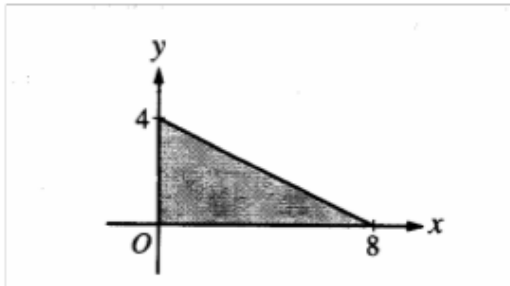


86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

92. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

86. C Each cross section is a semicircle with a diameter of y. The volume would be given by

$$\int_0^8 \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^8 \left(\frac{8-x}{2}\right)^2 dx = 16.755$$

92. D $\int_k^{\frac{\pi}{2}} \cos x\, dx = 0.1 \Rightarrow \sin\left(\frac{\pi}{2}\right) - \sin k = 0.1 \Rightarrow \sin k = 0.9$. Therefore $k = \sin^{-1}(0.9) = 1.120$.

87. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\,dy$

(B) $\int_0^2 (2-y)\,dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2\,dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2\,dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right)dx$

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

87. B Squares with sides of length x. Volume $= \int_0^2 x^2 dy = \int_0^2 (2-y)\,dy$

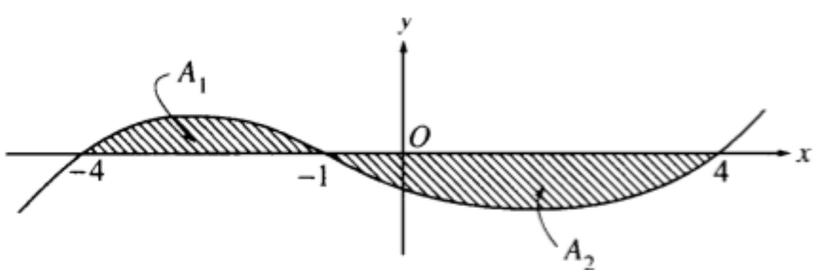
25. D The area is given by $\int_0^2 x^2 - (-x)\,dx = \left(\frac{1}{3}x^3 + \frac{1}{2}x^2\right)\Big|_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$.

83. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

84. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$



19. The graph of $y = f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$

(A) A_1 (B) $A_1 - A_2$ (C) $2A_1 - A_2$ (D) $A_1 + A_2$ (E) $A_1 + 2A_2$

83. C $\cos x = x$ at $x = 0.739085$. Store this in A. $\int_0^A (\cos x - x)\,dx = 0.400$

84. C Cross sections are squares with sides of length y.

Volume $= \int_1^e y^2 dx = \int_1^e \ln x\, dx = (x \ln x - x)\Big|_1^e = (e \ln e - e) - (0 - 1) = 1$

19. D $\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx = (A_1 - A_2) - 2(-A_2) = A_1 + A_2$

80. Let R be the region enclosed by the graph of $y=1+\ln\left(\cos^4 x\right)$, the x-axis, and the lines $x=-\frac{2}{3}$ and $x=\frac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

80. B The area is given by $\int_{-\frac{2}{3}}^{\frac{2}{3}} \left(1 + \ln(\cos^4 x)\right) dx = 0.919$

30. What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \frac{\pi}{3}$?

(A) $\frac{\pi}{\sqrt{3}}$

(B) π

(C) $\pi\sqrt{3}$

(D) $\frac{8\pi}{3}$

(E) $\pi \ln\left(\frac{1}{2} + \sqrt{3}\right)$

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

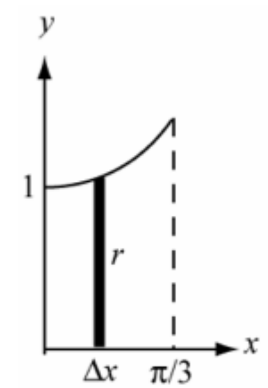
(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

23. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\frac{32\pi}{5}$ (B) $\frac{16\pi}{3}$ (C) $\frac{16\pi}{5}$ (D) $\frac{8\pi}{3}$ (E) π

