

Name: _____ Date: _____ Row: _____ Period: _____

APPLICATIONS OF DERIVATIVES

Work through the FRQ on a separate sheet of paper and show all your work for the MC questions.

MULTIPLE-CHOICE QUESTIONS

A calculator may not be used on the following questions.

- Let M represent the absolute maximum of $f(x)$ in an interval.
Let R represent a root of $f(x)$ in the given interval.
Let m represent the absolute minimum of $f(x)$ in the interval.
If $f(x) = x^3 - 3x^2$, then which of the following is true over the closed interval $-3 \leq x \leq 1$?
 - M and R occur at a critical point and m occurs at an endpoint.
 - M and m occur at critical points.
 - M , m , and R occur at endpoints of the given interval.
 - M occurs at an endpoint, whereas m and R occur at a critical point.
 - M and R occur at an endpoint, whereas m occurs at a critical point.

2. What value of c in the open interval $(0, 4)$ satisfies the Mean Value Theorem for $f(x) = \sqrt{3x+4}$?

(A) 0

(B) $\frac{3}{5}$

(C) $\frac{5}{3}$

(D) 2

(E) 3

3. If $f'(x) = \frac{(x+1)x^2}{(x-1)^{\frac{1}{3}}}$, then on which interval(s) is the continuous function $f(x)$ increasing?

(A) $(-1, 1)$

(B) $(-\infty, -1) \cup (1, \infty)$

(C) $(-\infty, 0) \cup (1, \infty)$

(D) $(-\infty, -1) \cup (0, \infty)$

(E) $(1, \infty)$

4. The points of inflection for $f(x)$ are at $x = p_1$ and $x = p_2$. Which of the following is (are) true?

I. The points of inflection for $f(x - a)$ are at $x = p_1 + a$ and $x = p_2 + a$.

II. The points of inflection for $bf(x)$ are at $x = bp_1$ and $x = bp_2$.

III. The points of inflection for $f(cx)$ are at $x = \frac{p_1}{c}$ and $x = \frac{p_2}{c}$.

(A) I only

(B) II only

(C) I and II only

(D) III only

(E) I and III only

5. Evaluate: $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 14}}{3 - 2x}$.

(A) $-\infty$

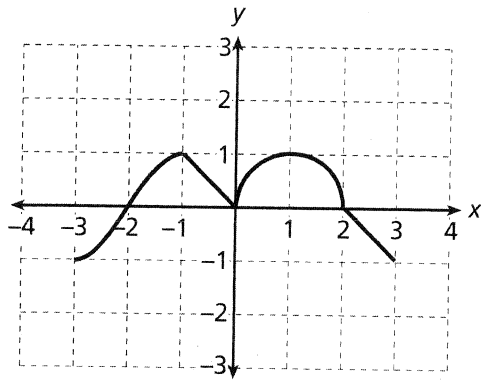
(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

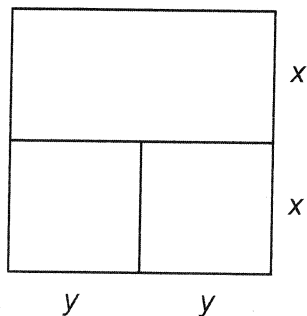
(D) $\frac{\sqrt{14}}{3}$

(E) ∞

6. The graph of $f'(x)$ is given below for $x \in [-3, 3]$. On which interval(s) is the function $f(x)$ both increasing and concave up?



- (A) $(-2, 2)$
 (B) $(-2, 0) \cup (0, 2)$
 (C) $(-3, -2)$
 (D) $(-2, -1) \cup (0, 1)$
 (E) None of these
7. A farmer has 100 yards of fencing to form two identical rectangular pens and a third pen that is twice as long as the other two pens, as shown in the diagram to the right. All three pens have the same width, x . Which value of y produces the maximum total fenced area?



