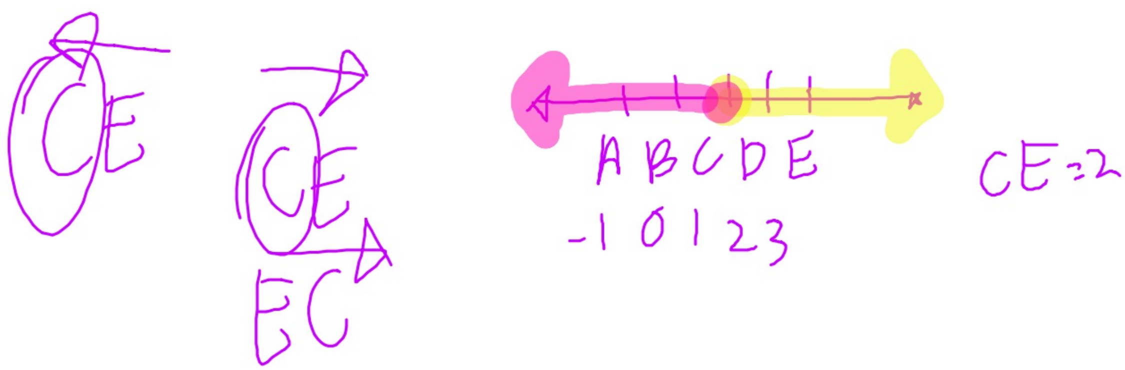
Lesson 1 - 6

Midpoint and Distance in the Coordinate Plane

Going Deeper

Essential question: *How can you find midpoints of segments and distances in the coordinate plane?*



CC.9-12.G.GPE.6

1 EXPLORE Finding Midpoints of Line Segments

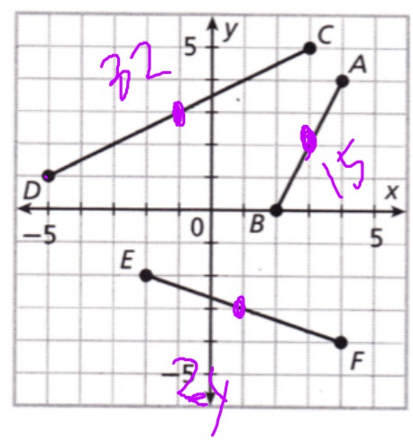
Follow the steps below for each of the given line segments.

A Use a ruler to measure the length of the line segment to the nearest millimeter.

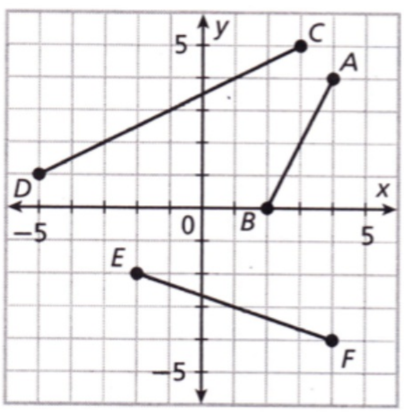
B Find half the length of the segment. Measure this distance from one endpoint to locate the midpoint of the segment. Plot a point at the midpoint.

C Record the coordinates of the segment's endpoints and the coordinates of the segment's midpoint in the table below.

D In each row of the table, compare the x-coordinates of the endpoints to the x-coordinate of the midpoint. Then compare the y-coordinates of the endpoints to the y-coordinate of the midpoint. Look for patterns.



Endpoint	Endpoint Coordinates	Endpoint	Endpoint Coordinates	Midpoint Coordinates
A	(4, 4)	*B*	(2, 0)	(3, 2)
C	(3, 5)	*D*	(−5, 1)	(−1, 3)
E	(−2, −2)	*F*	(4, −4)	(1, −3)



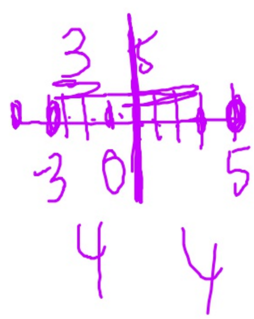
$3 + -5 = -2$

$\frac{-2}{2} = -1$

REFLECT

1a. Make a conjecture: If you know the coordinates of the endpoints of a line segment, how can you find the coordinates of the midpoint?

(5, 10)



(-3, 0)

1b. What are the coordinates of the midpoint of a line segment with endpoints at the origin and at the point (a, b)?

$$\left(\frac{a}{2}, \frac{b}{2}\right)$$

(a, b)

$(0, 0)$

The patterns you observed can be generalized to give a formula for the coordinates of the midpoint of any line segment in the coordinate plane.

