

Slopes of Lines

Section 3.5

$$y = mx + b \quad \frac{\text{Rise}}{\text{run}}$$

Goals:

1. Find slope of a line
2. Use slopes to determine if two lines are parallel, perpendicular, or neither

Same slope

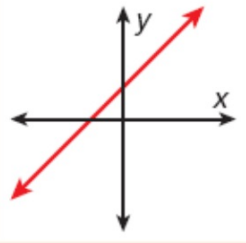
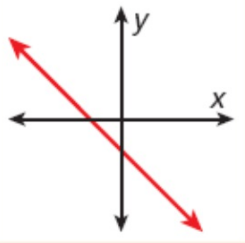
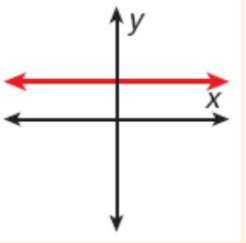
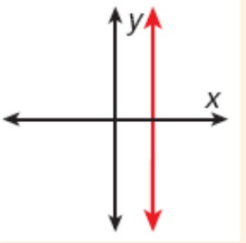
change sign (+/-)
flip it over (reciprocal)

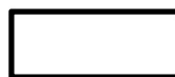






The **slope** of a line in a coordinate plane is a number that describes the **steepness** of the line. Any two points on a line can be used to determine the slope.

Summary: Slope of a Line			
Positive Slope	Negative Slope	Zero Slope	Undefined Slope
			



Slope of a Line

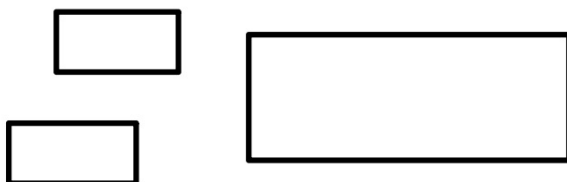
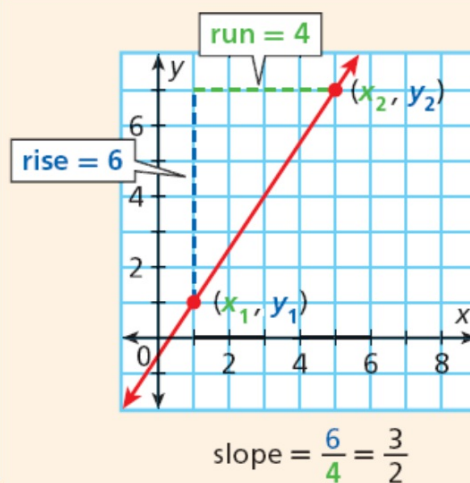
DEFINITION

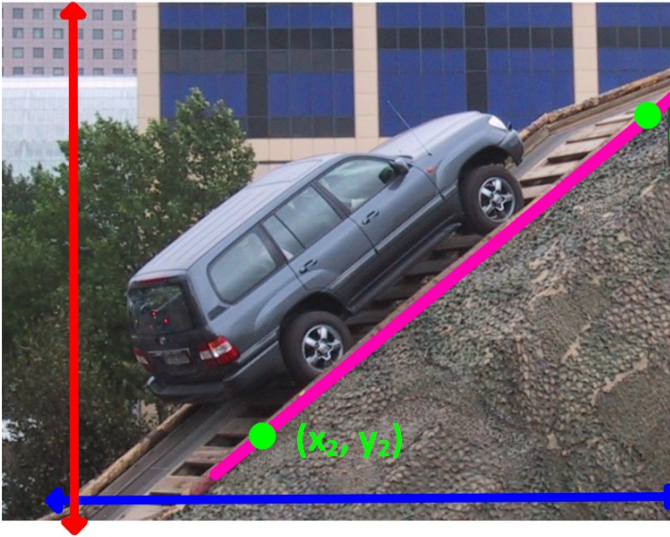
The **rise** is the difference in the y-values of two points on a line.

The **run** is the difference in the x-values of two points on a line.

