DATA ANALYSIS 7

A study is conducted relating AP Statistics exam scores to the total number of study hours for the AP Statistics class put in by students during the academic year, and the correlation is found to be .6 . Which of the following are true statements?

I. On the average, a 40 percent increase in study time results in a 24 percent increase in exam score.
II. Sixty percent of a student's exam score can be explained by the number of study hours.
III. Higher exam scores tend to be associated with higher numbers of study hours.

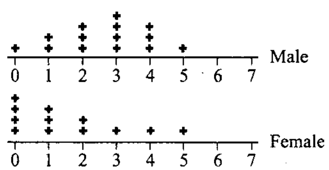
(A) I and II (B) I and III (C) II and III (D) I, II, and III

(E) None of the above gives the complete set of true responses.

Data Organization

DATA ANALYSIS 31

These dotplots for randomly selected male and female students at a particular high school show the number of times per week they eat at fast food restaurants.



Which of the following are true statements?

I. One distribution is roughly symmetric; the other is skewed left.
II. The difference in their means is less than the difference in their medians.
III. The ranges are the same.

(A) I and II (B) I and III (C) II and III (D) I, II, and III

(E) None of the above gives the complete set of true responses.

DATA ANALYSIS 30

Which of the following statements about the correlation r are true?

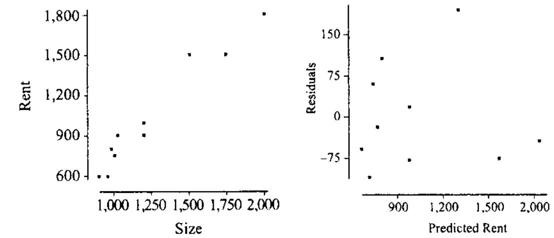
I. The correlation and the slope of the regression line always have the same sign.
II. A correlation of −.32 and a correlation of +.32 show the same degree of clustering around the regression line.
III. A correlation of .78 indicates a relationship that is 3 times as linear as one for which the correlation is .26.

(A) I and II (B) I and III (C) II and III (D) I, II, and III

(E) None of the above gives the complete set of true responses.

FREE RESPONSE 10

An SRS of apartment listings in a large northeastern city comparing monthly rent ($) versus size (ft^2) yields the following computer output:



Variable	Coef	s.e. Coef	t	p
Constant	-311.341	117.6	-2.65	0.0294
Size	1.07707	0.09047	11.9	0.0001

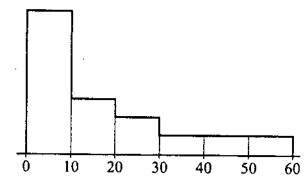
s = 102.4 R-squ = 94.7% R-squ(adj) = 94.0%

(a) Is a linear model appropriate for these data? Explain.

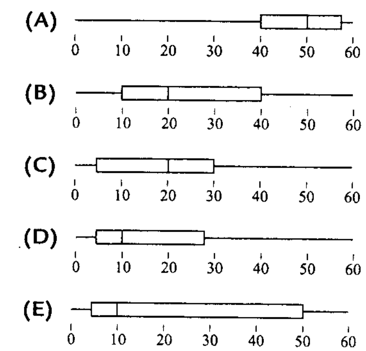
(b) Interpret the slope of the regression line.

(c) Interpret r^2 in context.

DATA ANALYSIS 3

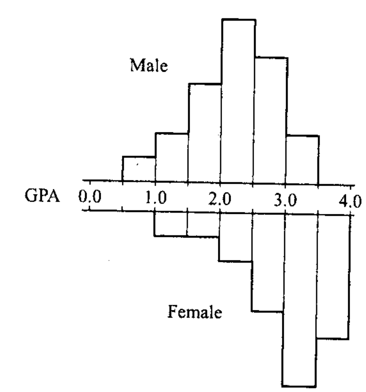


To which of the boxplots can the above histogram correspond?



FREE RESPONSE 8

The GPAs of random samples of 50 male and 50 female students at a large university are noted and summarized at the right.



Write a few sentences comparing the distributions of GPAs of male and female students at this university.

Answer: (E) Only III is true. The slope has the same sign, but generally not the same value, as the correlation. The coefficient of determination, giving the proportion of the y-variance that is predictable from a knowledge of x, is equal to r^2, not r. A positive correlation indicates that higher values of x tend to be associated with higher values of y.

Answer:

(a) Yes, the linear model is appropriate for these data. The scatterplot is approximately linear, and there is no apparent pattern in the residuals plot.

(b) The slope of the regression line is 1.08 meaning that for each additional square foot in size, the average rise in rent is \$1.08.

(c) The coefficient of determination is $r^2 = 94.7$ percent meaning that 94.7 percent of the variation in rent is explained by variation in size.

Data Organization

Answer: (C) The "Male" distribution is roughly symmetric, while the "Female" distribution is skewed right, not left. The medians are 3 and 1, respectively; the means are closer because the distribution with skew indicates the mean is greater than the median, while in the roughly symmetric distribution, the mean and median are close. Both have range $5 - 0 = 5$.