The Midpoint Formula

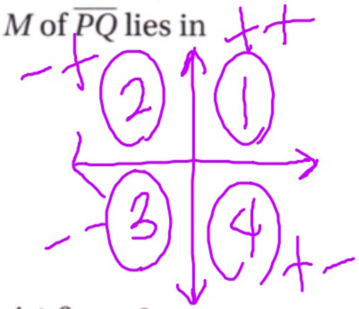
The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

CC.9-12.G.GPE.4

2 EXAMPLE Using the Midpoint Formula

$x_1\ y_1$ $x_2\ y_2$

$\overline{PQ}$ has endpoints $P(-4, 1)$ and $Q(2, -3)$. Prove that the midpoint M of $\overline{PQ}$ lies in Quadrant III.



A Use the given endpoints to identify x_1, x_2, y_1, and y_2.

$x_1 = -4$, $x_2 = 2$, $y_1 =$ 1, $y_2 =$ −3

B By the midpoint formula, the x-coordinate of M is $\frac{x_1 + x_2}{2} = \frac{-4 + 2}{2} = \frac{-2}{2} = -1$.

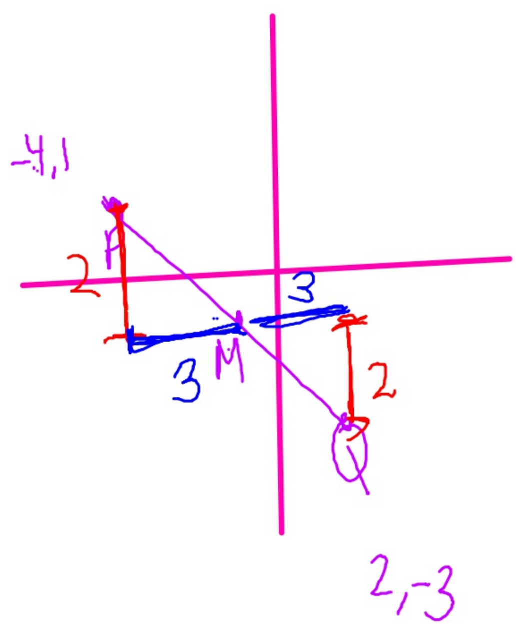
The y-coordinate of M is $\frac{y_1 + y_2}{2} = \frac{1 + (-3)}{2} = \frac{-2}{2} = -1$.

M lies in Quadrant III because (−1, −1)

(−1, −1) Both the x and y coodinates of the midpoint are negative

REFLECT

2a. What must be true about *PM* and *QM*? Show that this is the case.



PREP FOR CC.9-12.G.GPE.4

3 EXPLORE Finding a Distance in the Coordinate Plane

$\sqrt{a^2+b^2} = \sqrt{c^2}$

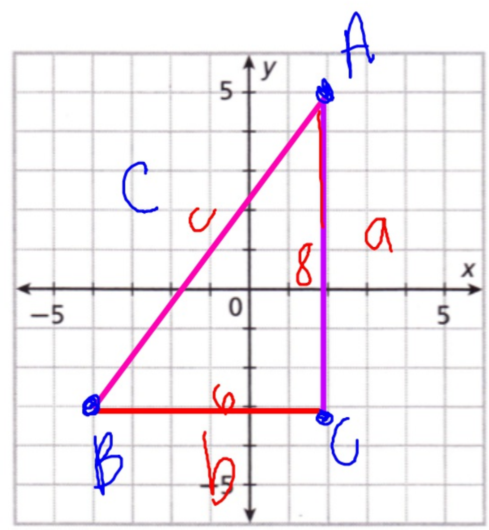
You can use the Pythagorean Theorem to help you find the distance between the points $A(2, 5)$ and $B(-4, -3)$.

$\sqrt{a^2+b^2} = c$

A Plot the points A and B in the coordinate plane at right.

B Draw $\overline{AB}$.

C Draw a vertical line through point A and a horizontal line through point B to create a right triangle. Label the intersection of the vertical line and the horizontal line point C.



D Each small grid square is 1 unit by 1 unit. Use this fact to find the lengths AC and BC.

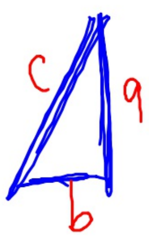
$AC =$ 8 $\qquad BC =$ 6

E By the Pythagorean Theorem, $AB^2 = AC^2 + BC^2$.
Complete the following using the lengths from Step D.

