

## Lesson 2 - 4

IF AND ONLY IF

IFF

# Biconditional Statements and Definitions

## Going Deeper

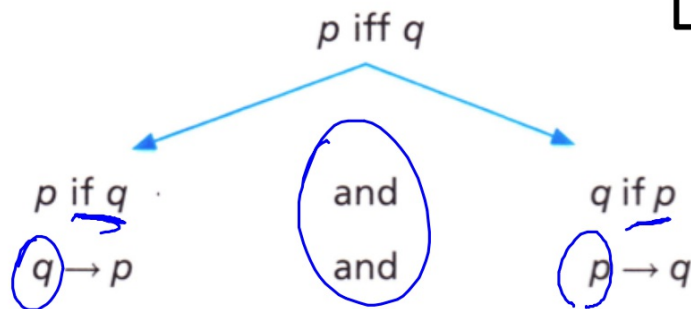
**Essential question:** *How can you analyze the truth of a biconditional statement?*

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When you combine a conditional statement and its converse, you create a biconditional statement.

A biconditional statement is a statement that can be written in the form " $p$  if and only if  $q$ ." This means "if  $p$ , then  $q$ " and "if  $q$ , then  $p$ ."

You have seen *If p, then q* and the notation  $p \rightarrow q$ . The notation **iff** is shorthand for **if and only if** and means a pair of statements.



The conjunction of these two statements is a biconditional statement. It is true only if both parts are true. It is false if either part is false.

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$p \leftrightarrow q$  means  $p \rightarrow q$  and  $q \rightarrow p$



### Writing Math

The biconditional "p if and only if q" can also be written as "p iff q" or  $p \leftrightarrow q$ .

CC.MP.3

**EXAMPLE**

**Analyzing Biconditionals and Definitions**

A student defined *square* this way:

A polygon is a square if and only if it has four right angles.

**A** Write the biconditional as a pair of conditional statements.

✓ If a polygon is a sq, then the polygon has four right angles.

If a polygon has 4 r, then the polygon is a square.

False  
rect


**Write the conditional statement and converse within the biconditional.**

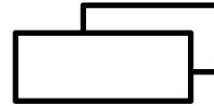
**A solution is neutral ↔ its pH is 7.**

✓ If a solution is neutral, then its pH is 7

✓ If a solution's pH is 7, then it's neutral



For a biconditional statement to be true,  both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.



**REFLECT**

- 1a.** Why is it necessary to prove only one of the two parts of a biconditional is false to conclude that the biconditional is false?

only need one to make entire Bicon. false  
false

Determine if the biconditional is true. If false, give a counterexample.

A rectangle has side lengths of 12 cm and 25 cm if and only if its area is  $300 \text{ cm}^2$ .



Conditional: If a rectangle has side lengths of 12 cm and 25 cm, then its area is  $300 \text{ cm}^2$ . *The conditional is true.*

Converse: If a rectangle's area is  $300 \text{ cm}^2$ , then it has side lengths of 12 cm and 25 cm. *The converse is false.*

If a rectangle's area is  $300 \text{ cm}^2$ , it could have side lengths of 10 cm and 30 cm. Because the



Determine if the biconditional is true. If false, give a counterexample.

$y = -5 \leftrightarrow y^2 = 25$

Conditional: If  $y = -5$ , then  $y^2 = 25$ .

*The conditional is true.*

Converse: If  $y^2 = 25$ , then  $y = -5$ .

*The converse is false.*

An angle is a right angle iff its measure is  $90^\circ$ .

Conditional: If an angle is a right angle, then its measure is  $90^\circ$ .

*The conditional is true.*

Converse: If the measure of an angle is  $90^\circ$ , then it is a right angle.

*The converse is true.*

### Example 4: Writing Definitions as Biconditional Statements

Write each definition as a biconditional.

A. A pentagon <sup>iff</sup> ~~is~~ a five-sided polygon.

A figure is a pentagon if and only if it is a 5-sided polygon.

B. A right angle measures  $90^\circ$ .

An angle is a right angle if and only if it measures  $90^\circ$ .

## Lesson Quiz

1. For the conditional "~~If~~ an angle is right, ~~then~~ its measure is  $90^\circ$ ," write the converse and a biconditional statement.

