Lesson 2 - 3 Using Deductive Reasoning to Verify Conjectures

Put the following statements in order to make a logical argument

Start: The apple in my lunch is rotten... A

If I eat rotten food, then I will get sick $A \rightarrow S$

If I get sick, then I will not go to school $S \rightarrow$ sc.

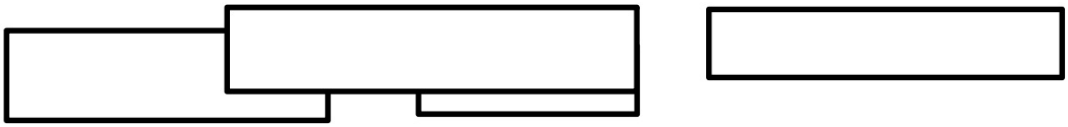
If I do not go to school, then I will not learn geometry

If I do not learn geometry, then I will earn a bad grade

If I earn a bad grade, then I will not be able to play sports

Deductive reasoning is the process of using logic to draw conclusions from given facts, definitions, and properties.

When several examples form a pattern and you assume the pattern will continue, you are applying *inductive reasoning*. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern. A statement you believe to be true based on inductive reasoning is called a **conjecture**.



Is the following an example of inductive or deductive reasoning?

On each of the first six days Jim attended his geometry class, Mrs. Lee, his geometry teacher, gave a homework assignment. Jim concludes that he will have geometry homework every day he has geometry class. Inductive

In the same geometry class, Maria reads the theorem "Vertical angles are congruent." She notices in a diagram that $\angle 1$ and $\angle 2$ are vertical angles. Maria concludes that $\angle 1 \cong \angle 2$. Deductive

Lara is told that $m(\angle A) = 150$ and $m(\angle B) = 30$. Since she knows the definition of supplementary angles, she concludes that $\angle A$ and $\angle B$ are supplementary. Deductive

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Jose did his assignment and found the sums of the exterior angles of several different polygons. Noticing the results were all the same, he concludes that the sum of the measures of the exterior angles of any polygon is $360°$. Inductive

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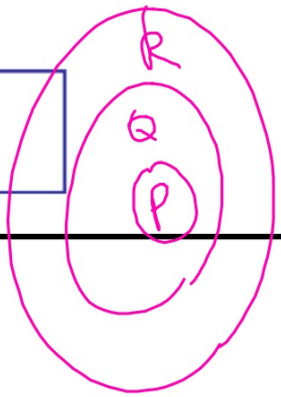
Kim observes that the sum of 2 and 4 is an even number, that the sum of 4 and 6 is an even number, and that the sum of 12 and 6 is also an even number. She concludes that the sum of two even numbers is always an even number. Inductive

Law of Syllogism

If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.



Determine if the conjecture is valid by the Law of Syllogism.

Given: If a figure is a kite, then it is a quadrilateral. If a figure is a quadrilateral, then it is a polygon.

Conjecture: If a figure is a kite, then it is a polygon.

Given: If a number is divisible by 2, then it is even. If a number is even, then it is an integer.

Conjecture: If a number is an integer, then it is divisible by 2.

NOT VALID (wrong) switched

Determine if the conjecture is valid by the Law of Syllogism.

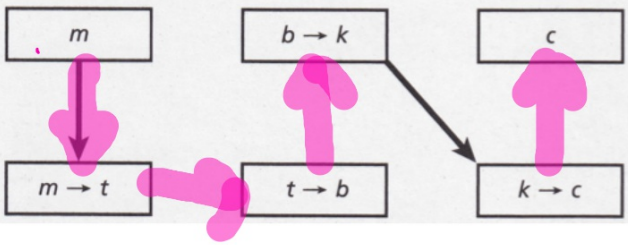
Given: If an animal is a mammal, then it has hair. If an animal is a dog, then it is a mammal.