$AB^2 = 8^2 + 6^2$

$c^2 = a^2 + b^2$

$\sqrt{c^2} = \sqrt{64 + 36}$

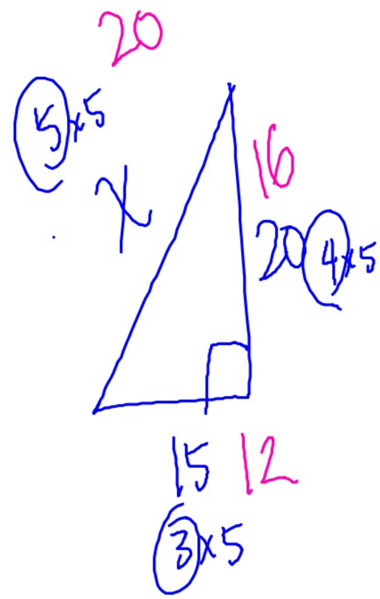


$c = \sqrt{100}$

$c = 10$

6 : 8 : 10

3 4 5



F Simplify the right side of the equation. Then solve for AB.

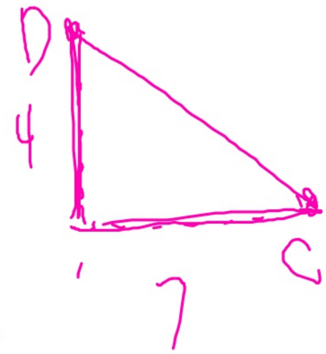
$AB^2 =$ 100, $AB =$ 10

REFLECT

3a. Explain how you solved for AB in Step F.

3b. Can you use the above method to find the distance between any two points in the coordinate plane? Explain.

$m = \frac{4}{-7}$



$\sqrt{16 + 49}$

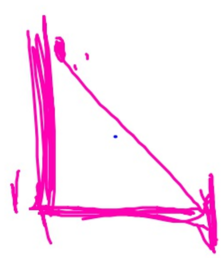
$\sqrt{65}$

The process of using the Pythagorean Theorem can be generalized to give a formula for finding the distance between two points in the coordinate plane.

The Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) in the coordinate plane is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



$\sqrt{6^2 + 5^2}$

$\sqrt{36 + 25}$

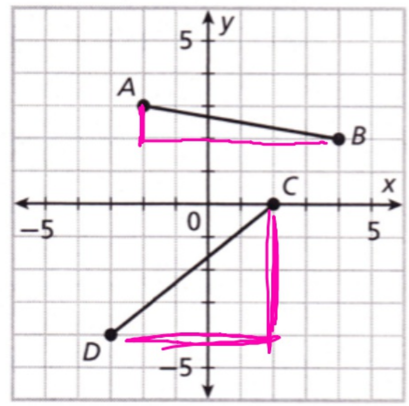
$\sqrt{61}$

$\sqrt{75}$ $\sqrt{25 \cdot 3}$ $5\sqrt{3}$

CC.9–12.G.GPE.4

4 EXAMPLE Using the Distance Formula

Prove that $\overline{CD}$ is longer than $\overline{AB}$.



A Write the coordinates of A, B, C, and D.

A(−2, 3), B (4,2), C (2,0), D (−3,−4)

B Use the distance formula to find AB and CD.

$$AB = \sqrt{[4-(-2)]^2 + (2-3)^2} = \sqrt{6^2 + (-1)^2} = \sqrt{36+1} = \sqrt{37}$$

$$CD = \sqrt{(-3 - 2)^2 + (4 - 0)^2} = \sqrt{(-5)^2 + (4)^2} = \sqrt{41}$$

So, $\overline{CD}$ is longer than $\overline{AB}$ because $\sqrt{41} > \sqrt{37}$

REFLECT

4a. When you use the distance formula, does the order in which you subtract the x-coordinates and the y-coordinates matter? Explain.

$(x - x)^2$

$1 - 4 = -3$

$4 - 1$ 3

PRACTICE

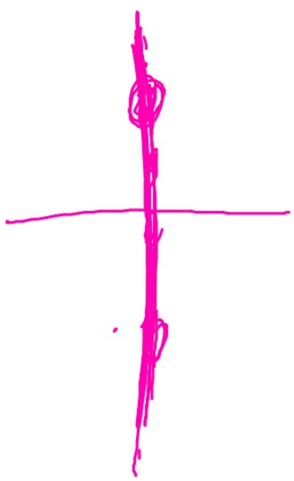
1. Find the coordinates of the midpoint of $\overline{AB}$ with endpoints $A(-10, 3)$ and $B(2, -2)$. $\left(-4, \frac{1}{2}\right)$

$$\frac{-10+2}{2} = \frac{-8}{2}$$

$$\frac{3-2}{2} = \frac{1}{2}$$

2. $\overline{RS}$ has endpoints $R(3, 5)$ and $S(-3, -1)$. Prove that the midpoint M of $\overline{RS}$ lies on the y-axis.

