14-3: Simplifying Expressions and Proving Identities

Simplify to a single trig function:

1. $\tan y(\tan y + \cot y)$

2. $$\frac{\sec y + \csc y}{1 + \tan y} = \frac{\cos y \sin y\left[\frac{1}{\cos y} + \frac{1}{\sin y}\right]}{\cos y \sin y\left[1 + \frac{\sin y}{\cos y}\right]}$$

$$= \frac{\sin y + \cos y}{\cos y \sin y + \sin^2 y}$$

$$= \frac{\overset{1}{\cancel{\sin y + \cos y}}}{\sin y\cancel{(\cos y + \sin y)}}$$

$$= \boxed{\csc y}$$

Prove:

3 $\dfrac{\sec^2 y - 1}{\sec^2 y} \equiv \sin^2 y$

4 $\dfrac{\cos y}{1 + \sin y} + \tan y \equiv \sec y$

$\dfrac{\cos y}{\cos y} \cdot \dfrac{\cos y}{1+\sin y} + \dfrac{\sin y}{\cos y}$

$\dfrac{\cos^2 y + \sin y(1+\sin y)}{\cos y(1+\sin y)}$

$\dfrac{\overbrace{\cos^2 y}^{(1-\sin^2 y)} + \sin y + \sin^2 y}{\cos y(1+\sin y)}$

$\dfrac{1 + \sin y}{\cos y(1+\sin y)}$

$\dfrac{1}{\cos y}$

$\dfrac{1}{\cos y}$