The **slope** of a line is the ratio of the rise to run. If (x_1, y_1) and (x_2, y_2) are any two points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

EXAMPLE





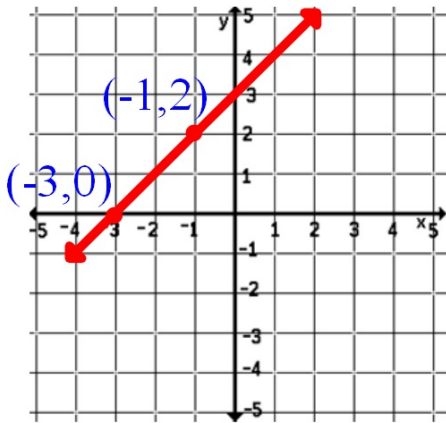
$$\text{Slope} = \frac{\text{Change in Y}}{\text{Change in X}}$$

$$= \frac{\text{Difference in Y}}{\text{Difference in X}}$$

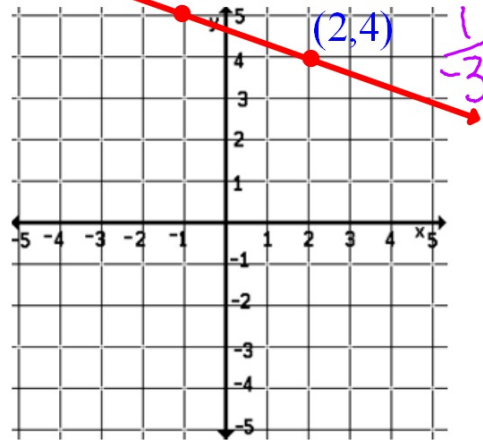
$$= \frac{(Y_2 - Y_1)}{(X_2 - X_1)} = \frac{-1}{3}$$

$$-\frac{1}{3} = -\frac{1}{3}$$

Find the slope of the line: $-\frac{1}{3}$

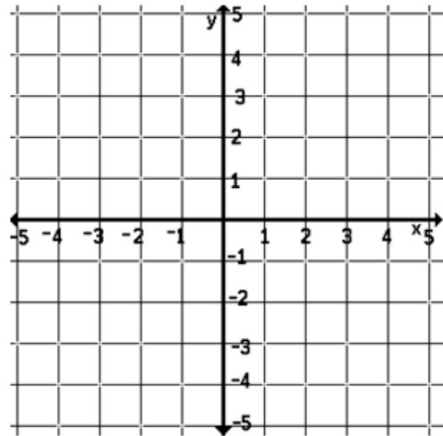


$$\frac{2}{2} = 1$$

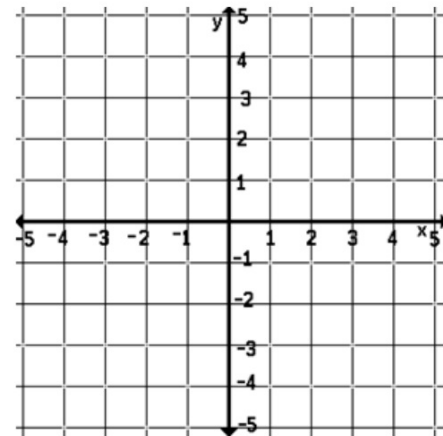


Graph the line containing these points and find their slopes.

$(-4,2)$ and $(2, -3)$



$(-2,3)$, $(3,5)$



Find the slope without graphing.