(0, #)

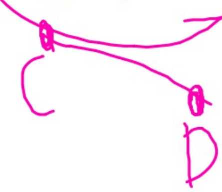


$$\frac{3-3}{2} = \frac{0}{2}$$

3. $\overline{CD}$ has endpoints $C(1, 4)$ and $D(5, 0)$. $\overline{EF}$ has endpoints $E(4, 5)$ and $F(2, -1)$. Prove that the segments have the same midpoint.

$\frac{1+5}{2}$ $\frac{4+0}{2}$

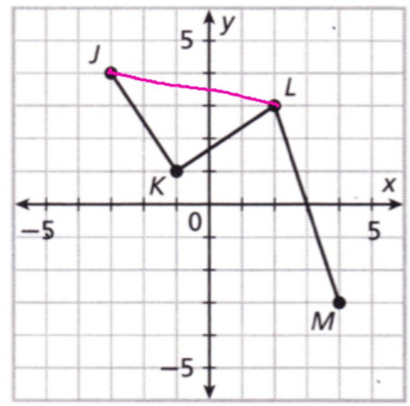
(3, 2)



$\frac{4+2}{2}$, $\frac{5-1}{2}$

Find mp of each

4. Find the distance between *J* and *L*.



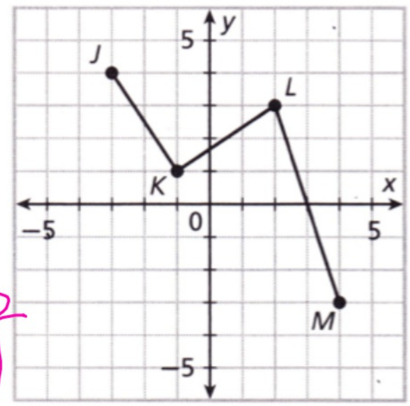
(-3,4) (2,3)

$$JL = \sqrt{(-3-2)^2 + (4-3)^2}$$

$$\sqrt{25+1}$$

$$JL = \sqrt{26}$$

5. Find the length of $\overline{LM}$. ______________



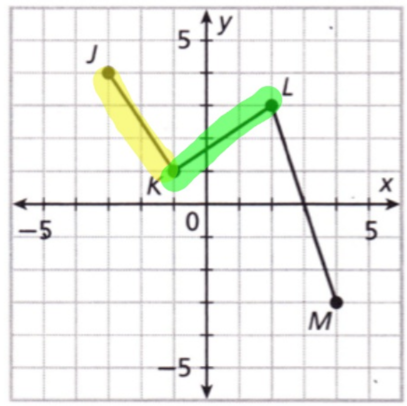
L(2,3) M(4,−3)

$$LM = \sqrt{(4-2)^2 + (3+3)^2}$$

$$\sqrt{4 + 36}$$

$$\sqrt{40} \quad 2\sqrt{10}$$

6. Prove that $JK = KL$.



7. $\overline{GH}$ has endpoint $G(2, 7)$ and midpoint $M(5, 1)$. Write two equations you can use to find the coordinates of the endpoint H. Then solve the equations and write the coordinates of H.

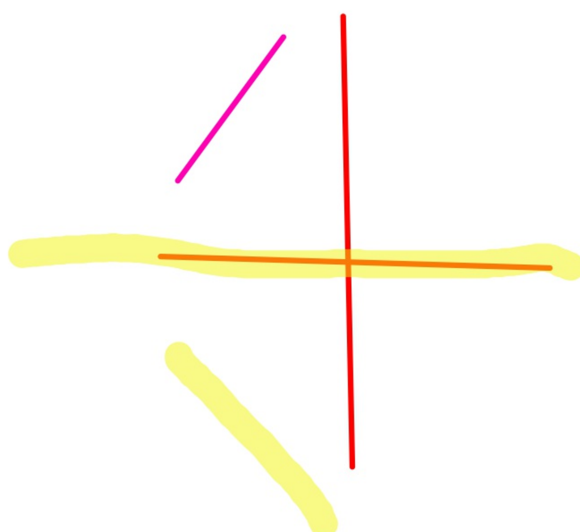
$D = \sqrt{(x-x)^2+(y-y)^2}$

G M H

(2, 7) (5, 1) ??

(8, -5)

8. A segment $\overline{AB}$ has endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$. The segment is reflected across the x-axis. Find the coordinates of the reflected segment. Then show the length of the reflected segment is the same as the length of the original segment. Explain your reasoning.



Homework 1.6
AP and PS (pages35-36)
Textbook Page 526 #1-12 even
(distance and midpoint)

Written Exercises

Find the distance between the two points. If necessary, you may draw graphs but you shouldn't need to use the distance formula.