Conjecture: If an animal is a dog, then it has hair. valid

Draw arrows to show the logical reasoning used to prove the statement. Some arrows are already drawn to help you.

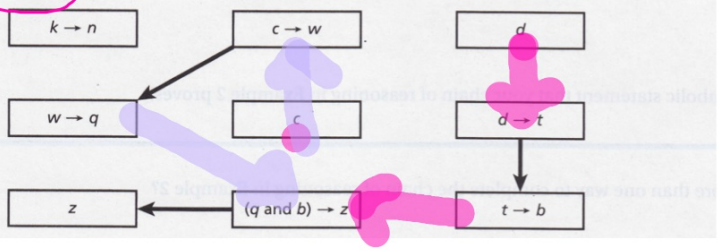
A **Given:** m, and the conditional statements in the boxes

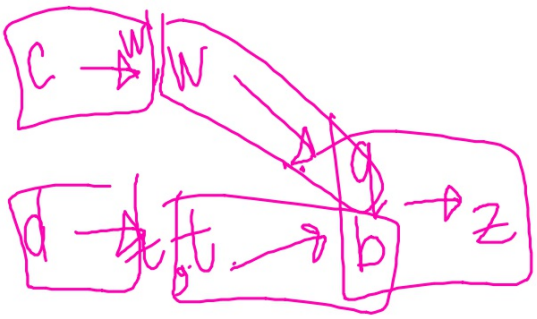
Prove: c



B **Given:** *c*, *d*, and the conditional statements in the boxes

Prove: *z*





REFLECT

1a. Write a symbolic statement that the chain of reasoning in each part proves.

Part A: m → c

Part B: c and d → z

1b. In a chain of reasoning, what do you call the statements that have arrows only pointing *away* from them? What do you call the statements that have arrows only pointing *toward* them?

Given, (IF) HYPOTHESIS ; Prove, conclusion (then)

1c. Are there any statements in Part B that are not used in the chain of reasoning? Explain.

k → n

1d. In the space below, arrange the statements from Part B and draw arrows connecting them in a way that makes the order of the reasoning clearer.

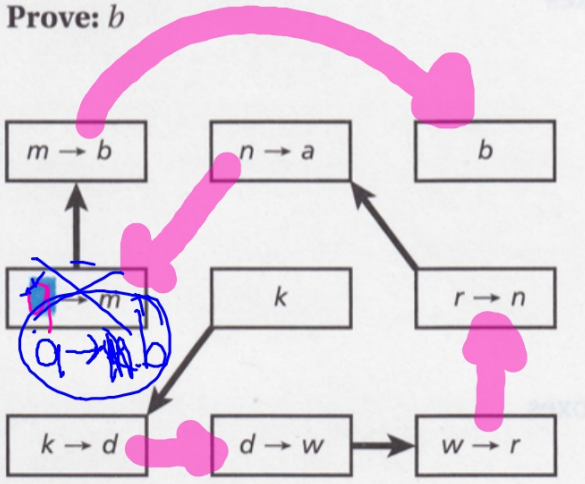
CC.MP.3

EXAMPLE Completing a Chain of Logical Reasoning

Draw arrows to show the logical reasoning used to prove the statement. Provide any missing information.

Given: k, and the conditional statements in the boxes

Prove: b



2a) $k \rightarrow B$

REFLECT

2a. Write a symbolic statement that your chain of reasoning in Example 2 proves.

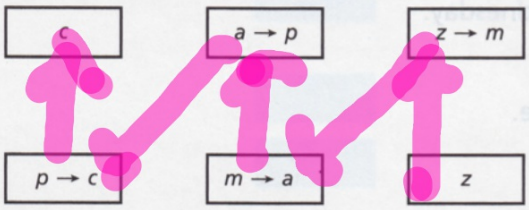
2b. Is there more than one way to complete the chain of reasoning in Example 2? Explain.

2c. In Example 2, suppose the box with the missing information is completely blank. Is there more than one way to complete the chain of reasoning? Explain.