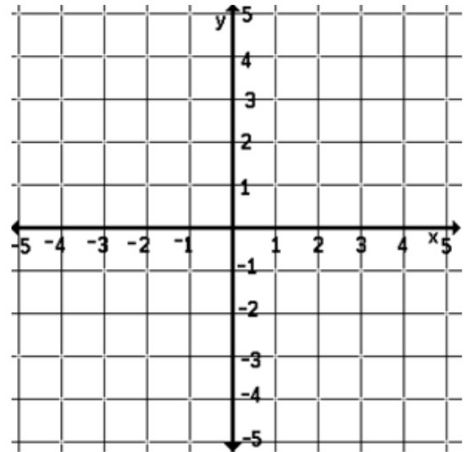
$(1,2)$ and $(3,5)$

$(-2,3)$ and $(7,1)$

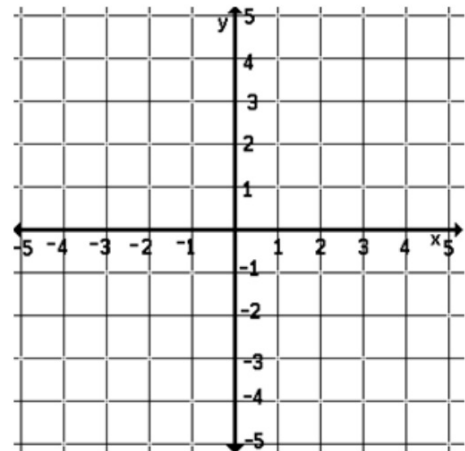
$(-4,-6)$ and $(3,-2)$

Find the slope of the line that contains the following points

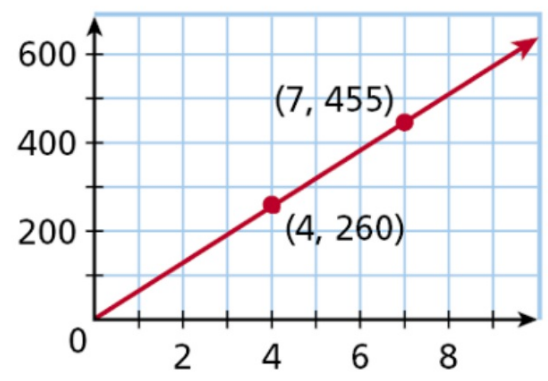
$(4,-6)$ and $(4,0)$



$(7,-2)$ and $(-3,-2)$



Justin is driving from home to his college dormitory. At 4:00 p.m., he is 260 miles from home. At 7:00 p.m., he is 455 miles from home. Graph the line that represents Justin's distance from home at a given time. Find and interpret the slope of the line.



$m =$

The slope is which means Justin is

Slopes of Parallel and Perpendicular Lines

3-5-1 Parallel Lines Theorem

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

3-5-2 Perpendicular Lines Theorem

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1 . Vertical and horizontal lines are perpendicular.



If a line has a slope of $\frac{a}{b}$, then the slope of a



perpendicular line is $-\frac{b}{a}$.

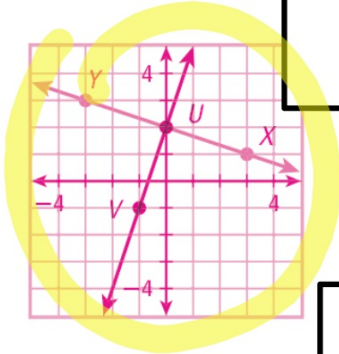
The ratios $\frac{a}{b}$ and $-\frac{b}{a}$ are called *opposite reciprocals*.



Graph each pair of lines. Use their slopes to determine whether they are parallel, perpendicular, or neither.

\overline{UV} and \overline{XY} for $U(0, 2)$,
 $V(-1, -1)$, $X(3, 1)$,
 and $Y(-3, 3)$

slope of $\overline{UV} = \frac{-1-2}{-1-0} = \frac{-3}{-1} = 3$
 slope of $\overline{XY} = \frac{3-1}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$



Perpendicular

\overline{GH} and \overline{IJ} for $G(-3, -2)$,
 $H(1, 2)$, $I(-2, 4)$, and $J(2, -4)$