30. C Each slice is a disk with radius $r = \sec x$ and width Δx.

$$\text{Volume} = \pi \int_0^{\pi/3} \sec^2 x \, dx = \pi \tan x \Big|_0^{\pi/3} = \pi\sqrt{3}$$



16. D The area of the region is given by $\int_{-2}^{2} (5-(x^2+1))dx = 2(4x-\frac{1}{3}x^3)\Big|_0^2 = 2\left(8-\frac{8}{3}\right) = \frac{32}{3}$

23. A Disks where $r = x$. $V = \pi\int_0^2 x^2 dy = \pi\int_0^2 y^4 dy = \frac{\pi}{5} y^5\Big|_0^2 = \frac{32\pi}{5}$

6. The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

30. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

1. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

6. D Area $= \int_3^4 \frac{1}{x-1}\,dx = \left(\ln|x-1|\right)\Big|_3^4 = \ln 3 - \ln 2 = \ln\frac{3}{2}$

30. C Each slice is a disk whose volume is given by $\pi r^2 \Delta x$, where $r = \sqrt{x}$.

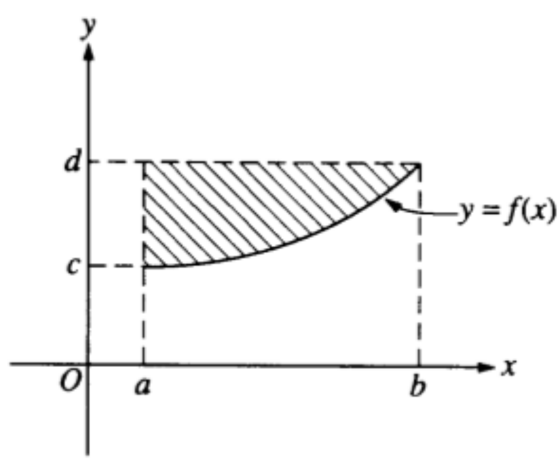
Volume $= \pi\int_0^3 (\sqrt{x})^2\,dx = \pi\int_0^3 x\,dx = \frac{\pi}{2}x^2\Big|_0^3 = \frac{9}{2}\pi$.

1. A $\int_0^1 (x - x^2)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

36. Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. The volume of the solid obtained by revolving R about the <u>x-axis</u> is given by

(A) $2\pi \int_0^{\frac{\pi}{2}} x \sin x \, dx$ (B) $2\pi \int_0^{\frac{\pi}{2}} x \cos x \, dx$ (C) $\pi \int_0^{\frac{\pi}{2}} (1 - \sin x)^2 \, dx$

(D) $\pi \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$ (E) $\pi \int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right) dx$



2. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)\,dy$ (B) $\int_a^b \left(d - f(x)\right) dx$ (C) $f'(b) - f'(a)$

(D) $(b-a)\left[f(b) - f(a)\right]$ (E) $(d-c)\left[f(b) - f(a)\right]$

36. E Disks: $\sum \pi(R^2 - r^2)\Delta x$ where $R = 1,\ r = \sin x$

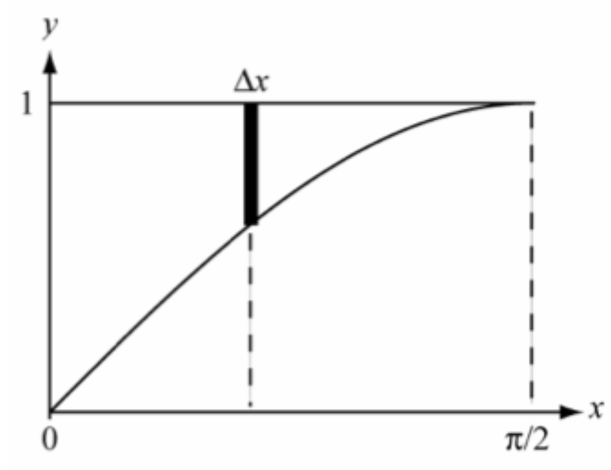
Volume $= \pi\int_0^{\pi/2}(1-\sin^2 x)\,dx$

Note that the expression in (E) can also be written as

$$\pi\int_0^{\pi/2}\cos^2 x\,dx = -\pi\int_{\pi/2}^{0}\cos^2\left(\tfrac{\pi}{2}-x\right)dx$$

$$= \pi\int_0^{\pi/2}\sin^2 x\,dx$$

and therefore option (D) is also a correct answer.