- (A) $\frac{25}{2}$
 (B) 10
 (C) $\frac{100}{11}$
 (D) $\frac{25}{3}$
 (E) None of these

8. For the function $f(x) = 12x^5 - 5x^4$, how many of the inflection points of the function are also extrema?
- (A) 4
 - (B) 3
 - (C) 2
 - (D) 1
 - (E) None
9. The position of an object moving along a straight line for $t \geq 0$ is given by $s_1(t) = t^3 + 2$, and the position of a second object moving along the same line is given by $s_2(t) = t^2$. If both objects begin at $t = 0$, at what time is the distance between the objects a minimum?
- (A) 2
 - (B) $\frac{50}{27}$
 - (C) $\frac{2}{3}$
 - (D) 0
 - (E) None of these

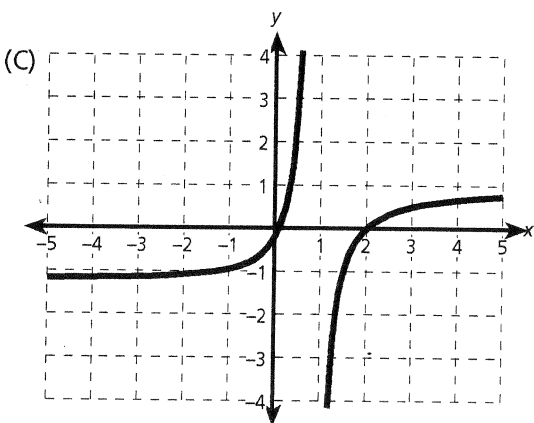
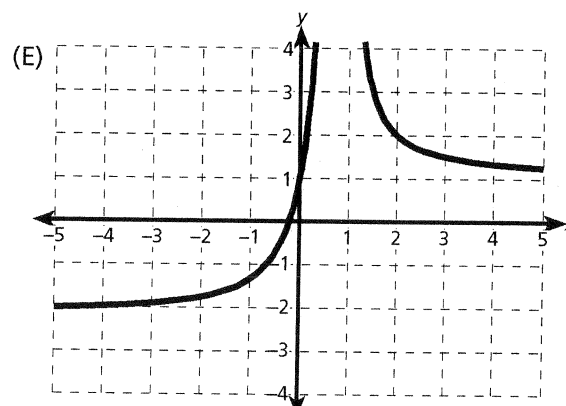
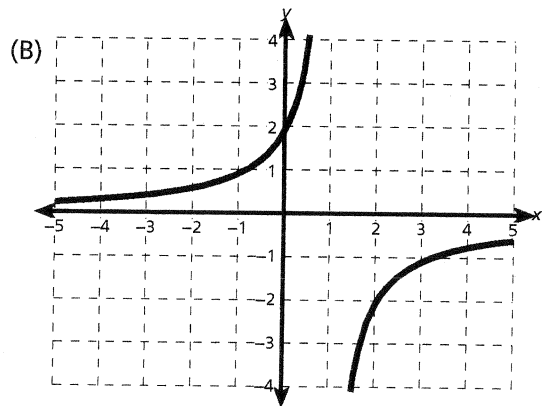
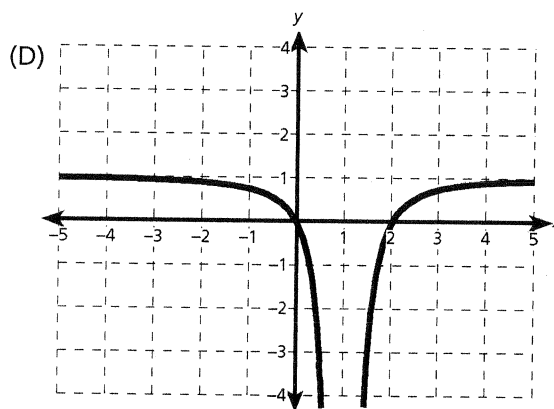
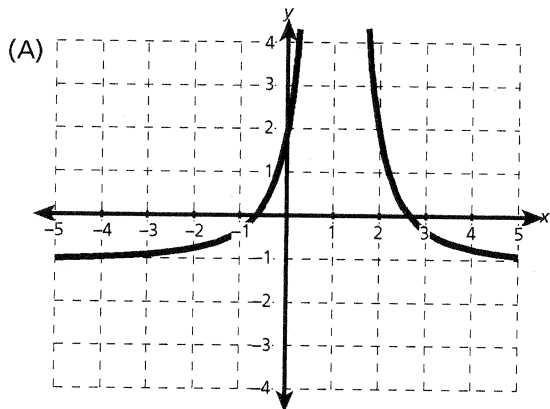
10. Given the following conditions for $f(x)$, which graph best illustrates $f(x)$?

