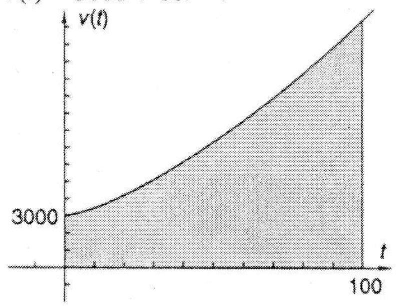
Name ____________________ Group ____________________ Date ____________

Exploration 33: Applications of Definite Integrals

Objective: Without looking at the text, learn an interpretation of $f(x)\ dx$ in a definite integral.

Spaceship Velocity and Displacement Problem: A spaceship is fired into orbit. As the last stage of the booster rocket is fired the spaceship is going 3000 feet per second (a bit under 2000 mph). Its velocity, $v(t)$, at t seconds since the booster was fired is given by

$v(t) = 3000 + 18t^{1.4}$.



1. On the graph, pick a sample point, t, somewhere on the t-axis between 0 and 100. Show the corresponding point $(t, v(t))$ on the graph.
2. Draw a narrow vertical strip of the region in such a way that the ordered pair $(t, v(t))$ is within the strip. Label the width of the strip dt.
3. If the strip is narrow (i.e., dt is small), the velocity throughout the time interval dt is not much different from the velocity $v(t)$ at the sample point. Explain why the distance traveled in this time interval is approximately equal to $v(t) \cdot dt$.
4. Write a Riemann sum that represents, approximately, the total distance the spaceship goes between $t = 0$ and $t = 100$.
5. Explain why the Riemann sum in Problem 4 is between the corresponding lower sum and upper sum. Based on this fact, why can you conclude that the limit of the Riemann sum is *exactly* equal to $\int_0^{100} v(t)\, dt$?
6. Find the distance traveled by the spaceship by evaluating the integral in Problem 5 using the fundamental theorem of calculus.
7. In order to orbit, the spaceship must be going at least 26,000 feet per second (about 17,500 mph). To the nearest second, at what time is it going that fast?
8. How far does the spaceship go from the time the last stage fires till it reaches orbital velocity?
9. Based on what you observed while doing this exercise, why do you suppose integrals are written in *differential* form, $\int f(x)\, dx$, instead of simply in derivative form, $\int f(x)$?