IFF



2. Determine if the biconditional "Two angles are complementary if and only if they are both acute" is true. If false, give a counterexample.

✓  $2\angle \rightarrow$  acute  
comp

(F) acute  $\rightarrow$  comp

3. Write the definition "An acute triangle is a triangle with three acute angles" as a biconditional.

$\angle 1 = 4^\circ$   
 $\angle 2 = 10^\circ$

A triangle is acute iff it has 3 acute  $\angle$ s.

## HW 2.4

AP (p69) and PS (p70)

Online Text p99 #10, 15, 16, 19

### PRACTICE AND PROBLEM SOLVING

Write the conditional statement and converse within each biconditional.

10. Three points are coplanar if and only if they lie in the same plane.

$P \rightarrow Q$

$Q \rightarrow P$

IF three points are coplanar, then they lie in the same plane

For each conditional, write the converse and a biconditional statement.

15. If a triangle contains a right angle, then it is a right triangle.

$Q \rightarrow P$

$P \text{ IFF } Q$

**Multi-Step** Determine if each biconditional is true. If false, give a counterexample.

16. Felipe is a swimmer if and only if he is an athlete.

Write each definition as a biconditional.

19. A catcher is a baseball player who is positioned behind home plate and who catches throws from the pitcher.  $\text{IFF}$

## Additional Practice

Write the conditional statement and converse within each biconditional.

1. The tea kettle is whistling if and only if the water is boiling.

Conditional: \_\_\_\_\_

Converse: \_\_\_\_\_

2. A biconditional is true if and only if the conditional and converse are both true.

Conditional: \_\_\_\_\_

Converse: \_\_\_\_\_

For each conditional, write the converse and a biconditional statement.

3. Conditional: If  $n$  is an odd number, then  $n - 1$  is divisible by 2.

Converse: \_\_\_\_\_

Biconditional: \_\_\_\_\_

4. Conditional: An angle is obtuse when it measures between  $90^\circ$  and  $180^\circ$ .

Converse: \_\_\_\_\_

Biconditional: \_\_\_\_\_

Determine whether a true biconditional can be written from each conditional statement. If not, give a counterexample.

5. If the lamp is unplugged, then the bulb does not shine.

\_\_\_\_\_

6. The date can be the 29th if and only if it is not February.

\_\_\_\_\_

Write each definition as a biconditional.

7. A cube is a three-dimensional solid with six square faces.

\_\_\_\_\_

\_\_\_\_\_

8. Tanya claims that the definition of *doofus* is "her younger brother."

\_\_\_\_\_

\_\_\_\_\_

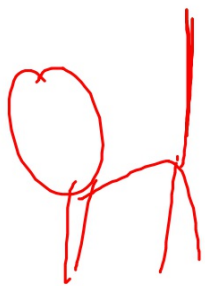
## Problem Solving

Use the table for Exercises 1–4. Determine if a true biconditional statement can be written from each conditional. If so, then write a biconditional. If not, then explain why not.

Mountain Bike Races	Characteristics
Cross-country	A massed-start race. Riders must carry their own tools to make repairs.
Downhill	Riders start at intervals. The rider with the lowest time wins.
Freeride	Courses contain cliffs, drops, and ramps. Scoring depends on the style and the time.
Marathon	A massed-start race that covers more than 250 kilometers.

false  
MS → XC

XC → MS



1. If a mountain bike race is mass-started, then it is a cross-country race.
2. If a mountain bike race is downhill, then time is a factor in who wins. (FF)

A mbrace is massstarted  
 iff it is a XC race

3. If a mountain bike race covers more than 250 kilometers, then it is a marathon race.
4. If a race course contains cliffs, drops, and ramps, then it is not a marathon race.

Choose the best answer.

5. The cat is the only species that can hold its tail vertically while it walks.
- A The converse of this statement is false.
  - y B The biconditional of this statement is false.
  - C The biconditional of this statement is true.
  - x D This statement cannot be written as a biconditional.
6. Which conditional statement can be used to write a true biconditional?
- F If you travel 2 miles in 4 minutes, then distance is a function of time.
  - G If the distance depends on the time, then distance is a function of time.
  - H If  $y$  increases as  $x$  increases, then  $y$  is a function of  $x$ .
  - J If  $y$  is not a function of  $x$ , then  $y$  does not increase as  $x$  increases.

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