2. B Summing pieces of the form: (vertical)·(small width), vertical $= (d - f(x))$, width $= \Delta x$

Area $= \int_a^b (d - f(x))\,dx$

25. The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{4\pi}{3}$ (B) $\frac{16\pi}{5}$ (C) $\frac{4}{3}$ (D) $\frac{16}{5}$ (E) $\frac{64}{5}$

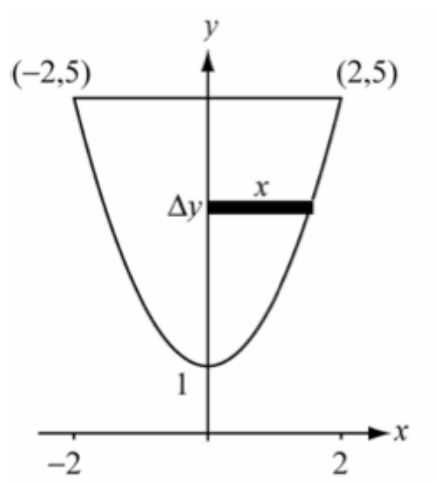
29. The region R in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the graph of $y = x^2 + 1$. The volume of the solid generated when R is revolved about the <u>y-axis</u> is

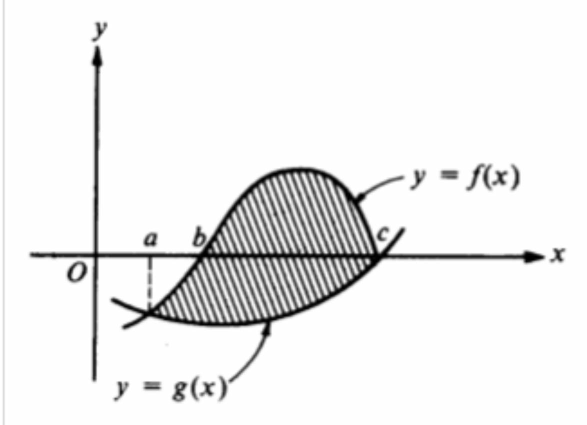
(A) 6π (B) 8π (C) $\frac{34\pi}{3}$ (D) 16π (E) $\frac{544\pi}{15}$

25. D Square cross-sections: $\sum y^2 \Delta x$ where $y = 4x^2$. Volume $= \int_0^1 16x^4\,dx = \frac{16}{5}x^4 \Big|_0^1 = \frac{16}{5}$.

29. B Disks: $\sum \pi x^2 \Delta y$ where $x^2 = y - 1$.

Volume $= \pi\int_1^5 (y-1)\,dy = \frac{\pi}{2}(y-1)^2\Big|_1^5 = 8\pi$





34. The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c (|f(x)| - |g(x)|)\,dx$

(B) $\int_b^c f(x)\,dx - \int_a^c g(x)\,dx$

(C) $\int_a^c (g(x) - f(x))\,dx$

(D) $\int_a^c (f(x) - g(x))\,dx$

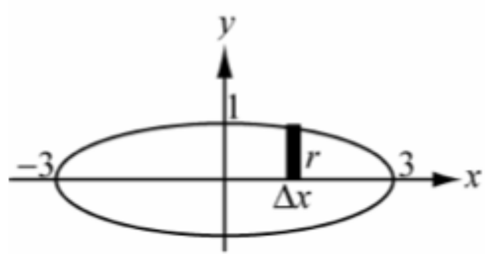
(E) $\int_a^b (g(x) - f(x))\,dx + \int_b^c (f(x) - g(x))\,dx$

43. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

34. D The interval is $x = a$ to $x = c$. The height of a rectangular slice is the top curve, $f(x)$, minus the bottom curve, $g(x)$. The area of the rectangular slice is therefore $(f(x) - g(x))\Delta x$. Set up a Riemann sum and take the limit as Δx goes to 0 to get a definite integral.

