18. Method A is a convenience sample. Method B is a cluster sample. Method C is a stratified sample. Method C is the most accurate because every orange is equally likely to be in the sample. If spoiled fruit is not evenly distributed over the different crates, Method B can give a biased result, hence it is less accurate. Method A is the easiest method to implement, but it is the least accurate.

19. Answers will vary.

20. The local newspaper's survey clearly shows the majority's preference. Start with the 56% of survey respondents who were in favor of the Northern Avenue location. Since there is a margin of error of $\pm$4%, the percent of the entire population in favor of Northern Avenue is between 52% and 60%. Of survey respondents, 44% are not in favor of the Northern Avenue location. Applying the margin of error means that between 40% and 48% of the entire population is not in favor of Northern Avenue. There is no overlap.

21. The sophomore class survey does not clearly indicate the majority's preference. Start with the 55% that prefer the museum. Since there is a margin of error of $\pm$6%, the actual percent of students the survey shows prefer going to the museum is between 49% and 61%. Of surveyed students, 45% prefer the amusement park. Applying the margin of error, between 39% and 51% of the entire population of students would prefer going to the amusement park. There is an overlap. Since the intervals overlap, the survey does not clearly show the population's preference.

22. The school newspaper's survey does not clearly indicate the majority's preference. Start with the 58% that planned to vote for Diedrich. Since there is a margin of error of $\pm$12%, the actual percent of students the survey shows will vote for Diedrich is between 46% and 70%. 42% of surveyed students plan to vote for LeBlanc. Applying the margin of error means LeBlanc will get between 30% and 54% of the vote. There is an overlap. LeBlanc has a more accurate understanding of the results as he recognizes that he still has a chance to win the election.

TEST PREP

23. B; only the shoppers that were motivated to sign up for the mailing list receive the survey, and only those motivated to return the survey are actually sampled.

24. A; Only those who are interested and able to answer the questionnaire will do so, so this is a convenience sample.

25. D; A population sample is a sample where every member of the population being sampled has a nonzero probability of being selected. In a convenience sample, the people who are not conveniently accessible have zero chance of being selected, so a convenience sample is not a probability sample.

26. H; self-selected and convenience samples are often biased; in a stratified sample, it may be difficult to separate the population into groups.

CHALLENGE AND EXTEND

27. Answers will vary. One way to approach the problem follows. For a simple random survey city officials need a list of all city residents. Survey respondents should be chosen at random from that list (random number generators, picking names out of a hat, etc.). For a systematic sample, city officials can go down the list and choose every 20th name, or every 50th name or every 3rd name (depending on how large a sample they want) and survey those residents.
For a stratified sample city officials might first group city residents by the neighborhoods they live in and randomly select a certain number of residents to survey from each neighborhood.

28a. A simple random survey of only 25 might wind up excluding one or more of the different specialties. The specialty with the fewest members, dermatologists, would be the most likely to be excluded.

b. Given that 40 out of 500, or 8%, of the doctors are dermatologists, 8% of the 25 doctors being surveyed should be dermatologists. Therefore, 2 dermatologists should be surveyed.

29. Questions b, c, and d are likely to produce biased results. Revised questions are:
For b: Do you prefer the taste of sample A or the taste of sample B?
For c: Would you rather read a book or watch a movie?
For d: Do you enjoy watching music videos?

8-6 BINOMIAL DISTRIBUTIONS

CHECK IT OUT!

1a. $(x - y)^5$

$= {}_5C_0x^5(-y)^0 + {}_5C_1x^4(-y)^1 + {}_5C_2x^3(-y)^2 + {}_5C_3x^2(-y)^3 + {}_5C_4x^1(-y)^4 + {}_5C_5x^0(-y)^5$

$= x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

b. $(a + 2b)^3$

$= {}_3C_0a^3(2b)^0 + {}_3C_1a^2(2b)^1 + {}_3C_2a^1(2b)^2 + {}_3C_3a^0(2b)^3$

$= 1 \cdot a^3 + 3 \cdot 2a^2b + 3 \cdot 4ab^2 + 1 \cdot 8b^3$

$= a^3 + 6a^2b + 12ab^2 + 8b^3$

2a. The probability that a student will be assigned to Counselor Jenkins is $\frac{1}{3}$.

$$P(2) = {}_3C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1$$
$$= 3\left(\frac{1}{9}\right)\left(\frac{2}{3}\right)$$
$$= \frac{2}{9} \approx 0.22$$

The probability that exactly 2 students will be assigned to Counselor Jenkins is about 22%.

b. The probability that Ellen will guess the right answer is $\frac{1}{4}$.

