25. $P(x < 16) = 0.16$ means the z value corresponding to this probability is -1.

$$-1 = \frac{16 - \mu}{\sigma}$$
$$-1\sigma = 16 - \mu$$

$P(x < 26) = 0.93$ means the z value corresponding to this probability is 1.5.

$$1.5 = \frac{26 - \mu}{\sigma}$$
$$1.5\sigma = 26 - \mu$$

Solve these two equations in two unknowns.

$$1.5\sigma = 26 - \mu$$
$$-1\sigma = 16 - \mu$$

Subtract the two equations.

$$2.5\sigma = 10$$
$$\sigma = 4$$

Substitute this value into one of the original equations to find μ.

$$-1(4) = 16 - \mu$$
$$-4 = 16 - \mu$$
$$-20 = -\mu$$
$$20 = \mu$$

26a. The transformation shifts the curve to the right by μ units.

b. The curve is compressed horizontally by a factor of σ.

c. $f(x)$ is the value at x of the normal curve with mean μ and standard deviation σ. The transformation $f\left(\frac{x - \mu}{\sigma}\right)$ shifts the normal curve so that the center is 0 and the standard deviation is 1. So $f(z)$ is the value of the standard normal curve that corresponds to $f(x)$.

8-8 ANALYZING DECISIONS

CHECK IT OUT!

1.

Value of Side	1	2	2	3	3	5
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$EV = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right)$$

$$EV = \frac{1 + 2 + 2 + 3 + 3 + 5}{6} = \frac{16}{6} \approx 2.67$$

2. EV(Route A) = 0.60(16) + 0.40(25) = 9.6 + 10 = 19.6 minutes; EV(Route B) = 0.20(40) + 0.80(10) = 8 + 8 = 16 minutes; EV(Route C) = 0.90(20) + 0.1(32) = 21.2 minutes. He should take Route B.

3. EV(College A) = 0.75 • 0.30 = 0.225; EV(College B) = 0.65 • 0.40 = 0.260; EV(College C) = 0.70 • 0.45 = 0.315. She has a higher probability of being accepted at College C with financial aid.

THINK AND DISCUSS

1. The probability of an event happening is only part of the expected value. The expected value takes into consideration other information, such as the numerical value of an outcome. For example, in a number cube with 3 sides labeled as 9 and the other 3 sides labeled 1, 2, and 3 respectively, the probability of landing on 10 is $\frac{1}{2}$ while the expected value of any roll of the number cube is 6. A probability is calculated for a single event. The expected value is a weighted average of all possible outcomes in an experiment.

2. Answers will vary. Possible answers may include a decision to buy a raffle ticket, join a group, buy materials, or buy lottery tickets.

3.

Outcome	A	B	C	
Value	10	5	1	
Probability	0.05	0.2	0.75	**Expected Value**
	0.5	1	0.75	2.25

EXERCISES

GUIDED PRACTICE

1. expected value

2.

Value of Side	5	5	5	6	10	11
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$EV = 5\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right) + 11\left(\frac{1}{6}\right)$$

$$EV = \frac{5 + 5 + 5 + 6 + 10 + 11}{6} = \frac{42}{6} = 7$$

3.

Value of Side	2	3	4	4	8	12
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$EV = 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 8\left(\frac{1}{6}\right) + 12\left(\frac{1}{6}\right)$$

$$EV = \frac{2 + 3 + 4 + 4 + 8 + 12}{6} = \frac{33}{6} = 5.5$$

4.

Value of Side	1	1	1	1	1	10
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$EV = 1\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right)$$

$$EV = \frac{1 + 1 + 1 + 1 + 1 + 10}{6} = \frac{15}{6} = 2.5$$

5. 7 + 5.5 = 12.5

6. EV(Route A) = (0.8)(14) + (0.2)(25) = 16.2 minutes
EV(Route B) = (0.6)(10) + (0.4)(30) = 18 minutes
I would recommend that Gloria take route A since doing so would take the shortest amount of time in

the long run.

7. EV(Category A) = (500)(.1) − (100)(.9) = − $40
EV(Category B) = (100)(.3) − (20)(.7) = $16
EV(Category C) = (50)(.6) − (0)(.4) = $30
Category C has the highest expected value.