Draw arrows to show the logical reasoning used to prove the statement. Provide any missing information.

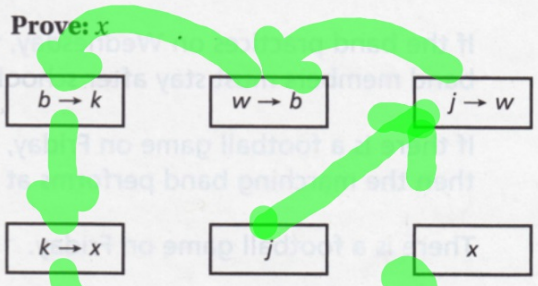
1. Given: z, and statements in boxes

Prove: c



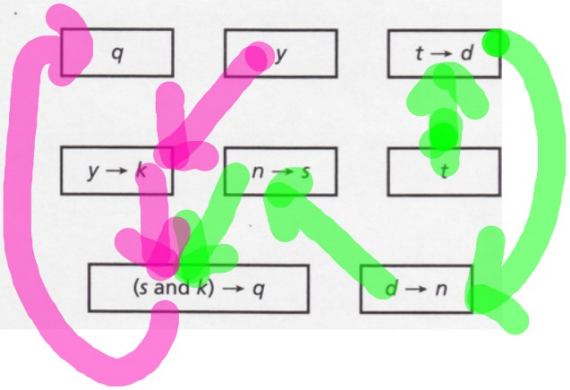
2. Given: j, and statements in boxes

Prove: x



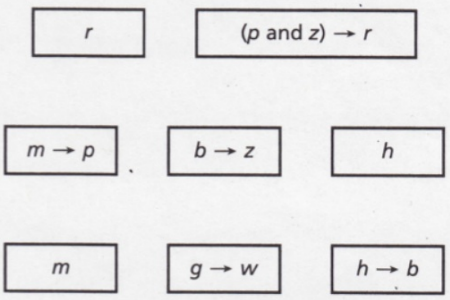
3. Given: t, y, and statements in boxes

Prove: q



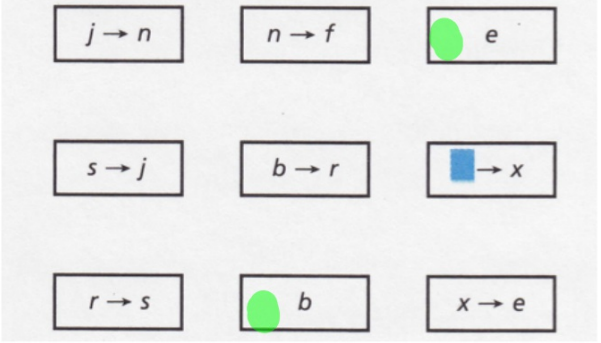
4. Given: h, m, and statements in boxes

Prove: r



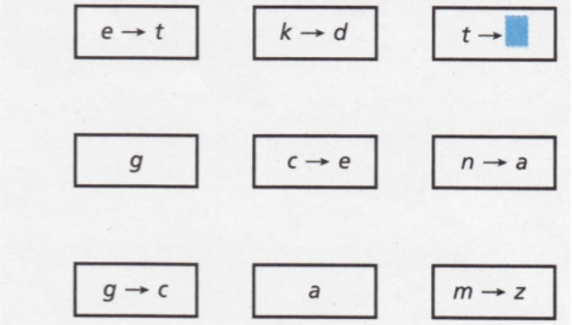
5. Given: b, and statements in boxes

Prove: e



6. Given: g, and statements in boxes

Prove: a



7. Write a symbolic statement for each conditional statement. Then represent the chain of reasoning using symbolic statements linked by arrows.