$P(2) + P(3) + P(4) + P(5)$
$= {}_5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 + {}_5C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 + {}_5C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 + {}_5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0$
$= 10\left(\frac{1}{16}\right)\left(\frac{27}{64}\right) + 10\left(\frac{1}{64}\right)\left(\frac{9}{16}\right) + 5\left(\frac{1}{256}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{1024}\right)$
$= \frac{47}{128} \approx 0.37$

The probability that Ellen will get at least 2 answers correct by guessing is about 37%.

3a. $P(\text{at least 2 correct})$
$= 1 - P(\text{0 or 1 corrrect})$
$= 1 - \left[{}_{20}C_0(0.25)^0(0.75)^{20} + {}_{20}C_1(0.25)^1(0.75)^{19}\right]$
$\approx 1 - \left[0.003 + 20(0.25)(0.004)\right]$
$= 1 - 0.023$
$= 0.977$

The probability that Wendy will get at least 2 correct answers by guessing is about 98%.

b. $P(\text{23 or fewer})$
$= 1 - P(\text{24 or 25})$
$= 1 - \left[{}_{25}C_{24}(0.98)^{24}(0.02)^1 + {}_{25}C_{25}(0.98)^{25}(0.02)^0\right]$
$\approx 1 - \left[25(0.6158)(0.02) + 0.6035\right]$
$= 1 - 0.9114$
$= 0.0886$

The probability that there are 23 or fewer acceptable parts is about 9%.

THINK AND DISCUSS

1. Possible answer: 1; a binomial experiment has only 2 outcomes, and the probability of those outcomes are p and q. So, $p + q = 1$.
2. ${}_nC_r$, p^r, and q^{n-r}
3.

Binomial Experiments	
Probability	**Example**
Probability of r successes in n trials	2 successes in 5 trials, where $p = 0.9$ $= {}_5C_2(0.9)^2(0.1)^3 \approx 0.0081$
Probability of at least r successes	At least 4 successes in 5 trials, where $p = 0.9$ $= {}_5C_4(0.9)^4(0.1)^1 + {}_5C_5(0.9)^5(0.1)^0 \approx 0.9185$
Probability of at most r successes	At most 1 success in 5 trials, where $p = 0.9$ $= {}_5C_0(0.1)^5 + {}_5C_1(0.9)^1(0.1)^4 \approx 0.00046$
Probability using a complement	At least 2 successes in 5 trials, where $p = 0.9$ $= 1 - p$ (0 or 1 success) $= 1 - 0.00046 \approx 0.99954$

EXERCISES

GUIDED PRACTICE

1. 2
2. $(x + 3)^4$
$= {}_4C_0x^43^0 + {}_4C_1x^33^1 + {}_4C_2x^23^2 + {}_4C_3x^13^3 + {}_4C_4x^03^4$
$= 1 \cdot x^4 + 4 \cdot 3x^3 + 6 \cdot 9x^2 + 4 \cdot 27x + 1 \cdot 81$
$= x^4 + 12x^3 + 54x^2 + 108x + 81$

3. $(3x + 5)^3$
$= {}_3C_0(3x)^35^0 + {}_3C_1(3x)^25^1 + {}_3C_2(3x)^15^2 + {}_3C_3(3x)^05^3$
$= 1 \cdot 27x^3 + 3 \cdot 45x^2 + 3 \cdot 75x + 1 \cdot 125$
$= 27x^3 + 135x^2 + 225x + 125$

4. $(p - 2)^6$
$= {}_6C_0p^6(-2)^0 + {}_6C_1p^5(-2)^1 + {}_6C_2p^4(-2)^2 + {}_6C_3p^3(-2)^3 + {}_6C_4p^2(-2)^4 + {}_6C_5p^1(-2)^5 + {}_6C_6p^0(-2)^6$
$= 1 \cdot p^6 + 6 \cdot \left(-2p^5\right) + 15 \cdot 4p^4 + 20 \cdot \left(-8p^3\right) + 15 \cdot 16p^2 + 6 \cdot (-32p) + 1 \cdot 64$
$= p^6 - 12p^5 + 60p^4 - 160p^3 + 240p^2 - 192p + 64$

5. $(x + y)^6$
$= {}_6C_0x^6y^0 + {}_6C_1x^5y^1 + {}_6C_2x^4y^2 + {}_6C_3x^3y^3 + {}_6C_4x^2y^4 + {}_6C_5x^1y^5 + {}_6C_6x^0y^6$
$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

6. $P(4) = {}_6C_4(0.30)^4(0.7)^2$
$= 15(0.0081)(0.49)$
≈ 0.060

The probability that exactly 4 athletes will be chosen is about 0.060.