Answer: (D) The value 10 seems to roughly split the area under the histogram in two, so the median is about 10. The area between 10 and 60 is split in two somewhere between 20 and 30, so Q_3 is between 20 and 30. Finally, boxplot D seems to best pick up the strong right skew.

Answer: (A) The slope and the correlation are related by the formula $b = r\frac{s_y}{s_x}$.

The standard deviations are always positive, so b and r have the same sign. Positive and negative correlations with the same absolute value indicate data having the same degree of clustering around their respective regression lines, one of which slopes up to the right and the other of which slopes down to the right. Even though $r = .78$ indicates a better fit with a linear model than $r = .26$ does, we cannot say that the linearity is threefold.

Answer: The comparison should address shape, center, and spread.

Shape: The female GPA distribution is skewed to the left (to the lower values), while the male GPA distribution is more bell-shaped.

Center: The center of the male GPA distribution is less than the center of the female GPA distribution (between 2.0 and 2.5 compared to just over 3.0).

Spread: The ranges of the two distributions are the same ($3.5 - 0.5 = 3.0$ and $4.0 - 1.0 = 3.0$).

FREE RESPONSE 36

A guidance counselor conducts a study to determine the effect of caloric intake at breakfast on alertness in first-period classes. She interviews an SRS of 20 students who claim to eat under 1,000 calories at breakfast, and an SRS of 20 students who claim to eat over 1,000 calories at breakfast. In each group, she measures alertness on a standard scientific scale.

(a) Explain why this is an observational study and not an experiment.

(b) Give an example of a possible confounding variable with an explanation in the context of this study.

(c) If the students who eat over 1,000 calories test higher on the alertness scale, is it reasonable to encourage all students to eat larger breakfasts?

(d) How could the counselor design a related experiment to study caloric intake at breakfast with alertness in first period?

FREE RESPONSE 28

High cholesterol levels can be reduced by low-fat diet or medication. Researchers would like to test the effectiveness of a new medication and to note whether the effectiveness, if any, is enhanced by diet. One hundred volunteers with high cholesterol levels, on no special diets, and not on medication, are recruited for a study.

(a) Conclusions will apply to what population?

(b) Explain how you would design a completely randomized experiment.

(c) How might you incorporate blocking and for what purpose?

(d) How might blinding be incorporated in this study and for what purpose?

FREE RESPONSE 48

Fifty employees operating sewing machines for a clothing manufacturer each consistently complete a number of pieces per week ranging from 25 to 40 as follows:

33	31	37	39	27	31	40	36	27	27
27	30	34	38	27	29	27	38	37	40
33	36	29	26	34	32	39	32	39	36
32	32	25	31	26	40	33	37	29	26
35	26	37	33	27	28	32	37	33	32

Consider these 50 data values to be a population with $\mu = 32.44$

(a) Using the following line from a random number table, explain and carry out a procedure to select a simple random sample of size 10 from our population.

77219 48190 20235 26836 23590 44492 14607 09431 75299 42662

(b) Determine $\bar{x}$ and s for your sample.

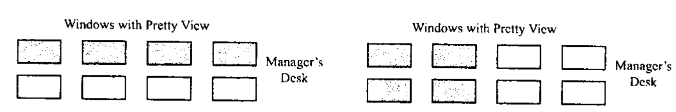
(c) Using your sample statistics, construct a 90 percent confidence interval for μ.

(d) Is μ in your interval? Is this unexpected?

FREE RESPONSE 44

A clothing manufacturer wishes to test which of two sewing machines leads to greater production. The workroom consists of eight large tables, and they plan to have everyone at a given table use the same machine. On one side of the room are windows with a pretty view, and on one side is the floor manager's desk.

(a) Suppose you decide to block using the Scheme *A* below (one block is white, one gray). How would you use randomization, and what is the purpose of the randomization?



Scheme *A* Scheme *B*

(b) Comment on the strength and weakness of the Scheme *A* as compared to blocking Scheme *B* (one block is white, one gray).

Experimental Design

Experimental design

Answer: (a) No treatment is being imposed on anyone.

(b) There are many possible answers. For example, it is possible that the students eating under 1,000 calories are those who sleep in late, have to rush to school without eating, and have not fully awakened during first period.

(c) No, because cause-and-effect conclusions cannot be drawn from observational studies.

(d) Randomly select a group of students who are told they must eat under 1,000 calories at breakfast, and randomly select another group who are told they must eat over 1,000 calories at breakfast. Compare alertness levels in first-period classes.

Answer: (a) Label the data values 01 to 50. Pick two digits at a time from the random table line, throwing out repeats and ignoring numbers over 50, until 10 numbers are selected. Note the data values corresponding to these numbers. In our case this results in selecting:

77219 48190 20235 26836 23590 44492 14607 09431 75299 42662

{21, 20, 23, 36, 04, 44, 14, 31, 29, 26} correspond to the data values {33, 40, 29, 40, 39, 33, 38, 32, 39, 32}, which becomes our sample.

(b) A calculator gives $\bar{x} = 35.5$ and $s = 4.089$.