2. (3, 3) and (−2, 3)
4. (0, 0) and (3, 4)

Use the distance formula to find the distance between the two points.

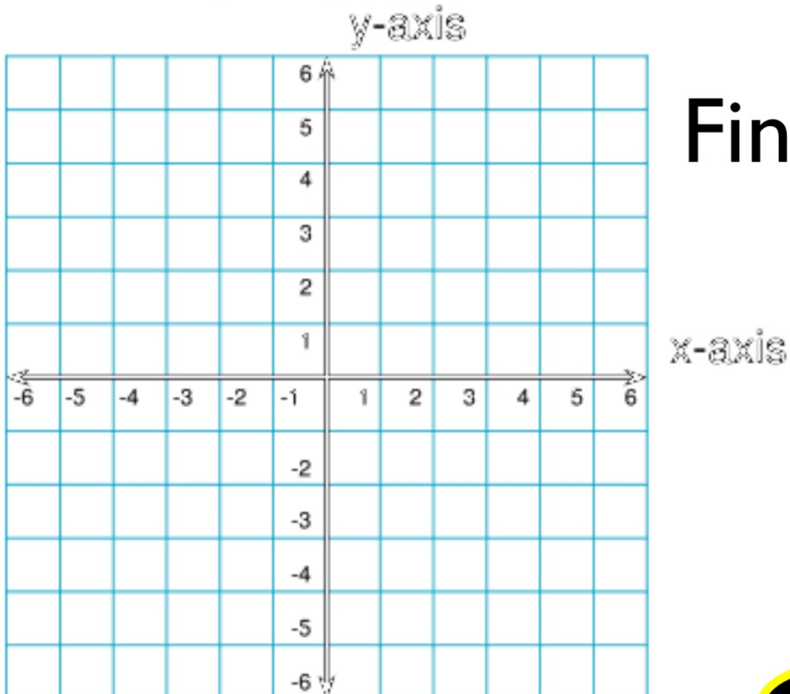
6. (3, 2) and (5, −2)
8. (12, −1) and (0, −6)

Find the distance between the points named. Use any method you choose.

10. (−2, −2) and (5, 7)
12. (−4, −1) and (−4, 3)

1-3: Find the distance between the two points.

2) $(5,8)$ and (-7,8)

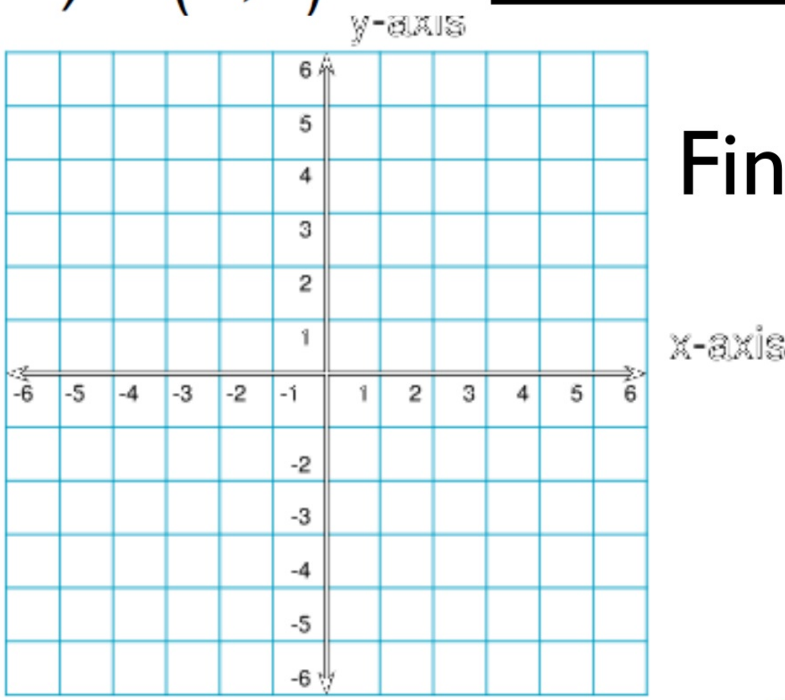


Find the midpoint

Answer

1-3: Find the distance between the two points.

