Law of Cosines (for non-right triangles only)
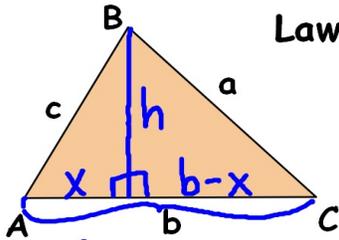
$$* \, a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$
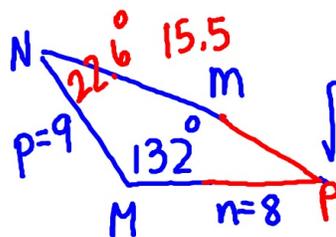
Proof:
$$a^2 = h^2 + (b-x)^2$$
$$a^2 = (c^2 - x^2) + b^2 - 2bx + x^2$$
$$a^2 = c^2 + b^2 - 2bx \qquad \left(\cos A = \frac{x}{c}, \text{ so } x = c(\cos A)\right)$$
$$a^2 = c^2 + b^2 - 2bc(\cos A)$$

ex. 1 Solve $\triangle MNP$, if $\angle M = 132°$, $n = 8$, $p = 9$.

$$m^2 = 9^2 + 8^2 - 2(9)(8)\cos 132°$$
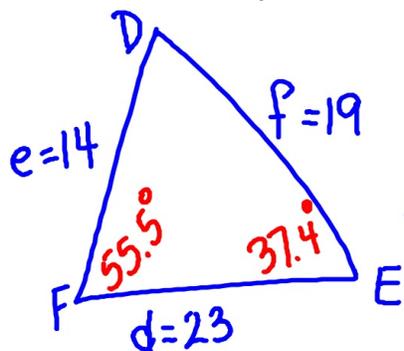$$\sqrt{m^2} = \sqrt{145 - 144\cos 132°}$$
$$M \approx 15.5$$

$$\frac{\sin 132°}{15.5} = \frac{\sin N}{8}$$

$$22.6° \approx \angle N$$
$$25.4° \approx \angle P$$

ex. 2 Solve $\triangle DEF$, if d = 23, e = 14, f = 19.

$$14^2 = 19^2 + 23^2 - 2(23)(19)\cos E$$

$$-694 = -874 \cos E$$

$$\cos^{-1}\left(\frac{694}{874}\right) = \angle E$$

$$37.4° \approx \angle E$$

$$\frac{\sin F}{19} = \frac{\sin 37.4°}{14}$$

$$\angle F \approx 55.5°$$

$$\angle D \approx 87.1°$$

(Triangle diagram: D at top, e=14 on left side, f=19 on right side, d=23 on bottom. Angle at F = 55.5°, angle at E = 37.4°)