43. B The cross-sections are disks with radius $r = y$ where $y = \frac{1}{3}\sqrt{9 - x^2}$.



$$\text{Volume} = \pi\int_{-3}^{3} y^2\,dx = 2\pi\int_0^3 \frac{1}{9}\left(9 - x^2\right)dx = \frac{2\pi}{9}\left(9x - \frac{1}{3}x^3\right)\Bigg|_0^3 = 4\pi$$

39. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{\left(1-e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1-e^{-3}$

21. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

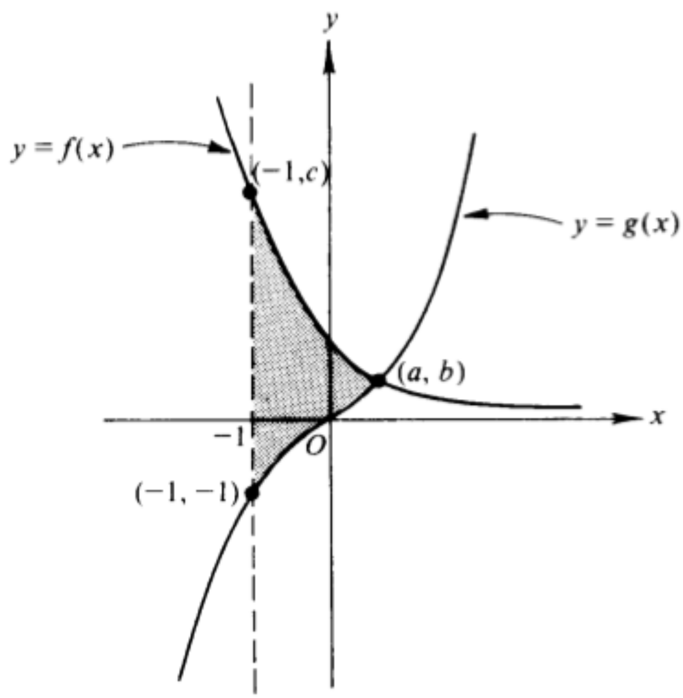
39. A Square cross sections: $\sum y^2 \Delta x$ where $y = e^{-x}$. $V = \int_0^3 e^{-2x}\,dx = -\frac{1}{2}e^{-2x}\Big|_0^3 = \frac{1}{2}\left(1 - e^{-6}\right)$

21. C $x = x^2 - 3x + 3$ at $x = 1$ and at $x = 3$.

Area $= \int_1^3 \left(x - \left(x^2 - 3x + 3\right)\right)dx = \int_1^3 \left(-x^2 + 4x - 3\right)dx = \left(-\frac{1}{3}x^3 + 2x^2 - 3x\right)\Big|_1^3 = \frac{4}{3}$

1. The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

(A) 36 (B) 23 (C) 20 (D) 17 (E) 9



5. The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point (a, b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

(A) $\int_0^a (f(x) - g(x))\,dx + \int_{-1}^0 (f(x) + g(x))\,dx$

(B) $\int_{-1}^b g(x)\,dx + \int_b^c f(x)\,dx$

(C) $\int_{-1}^c (f(x) - g(x))\,dx$

(D) $\int_{-1}^a (f(x) - g(x))\,dx$

(E) $\int_{-1}^a (|f(x)| - |g(x)|)\,dx$

1. D $\int_0^2 (4x^3+2)\,dx = (x^4+2x)\Big|_0^2 = (16+4)-(1+2) = 17$

5. D Area $= \int_{x_1}^{x_2} \left(\text{top curve} - \text{bottom curve}\right) dx,\ x_1 < x_2$; Area $= \int_{-1}^{a} \left(f(x) - g(x)\right) dx$