$f(x)$: The domain of the function is the real numbers, but $x \neq 1$;

$$\lim_{x \rightarrow -\infty} f(x) = -1; \quad \lim_{x \rightarrow 1^-} f(x) = \infty; \quad \lim_{x \rightarrow 1^+} f(x) = -\infty.$$

$f'(x) > 0$ for all x where $x \neq 1$, and $f'(x)$ does not exist at $x = 1$.

$f''(x) > 0$ for $x < 1$, $f''(x) < 0$ for $x > 1$, and $f''(x)$ does not exist at $x = 1$.



A calculator may be used for the following question.

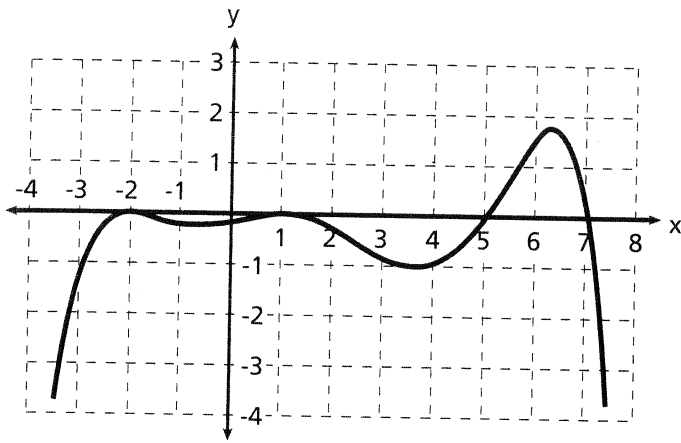
11. Let $f(x)$ be a function such that $f'(x) = \ln x \cdot \cos x + \frac{\sin x}{x}$. In the interval $0 < x < 3$, the graph of $f(x)$ has a point of inflection nearest $x =$
- (A) 0.352
 - (B) 1.101
 - (C) 2.128
 - (D) 2.259
 - (E) 2.901

A calculator may not be used on the following questions.

Questions 12 and 13 refer to the following information:

For time $0 \leq t \leq 10$, a particle moves along the x -axis with position given by $x(t) = t^3 - 7t^2 + 8t + 5$.

12. During what time intervals is the speed of the particle increasing?
- (A) $\frac{7}{3} < t \leq 10$ only
 - (B) $4 < t \leq 10$ only
 - (C) $0 \leq t < \frac{2}{3}$ and $\frac{7}{3} < t < 4$
 - (D) $0 \leq t < \frac{2}{3}$ and $4 < t \leq 10$
 - (E) $\frac{2}{3} < t < \frac{7}{3}$ and $4 < t \leq 10$
13. What is the position of the particle when it is farthest to the left?
- (A) -14
 - (B) -11
 - (C) $-\frac{47}{27}$
 - (D) 5
 - (E) $\frac{203}{27}$



14. Based on the graph of $g''(x)$ pictured above, how many points of inflection exist for the twice differentiable function $g(x)$ on the interval $-4 < x < 8$?
- (A) 5
 (B) 4
 (C) 3
 (D) 2
 (E) 1
15. A rectangle is drawn in the first quadrant so that it has two adjacent sides on the coordinate axes and one vertex on the curve $y = -\ln(x)$. Find the x coordinate of the vertex for which the area of the rectangle is a maximum.
- (A) $\frac{1}{2}$
 (B) $-\ln\left(\frac{1}{2}\right)$
 (C) $\frac{1}{e}$
 (D) e
 (E) $\frac{1}{e^2}$

FREE-RESPONSE QUESTION

This question does not require the use of a calculator.

1. The function $f(x)$ is defined as $f(x) = -2(x+2)(x-1)^2$ on the open interval $(-3, 3)$ as illustrated in the graph shown.
 - a. Determine the coordinates of the relative extrema of $f(x)$ in the open interval $(-3, 3)$.
 - b. Let $g(x)$ be defined as $g(x) = |f(x)|$ in the open interval $(-3, 3)$. Determine the coordinate(s) of the relative maxima of $g(x)$ in the open interval. Explain your reasoning.
 - c. For what values of x is $g'(x)$ not defined? Explain your reasoning.
 - d. Find all values of x for which $g(x)$ is concave down. Explain your reasoning.

