Review A

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① $2x^4 - 7x^3 + 13x^2 - 28x + 20$

$$\begin{array}{r|rrrrr} -1 & 2 & -7 & 13 & -28 & 20 \\ & & -2 & 9 & -22 & 50 \\ \hline & 2 & -9 & 22 & -50 & 70 \end{array}$$

Lower bound (alternating signs)

$$\begin{array}{r|rrrrr} 1 & 2 & -7 & 13 & -28 & 20 \\ & & 2 & -5 & 8 & -20 \\ \hline & 2 & -5 & 8 & -20 & \boxed{0} \end{array}$$

1 is a zero

$$\begin{array}{r|rrrrr} 2 & 2 & -7 & 13 & -28 & 20 \\ & & 4 & -6 & 14 & -28 \\ \hline & 2 & -3 & 7 & -14 & -8 \end{array}$$

$$\begin{array}{r|rrrrr} 3 & 2 & -7 & 13 & -28 & 20 \\ & & 6 & -3 & 30 & 6 \\ \hline & 2 & -1 & 10 & 2 & 26 \end{array}$$

$$\begin{array}{r|rrrrr} 4 & 2 & -7 & 13 & -28 & 20 \\ & & 8 & 4 & 68 & 160 \\ \hline & 2 & 1 & 17 & 40 & 180 \end{array}$$

upper bound all pos

$-1 < x < 4$

②

x	y
-1	70
0	20
1	0
2	-8
3	26
4	180

Since $f(2) = -8$ & $f(3) = 26$ there's a real root between 2 and 3

③ Possible rational roots:

$$\frac{\pm 1 \quad \pm 2 \quad \pm 4 \quad \pm 5 \quad \pm 10 \quad \pm 20}{\pm 1 \quad \pm 2}$$

$\pm 1 \quad \pm 2 \quad \pm 4 \quad \pm 5 \quad \pm 10 \quad \pm 20$

$\pm \frac{1}{2} \quad \pm \frac{5}{2}$

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(4) Because the upperbound is 4, there's no need to try 5, 10, 20.

Because the lowerbound is -1, there's no need to try $-2, -4, -5, -10, -20, -\frac{5}{2}$

Since according to the location principle, there's a real root between 2 & 3, try $\frac{5}{2}$

$$\begin{array}{c|ccccc} & 2x^4 & -7 & 13 & -28 & 20 \\ \frac{5}{2} & & 5 & -5 & 20 & -20 \\ \hline & 2x^3 & -2 & 8 & -8 & \boxed{0} \end{array}$$

→ $\frac{5}{2}$ is a zero

$$\begin{array}{c|cccc} & 2x^3 & -2 & 8 & -8 \\ 1 & & 2 & 0 & 8 \\ \hline & 2x^2 & 0 & 8 & \boxed{0} \end{array}$$

we know that 1 is also a zero

$2x^2 + 0x + 8 = 0$

$2x^2 + 8 = 0$

$2x^2 = -8 \rightarrow x^2 = -4 \rightarrow x = \pm\sqrt{-4} \rightarrow x = \pm 2i$

