+2} \quad 22) \underline{\hspace{2cm}}$$

- A) $\left[\frac{2}{5}, \infty\right)$

- B) $\left[-\frac{2}{5}, \infty\right)$

- C) $\left[-\infty, -\frac{2}{5}\right]$

- D) $\left[-\frac{2}{5}, \infty\right)$

Give an appropriate answer.

- 23) Find the value or values of c that satisfy $\frac{f(b) - f(a)}{b - a} = f'(c)$ for the function $f(x) = x^2 + 4x + 2$ on the interval $[-3, 2]$. 23) _____
- A) $-3, 2$ B) $-\frac{1}{2}$ C) $-\frac{1}{2}, \frac{1}{2}$ D) $0, -\frac{1}{2}$

Solve the problem.

- 24) Assume that a watermelon dropped from a tall building falls $y = 16t^2$ ft in t sec. Find the watermelon's average speed during the first 6 sec of fall. 24) _____
- A) 97 ft/sec B) 192 ft/sec C) 96 ft/sec D) 48 ft/sec
- 25) Find the points where the graph of the function has horizontal tangents. 25) _____
- $f(x) = 4x^2 + 3x + 2$
- A) $\left(\frac{3}{8}, -\frac{13}{8}\right)$ B) $(-11, 673)$ C) $(0, -2)$ D) $\left(-\frac{3}{8}, \frac{23}{16}\right)$
- 26) Given the distance function $s(t) = t^2 + 7t + 20$, where s is in feet and t is in seconds, find the velocity function, $v(t)$, and the acceleration function, $a(t)$. 26) _____
- A) $v(t) = 2t + 27; a(t) = 2$ B) $v(t) = 2t + 7; a(t) = 2t$
C) $v(t) = 2t + 7; a(t) = 2$ D) $v(t) = 2t + 7; a(t) = 0$

Suppose u and v are differentiable functions of x . Use the given values of the functions and their derivatives to find the value of the indicated derivative.

- 27) $u(1) = 4, u'(1) = -6, v(1) = 6, v'(1) = -2$. 27) _____
- $\frac{d}{dx}(uv)$ at $x = 1$
- A) 28 B) 44 C) -44 D) -36
- 28) $u(1) = 2, u'(1) = -7, v(1) = 6, v'(1) = -4$. 28) _____
- $\frac{d}{dx}\left(\frac{u}{v}\right)$ at $x = 1$
- A) $-\frac{17}{18}$ B) $-\frac{17}{8}$ C) $-\frac{25}{18}$ D) $-\frac{17}{3}$

Use analytic methods to find those values of x for which the given function is increasing and those values of x for which it is decreasing.

- 29) $f(x) = x^4 - 2$ 29) _____
- A) Increasing on $(-\infty, -1)$ and $(0, 1)$, decreasing on $(-1, 0)$ and $(1, \infty)$
B) Increasing on $(-1, 0)$ and $(1, \infty)$, decreasing on $(-\infty, -1)$ and $(0, 1)$
C) Increasing on $(-1, 1)$, decreasing on $(-\infty, -1)$ and $(1, \infty)$,
D) Increasing on $(-\infty, -1)$ and $(1, \infty)$, decreasing on $(-1, 1)$

Use the given substitution and the Chain Rule to find dy/dx .

- 30) $y = u^6$; $u = \cos x$ 30) _____
 A) $-6x^5 \sin(x^6)$ B) $-\sin(x^6)$ C) $-6 \cos^5 x \sin x$ D) $6 \cos^5 x \sin x$

Determine the limit algebraically, if it exists.

- 31) $\lim_{x \rightarrow 2} \sqrt{x-3}$ 31) _____
 A) 0 B) Does not exist C) -1 D) 1

Evaluate or determine that the limit does not exist for each of the limits (a) $\lim_{x \rightarrow d^-} f(x)$, (b) $\lim_{x \rightarrow d^+} f(x)$, and (c) $\lim_{x \rightarrow d} f(x)$

for the given function f and number d .

- 32) 32) _____

$$f(x) = \begin{cases} -4x - 2, & \text{for } x < 1, \\ 1, & \text{for } x = 1, \\ -5x + 9, & \text{for } x > 1 \end{cases}$$

$d = 1$

- | | |
|--------------------|--------------------|
| A) (a) 4 | B) (a) 4 |
| (b) -6 | (b) -6 |
| (c) -2 | (c) Does not exist |
| C) (a) -6 | D) (a) -6 |
| (b) 4 | (b) 4 |
| (c) Does not exist | (c) -2 |

Find a simple basic function as a right-end behavior model and a simple basic function as a left-end behavior model.

- 33) $y = \frac{x^2}{4} + e^x$ 33) _____
 A) $y = e^x$; $y = \frac{x^2}{4}$ B) $y = e^x$; $y = e^{-x}$ C) $y = e^{-x}$; $y = x^2$ D) $y = e^x$; $y = e^x$

Find the indicated derivative.