$P(4) + P(5) + P(6)$
$\approx 0.060 + {}_6C_5(0.30)^5(0.7)^1 + {}_6C_6(0.30)^6(0.7)^0$
$\approx 0.060 + 0.0102 + 0.0007$
≈ 0.070

The probability that at least 4 athletes are chosen is about 0.070.

7. $P(3) = {}_4C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^1$
$= 4\left(\frac{1}{125}\right)\left(\frac{4}{5}\right)$
$= \frac{16}{625} \approx 0.026$

The probability of getting exactly 3 coupons is about 0.026.

$P(\text{at least 2})$
$= 1 - P(\text{0 or 1})$
$= 1 - \left[{}_4C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 + {}_4C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^3\right]$
$= 1 - \left[\frac{256}{625} + 4\left(\frac{1}{5}\right)\left(\frac{64}{125}\right)\right]$
$= 1 - \frac{512}{625}$
$= \frac{113}{625} \approx 0.181$

The probability of getting at least 2 coupons is about 0.181.

8. $P(\text{at least 2 upside-down stamps})$
$= 1 - P(\text{0 or 1 upside-down stamps})$
$= 1 - \left[{}_{30}C_0(0.02)^0(0.98)^{30} + {}_{30}C_1(0.02)^1(0.98)^{29}\right]$
$\approx 1 - \left[0.545 + 30(0.02)(0.557)\right]$
$= 1 - 0.8792$
≈ 0.121

The probability of getting at least 2 boxes with an upside-down stamp is 0.121.

PRACTICE AND PROBLEM SOLVING

9. $(y + 5)^4$
$= {}_4C_0 y^4 5^0 + {}_4C_1 y^3 5^1 + {}_4C_2 y^2 5^2 + {}_4C_3 y^1 5^3 + {}_4C_4 y^0 5^4$
$= 1 \cdot y^4 + 4 \cdot 5y^3 + 6 \cdot 25y^2 + 4 \cdot 125y + 1 \cdot 625$
$= y^4 + 20y^3 + 150y^2 + 500y + 625$

10. $(2m - 1)^3$
$= {}_3C_0(2m)^3(-1)^0 + {}_3C_1(2m)^2(-1)^1 + {}_3C_2(2m)^1(-1)^2 + {}_3C_3(2m)^0(-1)^3$
$= 1 \cdot 8m^3 + 3 \cdot (-4m^2) + 3 \cdot 2m + 1 \cdot (-1)$
$= 8m^3 - 12m^2 + 6m - 1$

11. $(4 + 3x)^5$
$= {}_5C_0 4^5(3x)^0 + {}_5C_1 4^4(3x)^1 + {}_5C_2 4^3(3x)^2 + {}_5C_3 4^2(3x)^3 + {}_5C_4 4^1(3x)^4 + {}_5C_5 4^0(3x)^5$
$= 1 \cdot 1024 + 5 \cdot 768x + 10 \cdot 576x^2 + 10 \cdot 432x^3 + 5 \cdot 324x^4 + 1 \cdot 243x^5$
$= 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

12. $(2a + 3c)^3$
$= {}_3C_0(2a)^3(3c)^0 + {}_3C_1(2a)^2(3c)^1 + {}_3C_2(2a)^1(3c)^2 + {}_3C_3(2a)^0(3c)^3$
$= 1 \cdot 8a^3 + 3 \cdot 12a^2c + 3 \cdot 18ac^2 + 1 \cdot 27c^3$
$= 8a^3 + 36a^2c + 54ac^2 + 27c^3$

13. $P(6) + P(7) + P(8)$
$= {}_8C_6(0.83)^6(0.17)^2 + {}_8C_7(0.83)^7(0.17)^1 + {}_8C_8(0.83)^8(0.17)^1$
$\approx 28(0.327)(0.029) + 8(0.271)(0.17) + 0.225$
≈ 0.86
The probability that at least 6 students agree with the statement is about 0.86.