(5) $f(x) = 2x^4 - 7x^3 + 13x^2 - 28x + 20$

4 sign changes → 4, 2, 0 possible + roots

$f(-x) = 2(-x)^4 - 7(-x)^3 + 13(-x)^2 - 28(-x) + 20$

$= 2x^4 + 7x^3 + 13x^2 + 28x + 20$

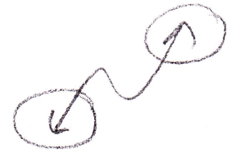
no sign changes → no neg roots

Total	+	−	i
4	4	0	0
	2	0	2
	0	0	4

→ this is the scenario

Review

⑥ $4x^3+x^2-18x$ → + coeff ; odd exp



$x(4x^2+x-18)$

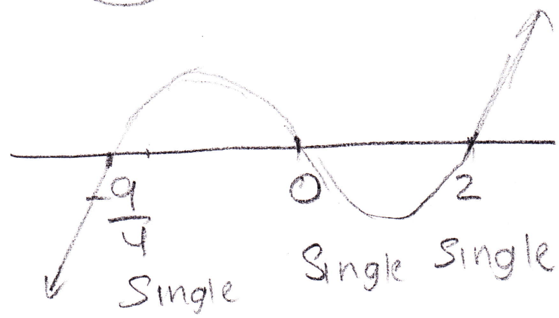
$4x \quad +9$

$x \quad -2$

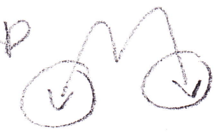
$x(4x+9)(x-2)$

$\downarrow \quad \downarrow \quad \downarrow$

$0 \quad -\frac{9}{4} \quad 2$



⑦ $-x^4+10x^2-24$ − coef ; even exp



$-x^2 \quad +2$

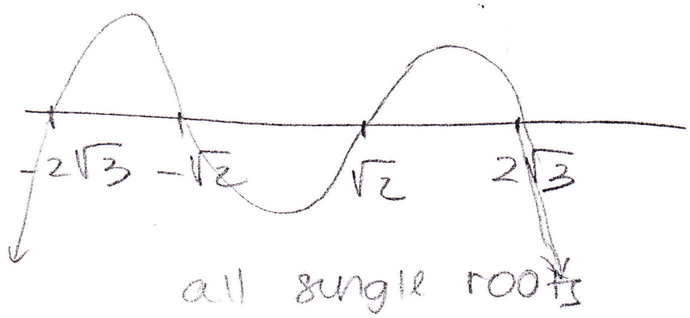
$x^2 \quad -12$

$(-x^2+2)(x^2-12)$

$x^2=2 \qquad x^2=12$

$x=\pm\sqrt{2} \qquad x=\pm 2\sqrt{3}$

$=\pm 1.41 \qquad \pm 3.46$



⑧ $6x^3-9x^2+x+0$

sum $= \frac{--9}{6} = \frac{3}{2}$; prod $= \frac{-0}{6} = 0$

⑨ $(5+\sqrt{2})+(5-\sqrt{2}) = 10$

$(5+\sqrt{2})(5-\sqrt{2}) = 25+5\sqrt{2}-5\sqrt{2}-2 = 23$

$x^2 -$ sum $+$ product

$x^2-10x+23=0$

(10) $(2-i\sqrt{5})+(2+i\sqrt{5})=4$

$(2-i\sqrt{5})(2+i\sqrt{5})=4+2i\sqrt{5}-2i\sqrt{5}-5i^2$

$=4+5=9$

x^2- sum $+$ product

$x^2-4x+9=0$

(11) $(7-i)+(7+i)=14$

$(7-i)(7+i)=49+7i-7i-i^2=50$

$(x^2-14x+50)(x-3)=$

x^3-14x^2+50x

$-3x^2+42x-150$

$x^3-17x^2+92x-150=0$

(12) $(1+i\sqrt{7})+(1-i\sqrt{7})=2$

$(1+i\sqrt{7})(1-i\sqrt{7})=1-7i^2=8$

x^2-2x+8

$(2+i)+(2-i)=4$

$(2+i)(2-i)=4-i^2=5$

x^2-4x+5

$(x^2-2x+8)(x^2-4x+5)$

$x^4-4x^3+5x^2$

$-2x^3+8x^2-10x$

$+8x^2-32x+40$

$x^4-6x^3+21x^2-42x+40=0$

Review

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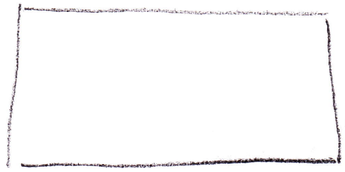
(13)



W=x

L = 40-x

$2L + 2W = 80$

$W = x \rightarrow 2L + 2x = 80$

$2L = 80 - 2x$

$L = \frac{80-2x}{2} = 40 - x$

$\text{Area} = x(40-x)$

zeros $\quad x=0 \quad x=40$

AOS $\quad x = \frac{0+40}{2} = 20$

$\rightarrow W = 20 \rightarrow L = 40 - 20 = 20$

$\text{Area} = 20(20) = 400$

(14)

original	change	new
80,000	−10000x	80000 −10000x
1.60	+.40x	1.6 + .4x

$f(x) = (80000 - 10,000x)(1.6 + .4x)$

$\downarrow$ 8 $\qquad \downarrow$ −4

$\text{AOS} = x = \frac{8 + -4}{2} = 2$

new customers = $80,000 - 10,000(2) = 60,000$

new price = $1.6 + .4(2) = \$2.40$

new revenue = $60,000(2.40) = \$144,000$