Band members must stay after school on Wednesday. — B

If the marching band performs at halftime, then the band practices on Wednesday. — H → W

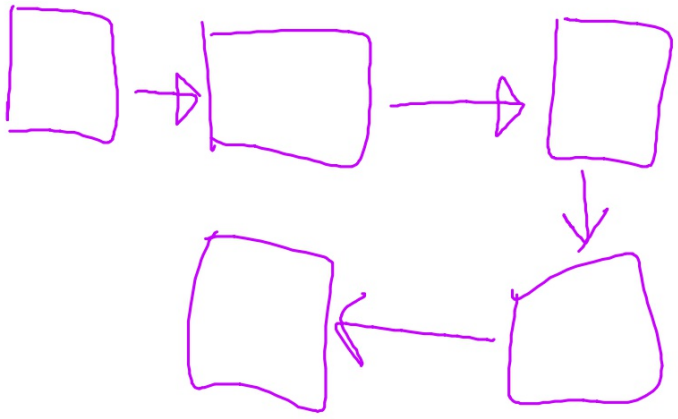
If the band practices on Wednesday, then band members must stay after school on Wednesday. — W → B

If there is a football game on Friday, then the marching band performs at halftime. — F → H

There is a football game on Friday. — F

given F plus others

Prove: B



The planet Yercon is full of unusual creatures. Here are some facts about the creatures.

If you are a zintoid, then you are a quorg.
If you are a meerk, then you are a ving.
If you are a quorg, then you are a wiklop.

1. Dorla is a meerk. What else must be true about Dorla?

2. Zim is a quorg. What else must be true about Zim?

3. In general, suppose you know that $p \rightarrow q$ is a true statement and that p is true. What can you conclude?

4. Use the facts about the creatures to write a new conditional statement about zintoids: If you are a zintoid, then __?__.

5. In general, suppose you know that $p \rightarrow q$ and $q \rightarrow r$ are true statements. What can you conclude?

The planet Yercon is full of unusual creatures. Here are some facts about the creatures.

> If you are a zintoid, then you are a quorg.
>
> If you are a meerk, then you are a ving.
>
> If you are a quorg, then you are a wiklop.

THINK AND DISCUSS

6. **Discuss** whether or not you can conclude that every quorg is also a zintoid.

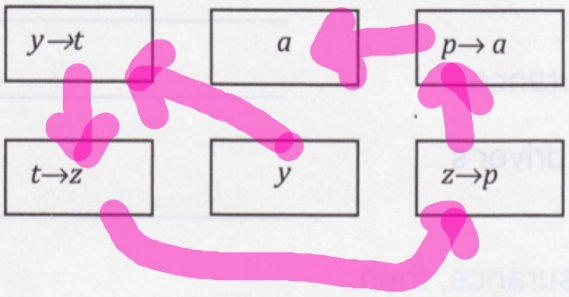
7. **Discuss** whether or not you can conclude that Rinn, who is a zintoid, is also a wiklop.

Additional Practice

HW 2.3 AP (p65) / PS (p66)

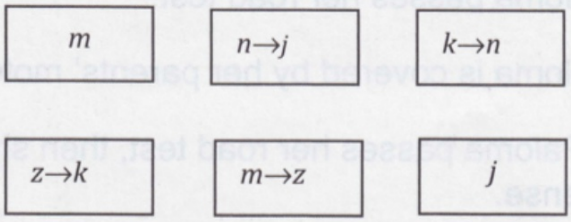
Draw arrows to show the logical reasoning used to prove the statement. Provide any missing information.

1. **Given:** y, and statements in boxes.
   Prove: a



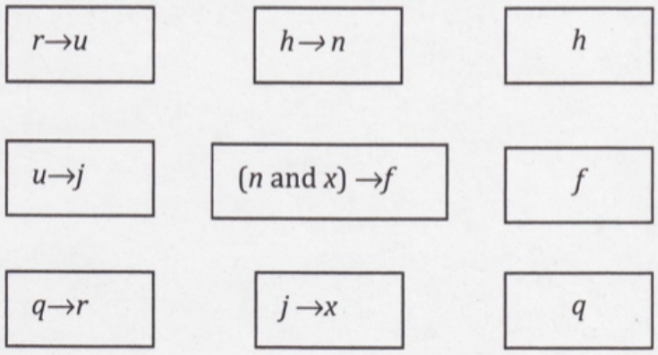
Draw arrows to show the logical reasoning used to prove the statement. Provide any missing information.