14. $P(2) = {}_5C_2(0.15)^2(0.85)^3$
$\approx 10(0.023)(0.614)$
≈ 0.14
The probability that exactly 2 marbles are black is about 0.14.
$P(\text{at least } 2)$
$= 1 - P(0 \text{ or } 1)$
$= 1 - \left[{}_5C_0(0.15)^0(0.85)^5 + {}_5C_1(0.15)^1(0.85)^4\right]$
$\approx 1 - [0.444 + 5(0.15)(0.522)]$
$= 1 - 0.8355$
≈ 0.16
The probability that at least 2 marbles are black is about 0.16.

15. $P(\text{2 girls and 1 boy}) = {}_3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$
$= 3\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)$
$= \frac{3}{8} = 0.375$
$P(\text{3 girls}) = {}_3C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^0$
$= \frac{1}{8} = 0.125$

16. $P(\text{at least } 4)$
$= 1 - P(0, 1, 2, \text{ or } 3)$
$= 1 - \left[{}_{15}C_0(0.25)^0(0.75)^{15} + {}_{15}C_1(0.25)^1(0.75)^{14} + {}_{15}C_2(0.25)^2(0.75)^{13} + {}_{15}C_3(0.25)^3(0.75)^{12}\right]$
$\approx 1 - 0.461$
≈ 0.54

17. $(x - y)^5$
$= {}_5C_0 x^5(-y)^0 + {}_5C_1 x^4(-y)^1 + {}_5C_2 x^3(-y)^2 + {}_5C_3 x^2(-y)^3 + {}_5C_4 x^1(-y)^4 + {}_5C_5 x^1(-y)^5$
$= x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

18. $(c + 6)^3$
$= {}_3C_0 c^3 6^0 + {}_3C_1 c^2 6^1 + {}_3C_2 c^1 6^2 + {}_3C_3 c^0 6^3$
$= 1 \cdot c^3 + 3 \cdot 6c^2 + 3 \cdot 36c + 1 \cdot 216$
$= c^3 + 18c^2 + 108c + 216$

19. $(4k - 1)^4$
$= {}_4C_0(4k)^4(-1)^0 + {}_4C_1(4k)^3(-1)^1 + {}_4C_2(4k)^2(-1)^2 + {}_4C_3(4k)^1(-1)^3 + {}_4C_4(4k)^0(-1)^4$
$= 1 \cdot 256k^4 + 4 \cdot (-64k^3) + 6 \cdot 16k^2 + 4 \cdot (-4k) + 1 \cdot 1$
$= 256k^4 - 256k^3 + 96k^2 - 16k + 1$

20. $(p + q)^7$
$= {}_7C_0 p^7q^0 + {}_7C_1 p^6q^1 + {}_7C_2 p^5q^2 + {}_7C_3 p^4q^3 + {}_7C_4 p^3q^4 + {}_7C_5 p^2q^5 + {}_7C_6 p^1q^6 + {}_7C_7 p^0q^7$
$= p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 + 35p^3q^4 + 21p^2q^5 + 7pq^6 + q^7$

21. $P(2) = {}_3C_2(0.8)^2(0.2)^1$
$= 3(0.64)(0.2)$
$= 0.384$

22. $P(1) = {}_5C_1(0.5)^1(0.5)^4$
$= 5(0.5)(0.0625)$
$= 0.15625$

23. $P(2) = {}_4C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2$
$= 6\left(\frac{1}{9}\right)\left(\frac{4}{9}\right)$
$= \frac{8}{27} \approx 0.30$

24. To seat all of the passengers who arrive, only 20 or less of the passengers who bought tickets need to show up.
$P(20 \text{ or less})$
$= 1 - P(21 \text{ or } 22)$
$= 1 - \left[{}_{22}C_{21}(0.91)^{21}(0.09)^1 + {}_{22}C_{22}(0.91)^{22}(0.09)^0\right]$
$\approx 1 - 0.40$
$= 0.60$
The probability that all of the passengers who arrive will have a seat is about 0.60.

25. $P(\text{4 males}) = {}_4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$
$= \frac{1}{16} = 0.0625$
$P(\text{at least 3 males})$
$= P(\text{3 males}) + P(\text{4 males})$
$= {}_4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 + \frac{1}{16}$
$= 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right) + \frac{1}{16}$
$= \frac{5}{16} \approx 0.3125$

26. $P(\text{more than 7 heads})$
$= P(8) + P(9) + P(10)$

$= {}_{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + {}_{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 + {}_{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0$

$= \frac{45}{1024} + \frac{5}{512} + \frac{1}{1024}$

$= \frac{7}{128} \approx 0.055$

27. P(at least 2 heads)

$= 1 - P(0 \text{ or } 1 \text{ heads})$

$= 1 - \left[{}_{10}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{10} + {}_{10}C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^9 \right]$