slope of $\overline{GH} =$
 slope of $\overline{IJ} =$

\vec{CD} and \vec{EF} for $C(-1, -3)$,
 $D(1, 1)$, $E(-1, 1)$, and $F(0, 3)$

slope of \vec{CD} =

slope of \vec{EF} =

\vec{WX} and \vec{YZ} for $W(3, 1)$,
 $X(3, -2)$, $Y(-2, 3)$, and
 $Z(4, 3)$

slope of \vec{WX} =

slope of \vec{YZ} =

**KL and MN for $K(-4, 4)$,
 $L(-2, -3)$, $M(3, 1)$, and
 $N(-5, -1)$**

slope of \overrightarrow{KL} =

slope of \overrightarrow{MN} =

**\overrightarrow{BC} and \overrightarrow{DE} for $B(1, 1)$,
 $C(3, 5)$, $D(-2, -6)$, and
 $E(3, 4)$**

slope of \overrightarrow{BC} =

slope of \overrightarrow{DE} =

Reteach

Slopes of Lines

The **slope** of a line describes how steep the line is. You can find the slope by writing the ratio of the **rise** to the **run**.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}$$

You can use a formula to calculate the slope m of the line through points (x_1, y_1) and (x_2, y_2) .

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

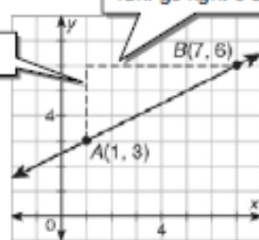
Change in y-values

Change in x-values

To find the slope of \overline{AB} using the formula, substitute $(1, 3)$ for (x_1, y_1) and $(7, 6)$ for (x_2, y_2) .

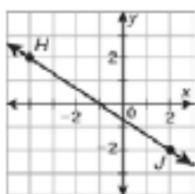
rise: go up 3 units

run: go right 6 units

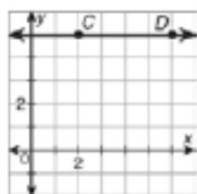


$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ &= \frac{6 - 3}{7 - 1} && \text{Substitution} \\ &= \frac{3}{6} && \text{Simplify.} \\ &= \frac{1}{2} && \text{Simplify.} \end{aligned}$$

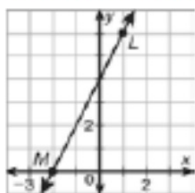
Use the slope formula to determine the slope of each line.



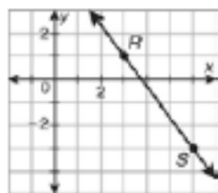
1. \overline{HJ}



2. \overline{CD}

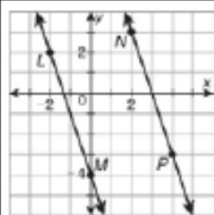


3. \overline{LM}



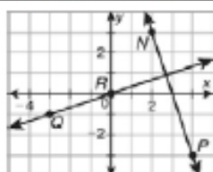
4. \overline{RS}

Slopes of Parallel and Perpendicular Lines



slope of $\overline{LM} = -3$
slope of $\overline{NP} = -3$

Parallel lines have the same slope.



slope of $\overline{NP} = -3$
slope of $\overline{QR} = \frac{1}{3}$

product of slopes:

$$-3\left(\frac{1}{3}\right) = -1$$

Perpendicular lines have slopes that are *opposite reciprocals*. The product of the slopes is -1 .

Use slopes to determine whether each pair of distinct lines is parallel, perpendicular, or neither.

5. slope of $\overline{PQ} = 5$

slope of $\overline{JK} = -\frac{1}{5}$

6. slope of $\overline{EF} = -\frac{3}{4}$

slope of $\overline{CD} = -\frac{3}{4}$

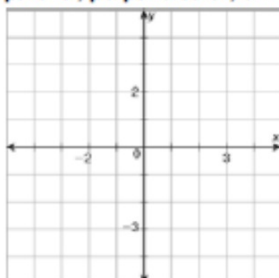
7. slope of $\overline{BC} = -\frac{5}{3}$

slope of $\overline{ST} = \frac{3}{5}$

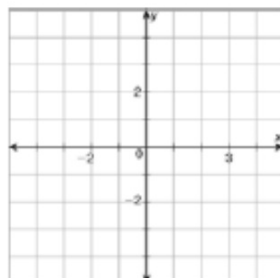
8. slope of $\overline{WX} = \frac{1}{2}$

slope of $\overline{YZ} = -\frac{1}{2}$

Graph each pair of lines. Use slopes to determine whether the lines are parallel, perpendicular, or neither.



