3.3b

23. $V(x) = (2x^3+3)(x^4-2x) = 2x^7 - x^4 - 6x$

$V'(x) = 14x^6 - 4x^3 - 6$

25. $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y+5y^3)$

$= (y^{-2} - 3y^{-4})(y+5y^3)$

$= y^{-1} + 5y - 3y^{-3} - 15y^{-1}$

$= -14y^{-1} + 5y - 3y^{-3}$

$F'(y) = 14y^{-2} + 5 + 9y^{-4}$

27. $g(x) = \dfrac{3x-1}{2x+1}$

$g'(x) = \dfrac{(2x+1)\frac{d}{dx}(3x-1) - (3x-1)\frac{d}{dx}(2x+1)}{(2x+1)^2}$

$6x+3-6x+2$

$= \dfrac{(2x+1)(3) - (3x-1)(2)}{(2x+1)^2}$

$= \dfrac{5}{(2x+1)^2}$

29. $y = \dfrac{x^3}{1-x^2}$

$y' = \dfrac{(1-x^2)\frac{d}{dx}(x^3) - x^3\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$

$= \dfrac{(1-x^2)(3x^2) - x^3(-2x)}{(1-x^2)^2}$

$3x^2 - 3x^4 + 2x^4$

$= \dfrac{3x^2 - x^4}{(1-x^2)^2} = \dfrac{x^2(3-x^2)}{(1-x^2)^2}$

31. $y = \dfrac{v^3 - 2v\sqrt{v}}{v} = \dfrac{v(v^2 - 2\sqrt{v})}{v} = v^2 - 2v = v^2 - 2v^{\frac{1}{2}}$

$y' = 2v - v^{-\frac{1}{2}}$

33. $y = \dfrac{t^2 + 2}{t^4 - 3t^2 + 1}$

$$y' = \frac{(t^4-3t^2+1)\frac{d}{dt}(t^2+2) - (t^2+2)\frac{d}{dt}(t^4-3t^2+1)}{(t^4-3t^2+1)^2}$$

$$= \frac{(t^4-3t^2+1)(2t) - (t^2+2)(4t^3-6t)}{(t^4-3t^2+1)^2}$$

$$= \frac{2t^5-6t^3+2t-(4t^5-6t^3+8t^3-12t)}{(t^4-3t^2+1)^2}$$

$$= \frac{-2t^5-4t^3-14t}{(t^4-3t^2+1)^2}$$

$$= \frac{-2t(t^4+4t^2+7)}{(t^4-3t^2+1)^2}$$

35. $y = ax^2 + bx + c$

$y' = 2ax + b$

37. $y = \dfrac{r^2}{1+\sqrt{r}} = \dfrac{r^2}{1+r^{\frac{1}{2}}}$

$$y' = \frac{(1+r^{\frac{1}{2}})\frac{d}{dr}(r^2) - r^2\frac{d}{dr}(1+r^{\frac{1}{2}})}{(1+r^{\frac{1}{2}})^2}$$

$$= \frac{(1+r^{\frac{1}{2}})(2r) - r^2(\frac{1}{2}r^{-\frac{1}{2}})}{(1+r^{\frac{1}{2}})^2}$$

$$= \frac{2r+2r^{\frac{3}{2}} - \frac{1}{2}r^{\frac{3}{2}}}{(1+r)^{\frac{1}{2}}}$$

$$= \frac{2r+\frac{3}{2}r^{\frac{3}{2}}}{(1+r)^{\frac{1}{2}}} = \frac{r(2+\frac{3}{2}r^{\frac{1}{2}})}{(1+r)^{\frac{1}{2}}}$$

39. $y = \sqrt[3]{t}\,(t^2 + t + t^{-1})$

$= t^{\frac{1}{3}}(t^2 + t + t^{-1})$

$= t^{\frac{7}{3}} + t^{\frac{4}{3}} + t^{\frac{-2}{3}}$

$y' = \frac{7}{3}t^{\frac{4}{3}} + \frac{4}{3}t^{\frac{1}{3}} - \frac{2}{3}t^{\frac{-5}{3}}$

$= \frac{1}{3}t^{\frac{-5}{3}}(7t^{\frac{9}{3}} + 4t^{\frac{6}{3}} - 2)$

$= \frac{1}{3}t^{\frac{-5}{3}}(7t^3 + 4t^2 - 2)$

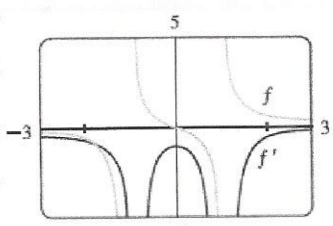
44. $f(x) = \dfrac{x}{x^2 - 1}$

$f'(x) = \dfrac{(x^2-1)\frac{d}{dx}(x) - x\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$

$= \dfrac{(x^2-1)(1) - x(2x)}{(x^2-1)^2}$

$= \dfrac{-x^2 - 1}{(x^2-1)^2}$

$= \dfrac{-(x^2+1)}{(x^2-1)^2}$



Notice that the slopes of all tangents to f are negative and $f'(x) < 0$ always.

53. $y = x + \sqrt{x} = x + x^{\frac{1}{2}}$ point $(1,2)$

$y' = 1 + \frac{1}{2}x^{-\frac{1}{2}}$

at $x = 1$ $y' = 1 + \frac{1}{2}(1)^{-\frac{1}{2}} = \frac{3}{2}$

tangent line:

at $x=1$ $y' = \frac{3}{2}$ pt $(1,2)$

$$y - 2 = \frac{3}{2}(x-1)$$

$$y = \frac{3}{2}x + \frac{1}{2}$$

normal line:

$m = -\frac{2}{3}$ pt $(1,2)$

$$y - 2 = -\frac{2}{3}(x-1)$$

$$y = -\frac{2}{3}x + \frac{8}{3}$$

56. $y = \frac{\sqrt{x}}{x+1} = \frac{x^{\frac{1}{2}}}{x+1}$ point $(4, 0.4)$

$$y' = \frac{(x+1)\frac{d}{dx}(x^{\frac{1}{2}}) - (x^{\frac{1}{2}})\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)(\frac{1}{2}x^{-\frac{1}{2}}) - x^{\frac{1}{2}}(1)}{(x+1)^2}$$

$$= \frac{\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} - x^{\frac{1}{2}}}{(x+1)^2}$$

$$= \frac{-\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}}{(x+1)^2}$$

(*) $$= \frac{\frac{1}{2}x^{-\frac{1}{2}}(-x+1)}{(x+1)^2}$$

$$= \frac{x^{-\frac{1}{2}}(-x+1)}{2(x+1)^2}$$

at $x = 4$ $y' = \frac{(4)^{-\frac{1}{2}}(-4+1)}{2(4+1)^2} = \frac{\frac{1}{2}(-3)}{50} = \frac{-3}{100}$

tangent line:

at $x = 4$ $y' = \frac{-3}{100}$ pt $(4, 0.4)$

$$y - 0.4 = \frac{-3}{100}(x-4)$$

$$y = \frac{-3}{100}x + \frac{13}{25}$$

normal line:

$m = \frac{100}{3}$ pt $(4, 0.4)$

$$y - 0.4 = \frac{100}{3}(x-4)$$

$$y = \frac{100}{3}x - \frac{1994}{15}$$

62. $s = 2t^3 - 7t^2 + 4t + 1$

ⓐ $v(t) = s' = 6t^2 - 14t + 4$

$a(t) = v'(t) = s'' = 12t - 14$

ⓑ $a(1) = 12(1) - 14 = -2 \ m/sec^2$

ⓒ