2. **Given:** m, and statements in boxes.
   Prove: j



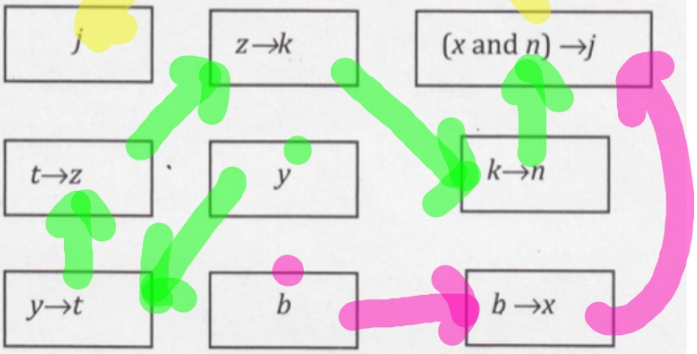
Draw arrows to show the logical reasoning used to prove the statement. Provide any missing information.

3. **Given:** h, q, and statements in boxes.
   Prove: f



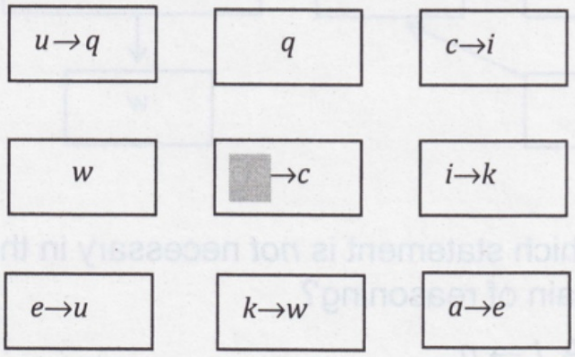
Draw arrows to show the logical reasoning used to prove the statement. Provide any missing information.

4. **Given:** b, y, and statements in boxes.
   Prove: j



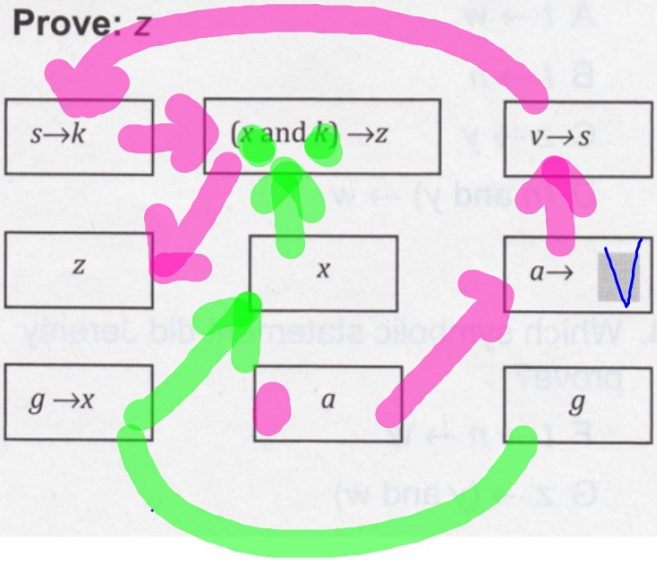
Draw arrows to show the logical reasoning used to prove the statement. Provide any missing information.

5. **Given:** *a*, and statements in boxes.
   Prove: *w*



Draw arrows to show the logical reasoning used to prove the statement. Provide any missing information.