9. \overline{FG} and \overline{HJ} for $F(-1, 2)$, $G(3, -4)$, $H(-2, -3)$, and $J(4, 1)$



10. \overline{RS} and \overline{TU} for $R(-2, 3)$, $S(3, 3)$, $T(-3, 1)$, and $U(3, -1)$

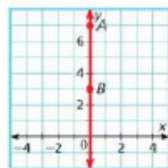
Independent Practice For Exercises	See Example
10–13	1
14	2
15–17	3

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

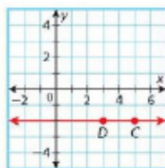
PRACTICE AND PROBLEM SOLVING

Use the slope formula to determine the slope of each line.

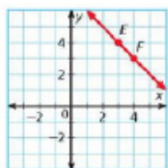
10. \overline{AB}



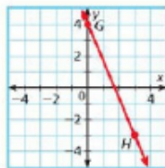
11. \overline{CD}



12. \overline{EF}



13. \overline{GH}



14. **Aviation** A pilot traveling at a constant speed flies 100 miles by 2:30 P.M. and 475 miles by 5:00 P.M. Graph the line that represents the pilot's distance flown. Find and interpret the slope of the line.

Graph each pair of lines. Use slopes to determine whether the lines are parallel, perpendicular, or neither.

15. \overline{AB} and \overline{CD} for $A(2, -1)$, $B(7, 2)$, $C(2, -3)$, and $D(-3, -6)$

16. \overline{XY} and \overline{ZW} for $X(-2, 5)$, $Y(6, -2)$, $Z(-3, 6)$, and $W(4, 0)$

17. \overline{JK} and \overline{L} for $J(-4, -2)$, $K(4, -2)$, and $L(-4, 6)$

18. **Geography** A point on a river has an elevation of about 1150 meters above sea level. The length of the river from that point to where it enters the sea is about 2400 km. Find and interpret the slope of the river.

For $F(7, 6)$, $G(-3, 5)$, $H(-2, -3)$, $I(4, -2)$, and $K(6, 1)$, find each slope.

19. \overline{FG}

20. \overline{GH}

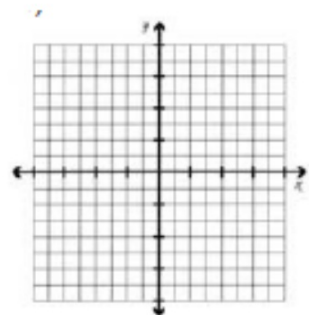
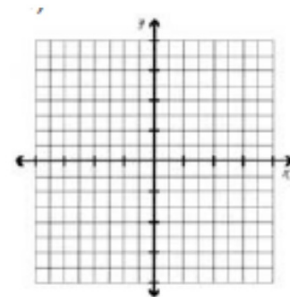
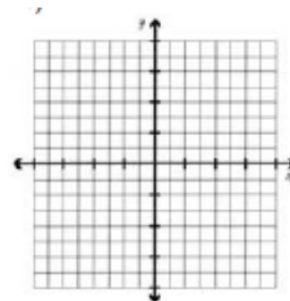
21. \overline{HK}

22. \overline{IK}

23. **Critical Thinking** The slope of \overline{AB} is greater than 0 and less than 1. Write an inequality for the slope of a line perpendicular to \overline{AB} .

24. **Write About It** Two cars are driving at the same speed. What is true about the lines that represent the distance traveled by each car at a given time?

25. A traffic engineer calculates the speed of vehicles as they pass a traffic light. While the light is green, a taxi passes at a constant speed. After 2 s the taxi is 132 ft past the light. After 5 s it is 330 ft past the light.
- Find the speed of the taxi in feet per second.
 - Use the fact that $22 \text{ ft/s} = 15 \text{ mi/h}$ to find the taxi's speed in miles per hour.



MULTI-STEP TEST PREP