- 34) Find y'' if $y = 6x \sin x$. 34) _____
 A) $y'' = -6x \sin x$ B) $y'' = -12 \cos x + 6x \sin x$
 C) $y'' = 6 \cos x - 12x \sin x$ D) $y'' = 12 \cos x - 6x \sin x$

Find the slope of the line tangent to the curve at the given value of x .

- 35) $f(x) = 2x^2 + 4x$; $x = 4$ 35) _____
 A) -48 B) 20 C) 26 D) 32

Solve the problem.

- 36) The profit in dollars from the sale of x thousand compact disc players is $P(x) = x^3 - 7x^2 + 6x + 7$. 36) _____
 Find the marginal profit when the value of x is 6.
 A) \$18 B) \$30 C) \$25 D) \$37

- 37) The position of a particle moving along a coordinate line is $s = \sqrt{4 + 12t}$, with s in meters and t in seconds. Find the particle's velocity at $t = 1$ sec. 37) _____
- A) $-\frac{1}{4}$ m/sec B) $\frac{3}{2}$ m/sec C) 3 m/sec D) $\frac{1}{8}$ m/sec

- 38) A ladder is slipping down a vertical wall. If the ladder is 13 ft long and the top of it is slipping at the constant rate of 2 ft/s, how fast is the bottom of the ladder moving along the ground when the bottom is 5 ft from the wall? 38) _____
- A) 4.8 ft/s B) 0.40 ft/s C) 2.4 ft/s D) 5.2 ft/s

Use logarithmic differentiation to find dy/dx .

- 39) $y = 98^x$ 39) _____
- A) $8 (\ln 9) 98^x$ B) $72 (\ln 8) 98^x$ C) $9 (\ln 8) 98^x$ D) $72 (\ln 9) 98^x$

Find dy/dx by implicit differentiation. If applicable, express the result in terms of x and y .

- 40) $7y^2 + 9x^2 - 5 = 0$ 40) _____
- A) $\frac{-9x^2}{14y}$ B) $\frac{-9x}{7y}$ C) $\frac{-9x}{7}$ D) $\frac{-18x + 5}{14y}$

Find dy/dx .

- 41) $y = (3x^3 + 8)(5x^7 - 9)$ 41) _____
- A) $12x^9 + 280x^6 - 81x^2$ B) $12x^9 + 280x^6 - 81x$
 C) $150x^9 + 280x^6 - 81x^2$ D) $150x^9 + 280x^6 - 81x$

- 42) $y = \frac{x + 8}{x - 8}$ 42) _____
- A) $\frac{2}{x - 8}$ B) $\frac{-16}{(x - 8)^2}$ C) $\frac{-8}{(x - 8)^2}$ D) $\frac{-16}{(x + 8)^2}$

- 43) $s = t^4 \tan t$ 43) _____
- A) $4t^3 \sec^2 t$ B) $-t^4 \sec^2 t + 4t^3 \tan t$
 C) $t^4 \sec^2 t + 4t^3 \tan t$ D) $t^4 \sec t \tan t + 4t^3 \tan t$

- 44) $y = \frac{6}{\sin x}$ 44) _____
- A) $6 \csc x \cot x - \sec^2 x$ B) $-6 \csc x \cot x$
 C) $6 \csc x \cot x$ D) $6 \cos x$

Find the value of df^{-1}/dx at $x = f(a)$.

- 45) $f(x) = 2x + 8, a = 1$ 45) _____
- A) 8 B) $\frac{1}{8}$ C) $\frac{1}{2}$ D) 2

Solve the problem.

46) If $y = x^2 - 2$, find an equation of the tangent line to the graph of y at $x = -4$. 46) _____

- A) $y = -4x - 18$ B) $y = -8x - 34$ C) $y = -8x - 36$ D) $y = -8x - 18$

47) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost: 47) _____

$$R(x) = 6x$$

$$C(x) = 0.001x^2 + 0.9x + 10.$$

- A) 3450 units B) 2550 units C) 5100 units D) 6900 units

48) An architect needs to design a rectangular room with an area of 89 ft^2 . What dimensions should he use in order to minimize the perimeter? Round to the nearest hundredth, if necessary. 48) _____

- A) $22.25 \text{ ft} \times 22.25 \text{ ft}$ B) $17.8 \text{ ft} \times 89 \text{ ft}$
C) $9.43 \text{ ft} \times 22.25 \text{ ft}$ D) $9.43 \text{ ft} \times 9.43 \text{ ft}$

Find dy/dx .

49) $y = \ln 9x$ 49) _____

- A) $\frac{1}{x}$ B) $\frac{1}{9x}$ C) $-\frac{1}{9x}$ D) $-\frac{1}{x}$

50) $y = \log(4x - 2)$ 50) _____

- A) $\frac{4x - 2}{4 \ln 10}$ B) $\frac{4}{\ln 10}$ C) $\frac{4}{(4x - 2) \ln 10}$ D) $\frac{1}{(4x - 2) \ln 10}$