PRACTICE AND PROBLEM SOLVING

8.

Value of Side	1	2	2	4	9	10
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$EV = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 9\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right)$

$EV = \frac{1+2+2+4+9+10}{6} = \frac{28}{6} \approx 4.67$

9.

Value of Side	1	1	1	5	5	5
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$EV = 1\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + 1\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right)$

$EV = \frac{1+1+1+5+5+5}{6} = \frac{18}{6} = 3$

10.

Value of Side	1	4	4	4	4	10
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$EV = 1\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right)$

$EV = \frac{1+4+4+4+4+10}{6} = \frac{27}{6} = 4.5$

11. 3 + 4.5 = 7.5

12. Expected Value without the warranty = (0.05)(−180) + (0.15)(−60) = − $18. Over the long run, it will cost $18 to repair or replace the camera. This is less than $50 cost to buy the warranty, so Tristan should not buy the warranty.

13. EV(raffle ticket) = (0.001)(2000) + (0.02)(150) = 5. A break even price for the high school would be $5.

14.

Value of Section	1	3	5	7	9
Probability	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$EV = 1\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right) + 5\left(\frac{1}{5}\right) + 7\left(\frac{1}{5}\right) + 9\left(\frac{1}{5}\right)$

$EV = \frac{1+3+5+7+9}{5} = \frac{25}{5} = 5$

15.

Value of Section	5	5	5	5	9	9	12	12
Probability	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$EV = 5\left(\frac{1}{8}\right) + 5\left(\frac{1}{8}\right) + 5\left(\frac{1}{8}\right) + 5\left(\frac{1}{8}\right) + 9\left(\frac{1}{8}\right) + 9\left(\frac{1}{8}\right)$
$+ 12\left(\frac{1}{8}\right) + 12\left(\frac{1}{8}\right)$

$EV = \frac{5+5+5+5+9+9+12+12}{8} =$
$\frac{62}{8} = 7.75$

16.

Value of Section	9	11	18
Probability	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$EV = 9\left(\frac{1}{3}\right) + 11\left(\frac{1}{3}\right) + 18\left(\frac{1}{3}\right)$

$EV = \frac{9+11+18}{3} = \frac{38}{3} \approx 12.67$

17. $37\left(\frac{1}{4}\right) + 13\left(\frac{1}{4}\right) + 10\left(\frac{1}{4}\right) + 0\left(\frac{1}{4}\right)$

$\frac{37+13+10+0}{4} = \frac{60}{4} = 15$

18.

Value of Section	12	100
Probability	$\frac{3}{4}$	$\frac{1}{4}$

$EV = 12\left(\frac{3}{4}\right) + 100\left(\frac{1}{4}\right)$

$EV = \frac{36+100}{4} = \frac{136}{4} = 34$

19.

Value of Section	10	12	16	20
Probability	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

$EV = 10\left(\frac{1}{3}\right) + 12\left(\frac{1}{3}\right) + 16\left(\frac{1}{6}\right) + 20\left(\frac{1}{6}\right)$

$EV = \frac{10+12}{3} + \frac{16+20}{6} = \frac{44+36}{6} = \frac{80}{6} \approx$
13.33

20.

Value of Section	16	20	8
Probability	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

$EV = 16\left(\frac{1}{4}\right) + 20\left(\frac{1}{4}\right) + 8\left(\frac{1}{2}\right)$

$EV = \frac{16+20+16}{4} = \frac{52}{4} = 13$

21. $x\left(\frac{3}{6}\right) + 2x\left(\frac{2}{6}\right) + 3x\left(\frac{1}{6}\right)$

$\frac{3x+4x+3x}{6} = \frac{10x}{6} = \frac{5x}{3}$

22.If you pulled all the marbles out of the bag you would get 40 points from the red, 36 points from the blue, and 20 points from the green. That is a total of 96 points. Divide this total by 30, and an approximate expected value is 3.

23. The expected loss on the policy is (\$10,000)(0.01) = \$100. To make a profit of 50%, it needs to charge twice the expected loss. Since 50% of \$200 is \$100, the insurance company should charge \$200.