3) $(2,3)$ and $\underline{(2,-3)}$

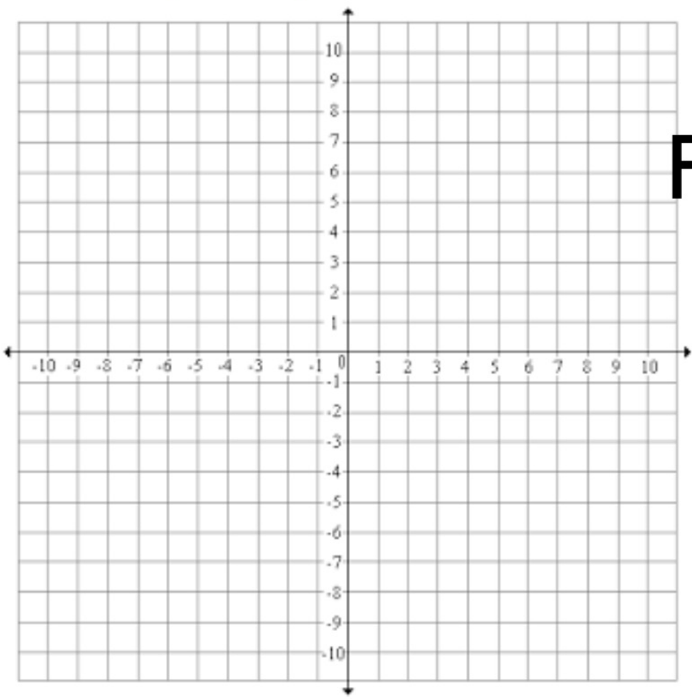


Find the midpoint

Answer

4-7: Find the distance between the two points.

4) $(4,2)$ and ___(1, -1)___

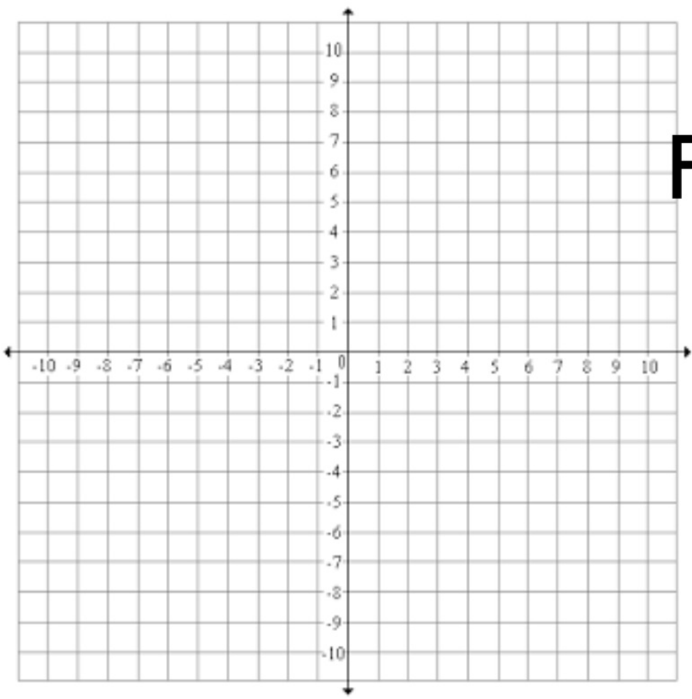


Find the midpoint

Answer

4-7: Find the distance between the two points.

5) $(-5, -5)$ and (7,1)



Find the midpoint

Answer

Additional Practice

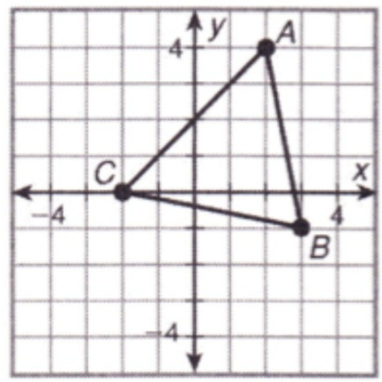
Find the coordinates of the midpoint of each segment.

1. $\overline{TU}$ with endpoints $T(5, -1)$ and $U(1, -5)$

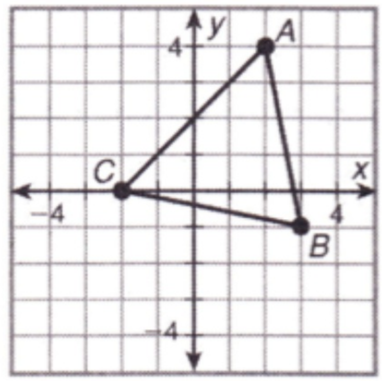
2. $\overline{VW}$ with endpoints $V(-2, -6)$ and $W(x + 2, y + 3)$

3. Y is the midpoint of $\overline{XZ}$. X has coordinates (2, 4), and Y has coordinates (–1, 1). Find the coordinates of Z.

