14-6: Sum and Difference Identities

trig. std. 10.0

May 24

★ identities 4-9 on trig identities list

Examples: find exact values in simplest form.

1 $\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

2 $\cos 105^\circ = \cos(60^\circ + 45^\circ)$ (Q2)

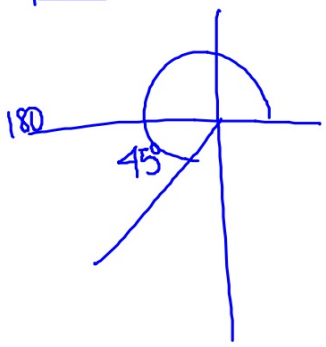
($\alpha = 60^\circ$, $\beta = 45^\circ$)

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

3 $\tan 285^\circ = \tan(225^\circ + 60^\circ)$ (Q4 − Q3) or $\tan(240^\circ + 45^\circ) = -\tan(75^\circ)$

($\alpha = 225^\circ$, $\beta = 60^\circ$)



$= \frac{\tan 225^\circ + \tan 60^\circ}{1 - \tan 225^\circ \tan 60^\circ} = \frac{1 + \sqrt{3}}{1 - (1)(\sqrt{3})}$

$= \frac{(1+\sqrt{3})}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{1+2\sqrt{3}+3}{1-3}$

$\frac{4+2\sqrt{3}}{-2} = -2 - \sqrt{3}$

Examples: simplify to a trig function of a single angle and find the exact value.

4 $\dfrac{\tan 80^\circ - \tan 20^\circ}{1 + \tan 80^\circ \tan 20^\circ} = \tan(80^\circ - 20^\circ) = \tan 60^\circ = \sqrt{3}$

5 $\sin \pi \cos \dfrac{\pi}{2} + \cos \pi \sin \dfrac{\pi}{2} = \sin\left(\pi + \dfrac{\pi}{2}\right) = \sin \dfrac{3\pi}{2} = -1$