24. Camilla multiplied each value by $\frac{1}{8}$ since there are eight sides on the die. But two of the sides are labeled with a 3 and 4 are labeled with a 6. These values should have been multiplied by $\frac{2}{8}$ and $\frac{4}{8}$, respectively.

25. a. Average travel time:

(0.6)(32) + (0.4)(58) = 42.4 minutes

b. The actual travel time will be close to either 32 minutes or 58 minutes, depending on whether Devon hits traffic or not. The expected value helps makes decisions between two or more routes, but the result is not necessarily close to the actual time on any given trip.

TEST PREP

26. D

$$EV(A) = \frac{1+4+4+4+10+15}{6} \approx 6.33$$

$$EV(B) = \frac{2+4+6+8+10+12}{6} = 7$$

$$EV(C) = \frac{3+5+7+9+11}{5} = 7$$

$$EV(D) = \frac{10+10+30}{3} \approx 16.67$$

27. C

$$EV = \frac{1+1+4+4+4+4}{6} = 3$$

28. C

CHALLENGE AND EXTEND

29. EV(Sales Projections) = (450)(0.25)(59) + (320)(0.6)(79) + (275)(0.15)(119) = \$26,714.25

30. Answers will vary. Possible answer: Mathematics can be used to make decisions about business, insurance, traffic, games, and so forth. This type of mathematics is expected value.

READY TO GO ON?

1. cluster

2. Yes. The interval for those not preferring a park bench is 62% ± 8% or 54% to 70%. The interval for those preferring a park bench is 38% ± 8% or 30% to 46%. The intervals do not overlap so the survey projects a clear decision.

3. $(m - 2n)^3$

$$= {}_3C_0 m^3(-2n)^0 + {}_3C_1 m^2(-2n)^1 + {}_3C_2 m^1(-2n)^2 + {}_3C_3 m^0(-2n)^3$$

$$= 1 \cdot m^3 + 3 \cdot \left(-2m^2n\right) + 3 \cdot 4mn^2 + 1 \cdot \left(-8n^3\right)$$

$$= m^3 - 6m^2n + 12mn^2 - 8n^3$$

4. $P(5) = {}_{10}C_5(0.25)^5(0.75)^5$

≈ 0.058

5. $P(\text{at least } 3)$

$= 1 - P(0, 1 \text{ or } 2)$

$$= 1 - \left[{}_{10}C_0(0.25)^0(0.75)^{10} + {}_{10}C_1(0.25)^1(0.75)^9 + {}_{10}C_2(0.25)^2(0.75)^8\right]$$

≈ 0.47

6. $P(5) = {}_5C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0 = \frac{1}{243} \approx 0.004$

7. $P(1) = {}_5C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^4 = \frac{80}{243} \approx 0.33$

8. $P(0) = {}_5C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^5 = \frac{32}{243} \approx 0.13$

9. $P(\text{at least } 1)$

$= 1 - P(0)$

$= 1 - \frac{32}{243}$

$= \frac{211}{243} \approx 0.87$

10. $z = \frac{x-\mu}{\sigma} = \frac{85-82}{6} = 0.5$. The corresponding z-value from a standard normal table is 0.69. The z-value in the table represents only the area to the right of the z-value. The probability that a randomly selected student scored more than 85 is represented by the area to the left of the z-value. Subtract 0.69 from the total area of 1 to find the area to the left of the z-value. Thus, 1 − 0.69 = 0.31.

11. Compare the areas under the standard normal curve to the number of data values.

- The area below the z-value is taken from a standard normal table.
- Calculate x by adding or subtracting the correct number of standard deviations to the mean.
- Calculate projected values by multiplying the area below z by the sample size and rounding to the nearest whole number.
- Count the data given to determine the actual number of values.

z	Area	x	Values below x	
			Projected	Actual
−2	0.02	20 − 2(2.45) = 15.1	0.02(24) ≈ 0	0
−1	0.16	20 − 1(2.45) = 17.55	0.16(24) ≈ 4	4
0	0.5	20 + 0(2.45) = 20	0.5(24) ≈ 12	10
1	0.84	20 + 1(2.45) = 22.45	0.84(24) ≈ 20	20
2	0.98	20 + 2(2.45) = 24.9	0.98(24) ≈ 24	23

The projected number of values that corresponds to each value of z is close to the actual number of data values. The data appear to be normally distributed.