6. **Given:** a, and statements in boxes.
Prove: z



Problem Solving

Given: R and I, plus others

Prove: D

Write a symbolic statement for each conditional statement. Then represent the chain of reasoning using symbolic statements linked by arrows.

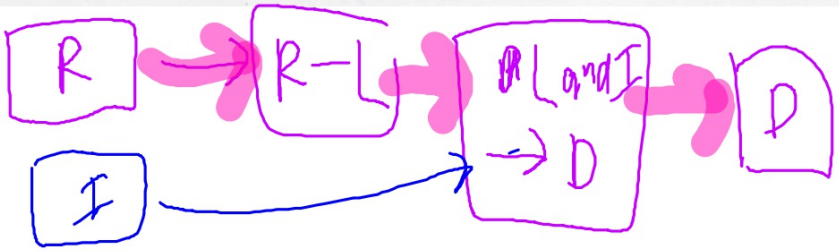
D 1. Paloma can drive her parents' car. — D

Paloma passes her road test. — R

Paloma is covered by her parents' motor vehicle insurance. — I

If Paloma passes her road test, then she will get her driver's license. — R → L

If Paloma has a driver's license and is covered by insurance, then she can drive her parents car. — I and L → D



Given *t* and *z*, and the conditional statements in the boxes shown, Jeremy drew arrows to complete the chain of reasoning. Select the best answer.

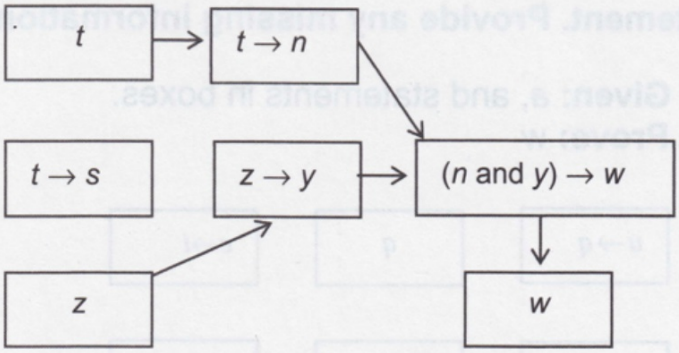
2. Which statement can you *not* assume to be true from the chain of reasoning?

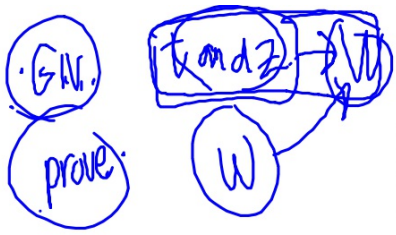
   A $t \rightarrow w$

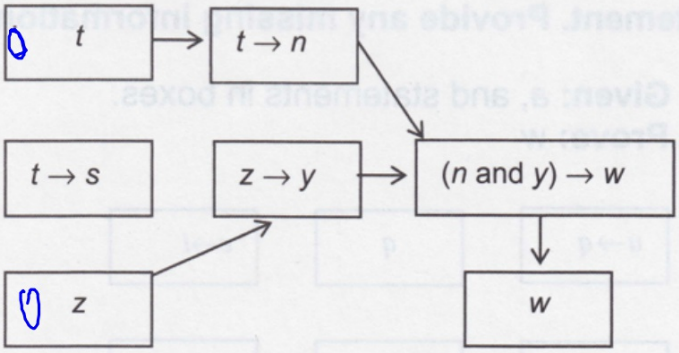
   B $t \rightarrow n$

   C $z \rightarrow y$

   D $(n \text{ and } y) \rightarrow w$







3. Which symbolic statement did Jeremy prove?

F $t \rightarrow n \rightarrow w$

G $z \rightarrow (y \text{ and } w)$

H $(t \text{ and } z) \rightarrow w$

J $(n \text{ and } y) \rightarrow w$

4. Which statement is *not* necessary in the chain of reasoning?

A $t \rightarrow n$

B $t \rightarrow s$

C z

D $z \rightarrow y$