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Using Deductive Reasoning to Verify Conjectures

Objective

Apply the Law of Detachment and the Law of Syllogism in logical reasoning.

Vocabulary

deductive reasoning

Deductive reasoning is the process of using logic to draw conclusions from given facts, definitions, and properties.

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Example 1A: Media Application

Is the conclusion a result of inductive or deductive reasoning?

There is a myth that you can balance an egg on its end only on the spring equinox. A person was able to balance an egg on July 8, September 21, and December 19. Therefore this myth is false.

Since the conclusion is based on a pattern of observations, it is a result of inductive reasoning.

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Example 1B: Media Application

Is the conclusion a result of inductive or deductive reasoning?

There is a myth that the Great Wall of China is the only man-made object visible from the Moon. The Great Wall is barely visible in photographs taken from 180 miles above Earth. The Moon is about 237,000 miles from Earth. Therefore, the myth cannot be true.

The conclusion is based on logical reasoning from scientific research. It is a result of deductive reasoning.

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Check It Out! Example 1

There is a myth that an eelskin wallet will demagnetize credit cards because the skin of the electric eels used to make the wallet holds an electric charge. However, eelskin products are not made from electric eels. Therefore, the myth cannot be true. Is this conclusion a result of inductive or deductive reasoning?

The conclusion is based on logical reasoning from scientific research. It is a result of deductive reasoning.

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In deductive reasoning, if the given facts are true and you apply the correct logic, then the conclusion must be true. The Law of Detachment is one valid form of deductive reasoning.

Law of Detachment

If $p \rightarrow q$ is a true statement and p is true, then q is true.

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Example 2A: Verifying Conjectures by Using the Law of Detachment

Determine if the conjecture is valid by the Law of Detachment.

Given: If the side lengths of a triangle are 5 cm, 12 cm, and 13 cm, then the area of the triangle is 30 cm^2 . The area of $\triangle PQR$ is 30 cm^2 .

Conjecture: The side lengths of $\triangle PQR$ are 5cm, 12 cm, and 13 cm.

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Example 2A: Verifying Conjectures by Using the Law of Detachment Continued

Identify the **hypothesis** and **conclusion** in the given conditional.

If the side lengths of a triangle are 5 cm, 12 cm, and 13 cm, then **the area of the triangle is 30 cm².**

The given statement “The area of $\triangle PQR$ is 30 cm²” matches the conclusion of a true conditional. But this does not mean the hypothesis is true. The dimensions of the triangle could be different. So the conjecture is not valid.

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Example 2B: Verifying Conjectures by Using the Law of Detachment

Determine if the conjecture is valid by the Law of Detachment.

Given: In the World Series, if a team wins four games, then the team wins the series. The Red Sox won four games in the 2004 World Series.

Conjecture: The Red Sox won the 2004 World Series.

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Example 2B: Verifying Conjectures by Using the Law of Detachment Continued

Identify the **hypothesis** and **conclusion** in the given conditional.

In the World Series, if a team wins four games, then the team wins the series.

The statement “The Red Sox won four games in the 2004 World Series” matches the hypothesis of a true conditional. By the Law of Detachment, the Red Sox won the 2004 World Series. The conjecture is valid.

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Check It Out! Example 2

Determine if the conjecture is valid by the Law of Detachment.

Given: If a student passes his classes, the student is eligible to play sports. Ramon passed his classes.

Conjecture: Ramon is eligible to play sports.

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Check It Out! Example 2 Continued

Identify the **hypothesis** and **conclusion** in the given conditional.

If a student passes his classes, then **the student is eligible to play sports.**

The statement “Ramon passed his classes” matches the hypothesis of a true conditional. By the Law of Detachment, Ramon is eligible to play sports. The conjecture is valid.

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Another valid form of deductive reasoning is the Law of Syllogism. It allows you to draw conclusions from two conditional statements when the conclusion of one is the hypothesis of the other.

Law of Syllogism

If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.

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Example 3A: Verifying Conjectures by Using the Law of Syllogism

Determine if the conjecture is valid by the Law of Syllogism.

Given: If a figure is a kite, then it is a quadrilateral. If a figure is a quadrilateral, then it is a polygon.

Conjecture: If a figure is a kite, then it is a polygon.

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Example 3A: Verifying Conjectures by Using the Law of Syllogism Continued

Let p , q , and r represent the following.

p : A figure is a kite.

q : A figure is a quadrilateral.

r : A figure is a polygon.

You are given that $p \rightarrow q$ and $q \rightarrow r$.

Since q is the conclusion of the first conditional and the hypothesis of the second conditional, you can conclude that $p \rightarrow r$. The conjecture is valid by Law of Syllogism.

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Example 3B: Verifying Conjectures by Using the Law of Syllogism

Determine if the conjecture is valid by the Law of Syllogism.

Given: If a number is divisible by 2, then it is even. If a number is even, then it is an integer.

Conjecture: If a number is an integer, then it is divisible by 2.

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Example 3B: Verifying Conjectures by Using the Law of Syllogism Continued

Let **x**, **y**, and **z** represent the following.

x: A number is divisible by 2.

y: A number is even.

z: A number is an integer.

You are given that **x** \rightarrow **y** and **y** \rightarrow **z**. The Law of Syllogism cannot be used to deduce that **z** \rightarrow **x**. The conclusion is not valid.

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Check It Out! Example 3

Determine if the conjecture is valid by the Law of Syllogism.

Given: If an animal is a mammal, then it has hair. If an animal is a dog, then it is a mammal.

Conjecture: If an animal is a dog, then it has hair.

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Check It Out! Example 3 Continued

Let x , y , and z represent the following.

x : An animal is a mammal.

y : An animal has hair.

z : An animal is a dog.

You are given that $x \rightarrow y$ and $z \rightarrow x$.

Since x is the conclusion of the second conditional and the hypothesis of the first conditional, you can conclude that $z \rightarrow y$. The conjecture is valid by Law of Syllogism.

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Example 4: Applying the Laws of Deductive Reasoning

Draw a conclusion from the given information.

A. Given: If $2y = 4$, then $z = -1$. If $x + 3 = 12$, then $2y = 4$. $x + 3 = 12$

Conclusion: $z = -1$.

B. If the sum of the measures of two angles is 180° , then the angles are supplementary. If two angles are supplementary, they are not angles of a triangle. $m\angle A = 135^\circ$, and $m\angle B = 45^\circ$.

Conclusion: $\angle A$ and $\angle B$ are not angles of a triangle.

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Check It Out! Example 4

Draw a conclusion from the given information.

Given: If a polygon is a triangle, then it has three sides.

If a polygon has three sides, then it is not a quadrilateral. Polygon P is a triangle.

Conclusion: Polygon P is not a quadrilateral.