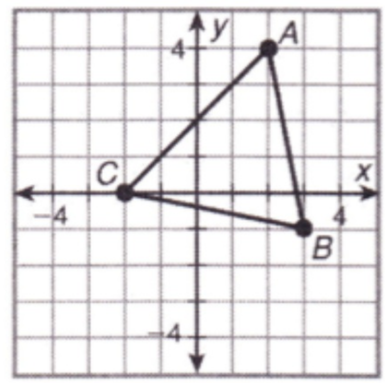
4. Find *AB*. _______________



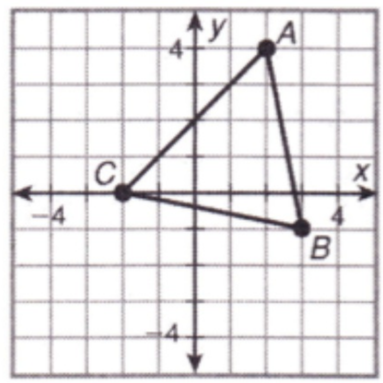
5. Find *BC*. _______________



6. Find *CA*. ________________



7. Name a pair of congruent segments.



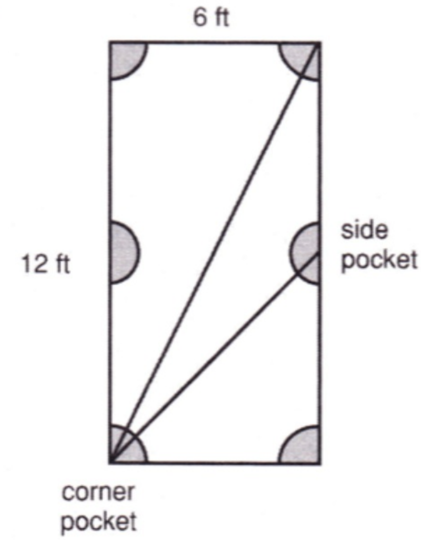
Find the distances.

8. Use the Distance Formula to find the distance, to the nearest tenth, between $K(-7, -4)$ and $L(-2, 0)$.

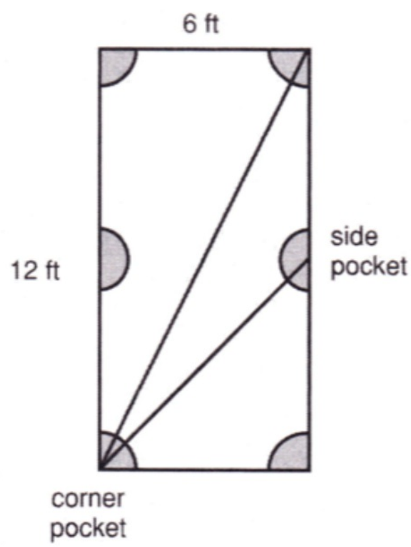
9. Use the Pythagorean Theorem to find the distance, to the nearest tenth, between $F(9, 5)$ and $G(-2, 2)$.

Snooker is a kind of pool or billiards played on a 6-foot-by-12-foot table. The side pockets are halfway down the rails (long sides).

10. Find the distance, to the nearest tenth of a foot, diagonally across the table from corner pocket to corner pocket.

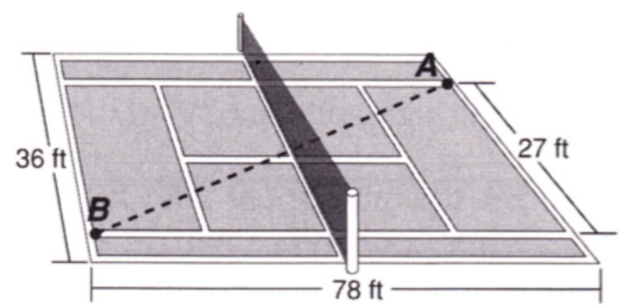


11. Find the distance, to the nearest tenth of an inch, diagonally across the table from corner pocket to side pocket.

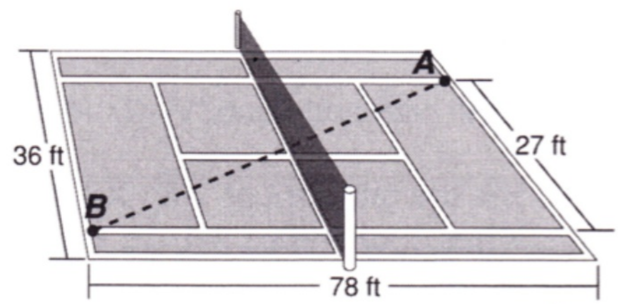


For Exercises 1 and 2, use the diagram of a tennis court

1. A singles tennis court is a rectangle 27 feet wide and 78 feet long. Suppose a player at corner *A* hits the ball to her opponent in the diagonally opposite corner *B*. Approximately how far does the ball travel, to the nearest tenth of a foot?



2. A doubles tennis court is a rectangle 36 feet wide and 78 feet long. If two players are standing in diagonally opposite corners, about how far apart are they, to the nearest tenth of a foot?