12. There is a 2% probability the dealer will have a $5,000 loss, and 0.02(5000) = $100. Because the cost of the insurance policy is $110, the dealer should not purchase the policy. Alternatively, the expected value can be calculated: $EV = 0.02(4890) + 0.98(-110) = \$97.80 + (-107.80) = -\$10.00$. Because the expected value is −$10.00, the dealer should not purchase the policy.

STUDY GUIDE: REVIEW

1. expected value
2. probability sample
3. hypothesis testing

8-1 MEASURES OF CENTRAL TENDENCY AND VARIATION

4. mean: 5.4; median: 6; mode: 8
5. mean: $13.\overline{3}$; median: 13; mode: 12, 13, 15
6. expected value
$= 0(0.65) + 1(0.22) + 2(0.1) + 3(0.03)$
$= 0.51$

8-2 DATA GATHERING

7. not representative; basketball players are likely to prefer sports
8. not representative; only homeowners are included in the sample, so not everyone in the city has a chance of being selected.
9. not representative; customers who prefer stores aren't likely to be at the website
10. $\frac{63}{100} = \frac{x}{1400}$; $x = 882$

8-3 SURVEYS, EXPERIMENTS, AND OBSERVATIONAL STUDIES

11. The treatment is working below fluorescent lights. The treatment group is those who are working below fluorescent lights. The control group is those who are working under incandescent lights.
12. The treatment is dogs receiving the company's brand of dog food. The treatment group is the households that were sent the company's brand. The control group is the households sent a brand other than the company's brand.

8-4 SIGNIFICANCE OF EXPERIMENTAL RESULTS

13. The vertical leap for both groups will be the same.

14. Comparing 5-number summaries:

	Without Inserts	With Inserts
Minimum	23	24
Q1	27	27.5
Median	30.5	30.5
Q3	34	35.5
Maximum	36	36

There is not a large difference, so we cannot reject the null hypothesis.

15. Yes; The null hypothesis is that the means of the population and sample are the same. Calculating the z-value: $\frac{390-400}{\frac{200}{\sqrt{3000}}} \approx -2.74$. Because $|z| \approx$

8-5

2.74 > 1.96, there is enough evidence to reject the claim.

SAMPLING DISTRIBUTIONS

16. Yes. The interval for Matthews is 62% ± 11%, or 51% to 73%. The interval for Harris is 38% ± 11%, or 27% to 49%. The intervals do not overlap, so the survey projects a clear winner.
17. No. The interval for vanilla is 53% ± 5%, or 48% to 58%. The interval for chocolate is 47% ± 5%, or 42% to 52%. The intervals overlap, so the survey does not project a clear winner.

8-6

BINOMIAL DISTRIBUTIONS

19. $(5 + 2x)^3$
$= {}_3C_0 5^3(2x)^0 + {}_3C_1 5^2(2x)^1 + {}_3C_2 5^1(2x)^2 + {}_3C_3 5^0(2x)^3$
$= 1 \cdot 125 + 3 \cdot 50x + 3 \cdot 20x^2 + 1 \cdot 8x^3$
$= 125 + 150x + 60x^2 + 8x^3$

20. $(x - 2y)^4$
$= {}_4C_0 x^4(-2y)^0 + {}_4C_1 x^3(-2y)^1 + {}_4C_2 x^2(-2y)^2 + {}_4C_3 x^1(-2y)^3 + {}_4C_4 x^0(-2y)^4$
$= 1 \cdot x^4 + 4 \cdot \left(-2x^3y\right) + 6 \cdot 4x^2y^2 + 4 \cdot \left(-8xy^3\right) + 1 \cdot 16y^4$
$= x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$

21. expected value $= 75 \cdot 0.65$
$= 48.75$
standard deviation $= \sqrt{75(0.65)(0.35)}$
≈ 4.13

22. $P(3) = {}_8C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^5$
$= 56\left(\frac{1}{216}\right)\left(\frac{3125}{7776}\right) \approx 0.10$
P(at least 2)
$= 1 - P(0 \text{ or } 1)$
$= 1 - \left[{}_8C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^8 + {}_8C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^7\right]$
≈ 0.40