35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

34. The area of the region in the first quadrant that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

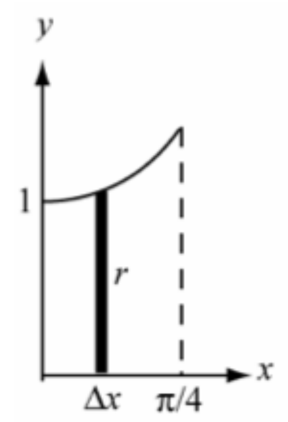
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$

45. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

35. C Washers: $\sum \pi r^2 \Delta x$ where $r = y = \sec x$.

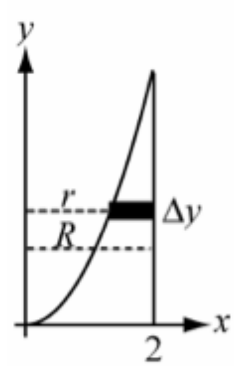
$$\text{Volume} = \pi \int_0^{\frac{\pi}{4}} \sec^2 x\, dx = \pi \tan x \Big|_0^{\pi/4} = \pi(\tan \frac{\pi}{4} - \tan 0) = \pi$$



34. A $\int_0^1 \left((x+8)-(x^3+8)\right) dx = \int_0^1 (x - x^3)\, dx = \left(\frac{1}{2}x^2 - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{1}{4}$

45. A Washers: $\sum \pi\left(R^2 - r^2\right)\Delta y$ where $R = 2,\ r = x$

$$\text{Volume} = \pi \int_0^4 \left(2^2 - x^2\right) dy = \pi \int_0^4 (4 - y)\, dy = \pi\left(4y - \frac{1}{2}y^2\right)\Big|_0^4 = 8\pi$$



15. The area of the region bounded by the lines $x=0$, $x=2$, and $y=0$ and the curve $y=e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

17. What is the area of the region completely bounded by the curve $y=-x^2+x+6$ and the line $y=4$?

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

15. C $\text{Area} = \int_0^2 e^{\frac{x}{2}}\,dx = 2e^{\frac{x}{2}}\Big|_0^2 = 2(e-1)$

17. C Determine where the curves intersect. $-x^2 + x + 6 = 4 \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0 \Rightarrow x = -1,\ x = 2$. Between these two x values the parabola lies above the line $y = 4$.

$\text{Area} = \int_{-1}^{2}\left((-x^2 + x + 6) - 4\right)dx = \left(-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right)\Big|_{-1}^{2} = \frac{9}{2}$

13. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

23. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m.
(B) increases as m increases.
(C) decreases as m increases.
(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

24. Let R be the region in the first quadrant bounded by the x-axis and the curve $y = 2x - x^2$. The volume produced when R is revolved about the x-axis is

(A) $\frac{16\pi}{15}$

(B) $\frac{8\pi}{3}$

(C) $\frac{4\pi}{3}$

(D) 16π

(E) 8π

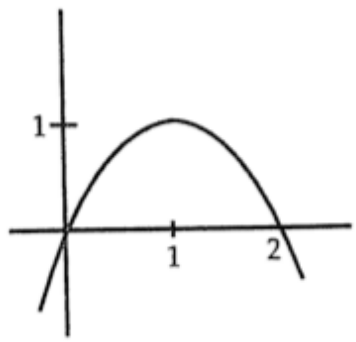
No Calculators

9. The area of the region completely bounded by the curve $y = -x^2 + 2x + 4$ and the line $y = 1$ is

(A) 8.7
(B) 9.7
(C) 10.7
(D) 11.7
(E) 12.7

Calculator Active

24. A p. 53



The volume of this solid formed by revolving about the x-axis is calculated using disks.