$= 1 - \frac{11}{1024}$

$= \frac{1013}{1024} \approx 0.989$

28. P(5 heads)

$= {}_{10}C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5$

$= \frac{63}{256} \approx 0.25$

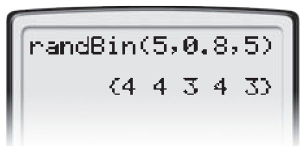
29. P(0 or 1 defective)

$= {}_8C_0(0.05)^0(0.95)^8 + {}_8C_1(0.05)^1(0.95)^7$

≈ 0.94

The probability of no more than 1 defective part in a box of 8 is about 0.94.

30a. Possible answer: randBin(5, 0.8, 5)



b. P(at least 4)

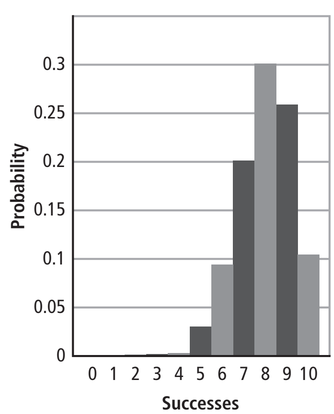
$= P(4) + P(5)$

$= {}_5C_4(0.8)^4(0.2)^1 + {}_5C_5(0.8)^5(0.2)^0$

≈ 0.74

c. Possible answer: $0.6 < 0.74$

31.



The bar heights increase nearly exponentially from 0 successes to 7 successes, maximize at 8, and drop at 10. The expected value is 8.

32. 3 of one and 1 of the other:

$2 \cdot \left({}_4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 \right) = 2 \cdot \frac{4}{16} = 0.5$

2 of each: $\frac{6}{16} = .375$

33a. $P(\text{rain}) = \frac{82}{365} \approx 0.22$

b. $P(3 \text{ rainy days}) = {}_7C_3(0.22)^3(0.78)^4$

≈ 0.14

c. P(at least 3 rainy days)

$= 1 - P(0, 1, \text{ or } 2 \text{ rainy days})$

$= 1 - \left[{}_7C_0(0.22)^0(0.78)^7 + {}_7C_1(0.22)^1(0.78)^6 + {}_7C_2(0.22)^2(0.78)^5 \right]$

≈ 0.19

34. The trials are dependent. For a binomial experiment the trials must be independent.

35. P(delayed at least 3 times)

$= P(3) + P(4)$

$= {}_4C_3(0.2046)^3(0.7954)^1 + {}_4C_4(0.2046)^4(0.7954)^0$

≈ 0.03

36a. P(home run)

$= {}_6C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^5$

$= \frac{3}{32} \approx 0.09375$

b. P(out)

$= {}_6C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^6 + {}_6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4 + {}_6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3$

$= \frac{1}{64} + \frac{15}{64} + \frac{5}{16}$

$= \frac{9}{16} \approx 0.5625$

c. P(hit)

$= {}_6C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^4 + {}_6C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1 + {}_6C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0 + \frac{3}{32}$

$= \frac{15}{64} + \frac{3}{32} + \frac{1}{64} + \frac{3}{32}$

$= \frac{7}{16} \approx 0.4375$

d. They are complements.

37. P(fewer than 3)

$= {}_5C_0(0.45)^0(0.55)^5 + {}_5C_1(0.45)^1(0.55)^4 + {}_5C_2(0.45)^2(0.55)^3$

≈ 0.59

38. Possible answer: in 20 coin flips, the probability of getting fewer than 18 heads

39. P(2 or fewer)

$\approx 0.017 + 0.08 + 0.195$

≈ 0.3

40. Possible answer: ≈ 0.33; $P(0 \text{ successes}) \approx 0.017$. So, solve $x^{10} = 0.017$ to find the complement, ≈ 0.67, and subtract from 1.

