4.2/4.3 Matrix Multiplication / Determinants

Matrix Multiplication: $A_{m \times n} \cdot B_{n \times p} = (AB)_{m \times p}$

c r

1

$$\begin{bmatrix} -1 & 5 \\ 5 & 2 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -4+30 & 3+40 \\ 20+12 & -15+16 \\ 0-24 & 0-32 \end{bmatrix} = \begin{bmatrix} 26 & 43 \\ 32 & 1 \\ -24 & -32 \end{bmatrix}$$

$3 \times 2 \quad 2 \times 2$

3×2

2

$$\begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 4 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 6 \\ -2 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ -2 & 7 \\ 0 & -15 \end{bmatrix}$$

$3 \times 3 \quad 3 \times 2$

3×2

3 $A = \begin{bmatrix} 2 & -2 \\ 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

$2A(B+C) =$ $\begin{bmatrix} 4 & -4 \\ 2 & 8 \end{bmatrix}$ $\begin{bmatrix} 0 & 4 \\ -1 & -3 \end{bmatrix}$ $=$ $\begin{bmatrix} 4 & 28 \\ -8 & -16 \end{bmatrix}$

2A (B+C)

2 x 2 2 x 2 2 x 2

A determinant is a real number associated with a square matrix.

For a 2 x 2 matrix:

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

4 Find the determinant of $\begin{bmatrix} -5 & 7 \\ -2 & 9 \end{bmatrix}$

$$= \begin{vmatrix} -5 & 7 \\ -2 & 9 \end{vmatrix} = -45 + 14 = -31$$

For a 3 x 3 matrix:

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

$= aei + bfg + cdh - gec - hfa - idb$

5 Find the determinant of $\begin{bmatrix} 1 & 3 & -5 \\ 0 & -2 & 4 \\ -7 & 8 & 9 \end{bmatrix}$

$= -64$

$$\begin{vmatrix} 1 & 3 & -5 \\ 0 & -2 & 4 \\ -7 & 8 & 9 \end{vmatrix} \begin{matrix} 1 & 3 \\ 0 & -2 \\ -7 & 8 \end{matrix}$$

$= -18 - 84 + 0 + 70 - 32 + 0$