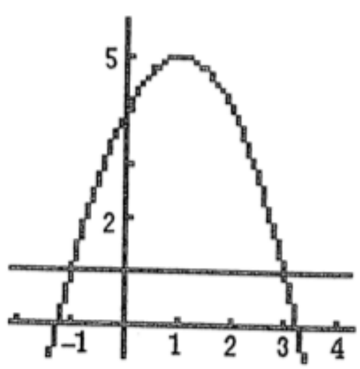
$$\pi \int_0^2 (2x - x^2)^2\, dx = \pi \int_0^2 (4x^2 - 4x^3 + x^4)\, dx$$

$$= \pi \left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]_0^2$$

$$= \pi \left[\frac{32}{3} - 16 + \frac{32}{5} \right]$$

$$= \pi \frac{160 - 240 + 96}{15} = \frac{16\pi}{15}$$

9. C p. 57



First determine the intersection points of the two functions.

$$-x^2 + 2x + 4 = 1$$
$$0 = x^2 - 2x - 3$$
$$0 = (x - 3)(x + 1)$$
$$x = -1,\ 3$$

The area is then

$$\int_{-1}^{3} (\text{top function} - \text{bottom function})\, dx.$$

$$\int_{-1}^{3} ((-x^2 + 2x + 4) - 1)\, dx = \int_{-1}^{3} (-x^2 + 2x + 3)\, dx \approx 10.667$$

26. The base of a solid is the region in the first quadrant bounded by the line $x = 2y + 4$ and the coordinate axes. What is the volume of the solid if every cross section perpendicular to the x-axis is a semicircle ?

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{3}$ (C) $\frac{8\pi}{3}$ (D) $\frac{32\pi}{3}$ (E) $\frac{64\pi}{3}$

26. Since every cross section is a semicircle, Area $= \frac{1}{2}\pi(\text{radius})^2 = \frac{1}{2}\pi\left(\frac{1}{2}y\right)^2 = \frac{\pi}{8}y^2$.

$$\text{Volume} = \int_0^4 \frac{\pi}{8}\left(\frac{4-x}{2}\right)^2 dx \quad (\text{since } x + 2y = 4 \Rightarrow 2y = 4 - x \text{ or } y = \tfrac{4-x}{2})$$

$$= \frac{\pi}{32}\int_0^4 (4-x)^2\, dx$$

$$= \frac{\pi}{32}\int_0^4 (16 - 8x + x^2)\, dx$$

$$= \frac{\pi}{32}\left(16x - 4x^2 + \frac{x^3}{3}\right)\Bigg|_0^4$$

$$= \frac{\pi}{32}\left[\left(64 - 64 + \frac{64}{3}\right) - (0)\right]$$

$$= \frac{\pi}{32}\left(\frac{64}{3}\right) = \frac{2\pi}{3}$$

The correct choice is (A).

32. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = 0$ and $x = \pi/2$ is divided into two regions by the line $x = c$. If the area of the region for $0 \le x \le c$ is equal to the area of the region for $c \le x \le \pi/2$, then c must be

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{9}$

(E) $\frac{5\pi}{18}$

No Calculators

11. Let R be the region in the first quadrant bounded above by the graph of $f(x) = 2\text{Arctan } x$ and below by the graph of $y = x$. What is the volume of the solid generated when R is rotated about the x-axis?

(A) 1.21 (B) 2.68 (C) 4.17 (D) 6.66 (E) 7.15

32. Since the area of the region for $0 \le x \le c$ is equal to the area of the region for $c \le x \le \frac{\pi}{2}$,

$$\int_0^c \cos x \, dx = \int_c^{\frac{\pi}{2}} \cos x \, dx$$

$$\sin x \Big|_0^c = \sin x \Big|_c^{\frac{\pi}{2}}$$

$$\sin c = 1 - \sin c$$

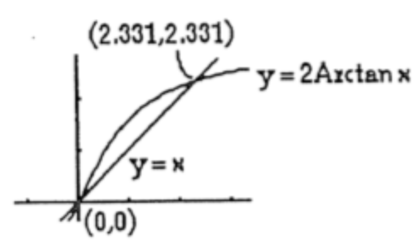
$$2 \sin c = 1$$

$$\sin c = \tfrac{1}{2} \Rightarrow c = \tfrac{\pi}{6}$$

The correct choice is (B).

11. E p. 13

The intersection points are found with the calculator.



Using the washer method, $V = \pi \cdot \int_0^{2.331} \left[(2\text{Arctan } x)^2 - x^2 \right] dx \approx 7.15$