TEST PREP

41. B.
Binomial trials are independent.

42. H
${}_2C_1(0.4)^1(0.6)^1$
$= 0.48$

43. B.
It matches the Binomial trial formula.

44. P(0 or 1 impefect parts)

$= {}_{10}C_0(0.04)^0(0.94)^{10} + {}_{10}C_0(0.04)^0(0.94)^{10}$

$\approx 0.94 = 94\%$

45. $P(3 \text{ or more})$
$= 1 - P(0, 1, \text{ or } 2)$
$= 1 - \left[{}_{10}C_0(0.188)^0(0.812)^{10} + {}_{10}C_1(0.188)^1(0.812)^9 + {}_{10}C_2(0.188)^2(0.812)^8 \right]$
≈ 0.29

CHALLENGE AND EXTEND

46a. 65; number of people × probability left-handed

b. standard deviation $= \sqrt{npq}$
$= \sqrt{650(0.1)(0.9)} \approx 7.6485$
So, n is approximately between 57.3515 and 72.6485
So, $\{ n \mid 58 \le n \le 72 \}$.

47a. $P(\text{at least one 1 in 6 rolls})$
$= 1 - P(\text{zero 1s})$
$= 1 - {}_6C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^6$
≈ 0.67

b. $P(\text{at least two 1s in 12 rolls})$
$= 1 - P(\text{zero or one 1})$
$= 1 - \left[{}_{12}C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} + {}_{12}C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} \right]$
≈ 0.62

48. $P(\text{at least } 4)$
$= 1 - P(\text{at most } 3)$
enter: $1 - \text{binomcdf}(20, 0.4, 3) \approx 0.984$

49. ${}_nC_r + {}_nC_{r+1}$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)![n-(r+1)]!}$$
$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$$
$$= \frac{n!}{r!(n-r)(n-r-1)!} + \frac{n!}{(r+1)r!\,(n-r-1)!}$$
$$= \left(\frac{r+1}{r+1}\right)\frac{n!}{r!(n-r)(n-r-1)!} + \left(\frac{n-r}{n-r}\right)\frac{n!}{(r+1)r!(n-r-1)!}$$
$$= \frac{n!(r+1)}{(r+1)!(n-r)(n-r-1)!} + \frac{n!(n-r)}{(r+1)r!(n-r)!}$$
$$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$$
$$= \frac{n!(n+1)}{(r+1)!(n-r)!}$$
$$= \frac{(n+1)!}{(r+1)!(n-r)!}$$
$$= {}_{n+1}C_{r+1}$$

50a. $P(1) = {}_2C_1p^1(1-p)^1$
The equation is: $2p(1-p) = 0.4$
$2p - 2p^2 = 0.4$
$-2(p^2 - p + 0.2) = 0.$
$p = \frac{1 \pm \sqrt{1 - 4(0.2)}}{2} \approx 0.72 \text{ or } \approx 0.28$
So, $p \approx 0.72$ or ≈ 0.28

b. $P(2) = {}_2C_2(0.72)^2(0.28)^0$
≈ 0.52 or ≈ 0.076 if $p \approx 0.28$.

8-7 FITTING TO A NORMAL DISTRIBUTION

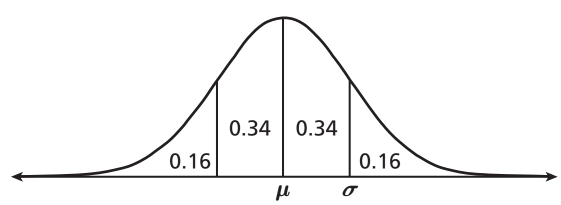
CHECK IT OUT!

1. Looking at the graph, there approximately 19 blocks under the curve that are less than 400. Therefore, the probability is about 0.19.
2. The probability that $x > 106$ is desired. The following is the z score:
$z = \frac{106 - 142}{18} = \frac{-36}{18} = -2$
Use the Standard Normal Values table to find the area under the curve for all values greater than −2, which is $1 - 0.02 = 0.98$. The probability of scoring more than 106 is about 0.98.
3. Because of symmetry of the normal distribution, there should be approximately the same number of values above and below the mean. However, the table has 14 values below the mean and 4 values above the mean. Therefore, the data does not appear to be normally distributed. The table below confirms that the data values are not normally distributed.

z	Area	X	Values below x	
			Projected	Actual
−2	0.02	5	0	0
−1	0.16	21	3	0
0	0.5	37	9	14
1	0.84	53	15	15
2	0.98	69	18	18

THINK AND DISCUSS

1. Subtract 50 from x, and divide the result by 5.
2. The total area under the normal curve is 1, so if the area for x-values less than a is p, then the area for x-values greater than a is $1 - p$.
3.



EXERCISES

GUIDED PRACTICE

1. The standard normal value of a statistic is found by subtracting the mean from the statistic and dividing the result by the standard deviation.
2. There are approximately 83 boxes underneath the curve and to the right of 6. Therefore, the probability is approximately 0.83.