A map of an amusement park is shown on a coordinate plane, where each square of the grid represents 1 square meter. The water ride is at (–17, 12), the roller coaster is at (26, –8), and the Ferris wheel is at (2, 20). Find each distance to the nearest tenth of a meter.

3. What is the distance between the water ride and the roller coaster?

A map of an amusement park is shown on a coordinate plane, where each square of the grid represents 1 square meter. The water ride is at (–17, 12), the roller coaster is at (26, –8), and the Ferris wheel is at (2, 20). Find each distance to the nearest tenth of a meter.

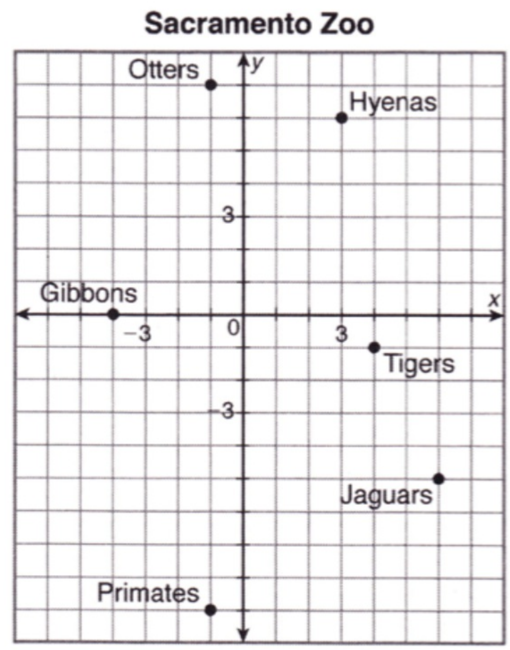
4. A caricature artist is at the midpoint between the roller coaster and the Ferris wheel. What is the distance from the artist to the Ferris wheel?

Use the map of the Sacramento Zoo on a coordinate plane for Exercises 5–7. Choose the best answer.

5. To the nearest tenth of a unit, how far is it from the tigers to the hyenas?

   A 5.1 units C 9.9 units

   B 7.1 units D 50.0 units



6. Between which of these exhibits is the distar the least?

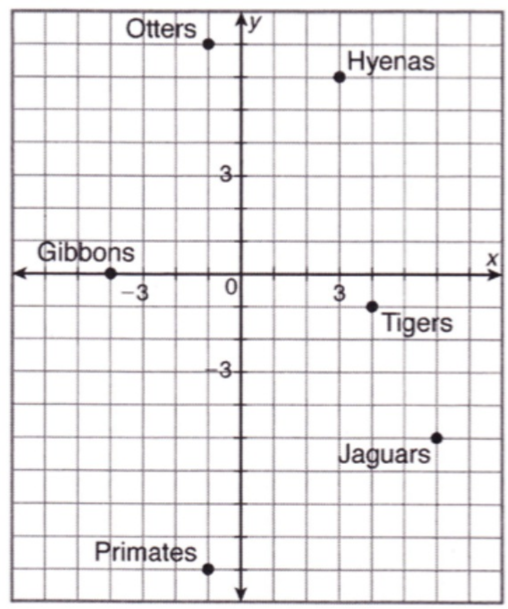
F tigers and primates

G hyenas and gibbons

H otters and gibbons

J tigers and otters

Sacramento Zoo



7. Suppose you walk straight from the jaguars to the tigers and then to the otters. What is the total distance to the nearest tenth of a unit?

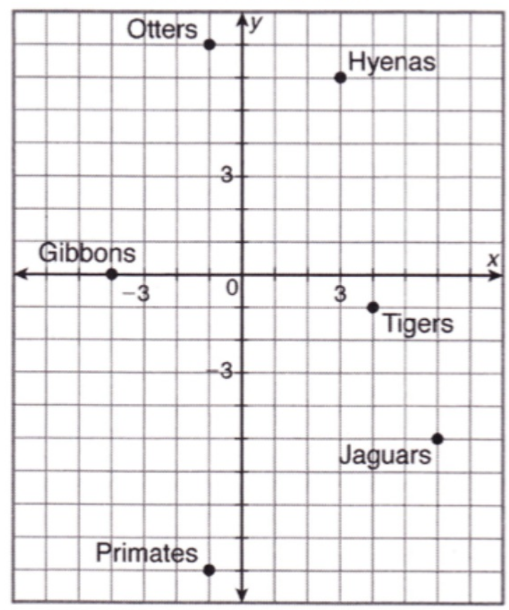
A 11.4 units

B 13.0 units

C 13.9 units

D 14.2 units

Sacramento Zoo



A Yes

B No