(c) With df = 10 − 1 = 9, the critical t-values are ± `invT(.95, 9)` = ± 1.833, and so the 90 percent confidence interval is $35.5 \pm 1.833\left(\frac{4.089}{\sqrt{10}}\right) = 35.5 \pm 2.37$ or between 33.13 and 37.87. (`TInterval` gives the same result even quicker.)

(d) $\mu = 32.44$ is not in the interval, but this is not unexpected, since we were only 90 percent confident that our interval contained μ.

Answer: (a) Conclusions will apply to non-dieters with high cholesterol levels who are not taking medications.

(b) The design should incorporate random assignment of the volunteers to four treatment groups (medication and diet, medication and no diet, placebo and diet, placebo and no diet) and a measurement and comparison of cholesterol levels.

(c) There are many possible answers, but each should be explained in the context of the problem. For example, you could block on gender or age because men versus women or old versus young might respond differently to medication or diet. In any case, give the scheme of first splitting volunteers into separate blocks (such as gender or age) and then randomly assigning subjects in each block to the four treatment groups.

(d) Blinding is incorporated through the use of a placebo if there is a possibility some volunteers might be able to subconsciously influence their cholesterol levels if they knew for sure they were taking the medication and thus expected results.

Answer: (a) In each block, two of the tables will be randomly assigned to receive one of the machines, while the remianing two tables in the block will receive the other machine. Randomization of machines to tables within each block should reduce bias due to confounding variables associated with the tables that might be related to productivity. In particular, the randomization in blocks in the Scheme *A* should even out the effect of the distance tables are from the manager's desk.

(b) The Scheme *A* creates homogeneous blocks with respect to window exposure; the Scheme *B* creates homogeneous blocks with respect to distance from the manager's desk. Randomization of machines to tables within blocks in the Scheme *A* should even out effects of distance to manager's desk, while randomization of machines to tables within blocks in the Scheme *B* should even out effects of window exposure.

Inference

FREE RESPONSE 21

Researchers wish to determine if the stimulant, caffeine, enhances athletic performance. Ten short distance runners are timed in the 100-yard dash on two successive days. Each day they are given either 300 mg caffeine or a placebo. For each runner, a coin toss determines which identical looking pill is taken which day. Their times (in seconds) are as follows:

Runner:	1	2	3	4	5	6	7	8	9	10
With placebo:	10.2	9.8	9.9	10.4	10.2	9.8	10.1	10.7	9.7	9.8
With caffeine:	10.2	9.6	9.8	10.1	9.8	10.0	10.2	10.4	9.6	9.8

(a) Do the data suggest that short distance runners improve their times when using caffeine? Perform an appropriate statistical test.

(b) Does knowing a runner's time without drugs help predict his time using caffeine? Perform an appropriate statistical test.

FREE RESPONSE 22

An SRS of 100 students at schools using an innovative math program scored an average of 357 with a standard deviation of 54 on a state test; an SRS of 150 students at schools using a traditional approach scored an average of 343 with a standard deviation of 62 on the same state test.

(a) Is there evidence that students using the innovative approach have a higher average score than students using the traditional approach. Give statistical justification for your answer.

(b) Suppose a study using this design resulted in a *P*-value less than .01. Would it be reasonable for all school boards to push for adoption of the innovative approach? Explain.

(c) Assuming standard deviations of 54 and 62 as above, how large a sample (same number for both) should be used to be 95 percent sure of knowing the difference in scores to within 10.

FREE RESPONSE 18

A senator's approval rating stood at 76 percent before she took a crucial vote.

(a) Her staff believes the rating is still around 76 percent. To confirm this, how large an SRS should they sample to obtain a 90 percent confidence interval estimate with a margin of error ≤ 3 percent?

(b) Her staff randomly samples 600 people and finds 440 people approve of the senator's job performance. Is there evidence that the rating has changed from 76 percent. Perform an appropriate statistical test.

(c) In part (b), suppose the staff suspects the rating has gone down. Is there evidence the approval rating has slipped down from 76 percent? Perform an appropriate statistical test.

(d) Are the answers in (b) and (c) contradictory? Explain.

FREE RESPONSE 15

In a study of detergents, each of 300 volunteer households was randomly (coin toss) given a box of detergent *A* or *B*. After using their given detergent for several loads of clothes, each household gave a score from 1 to 5 with 1 = poor and 5 = great for their detergent. The results:

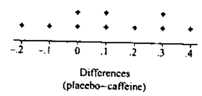
Detergent score	1	2	3	4	5
Detergent *A* frequency	16	41	53	38	13
Detergent *B* frequency	45	50	29	11	4

(a) Use a graphical display to compare the scores received. Write a few sentences, based on your graphs, to compare detergents *A* and *B*.

(b) Does there appear to be significantly greater satisfaction with detergent *A* as compared to *B*? Give a statistical justification for your answer.

Inference

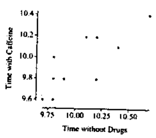
Answer: (a) The proper test is a matched-pair t-test on the set of differences, H_0: $\mu_D = 0$, H_a: $\mu_D > 0$. A dotplot of the differences is roughly symmetric and bell-shaped with no outliers or extreme skewness:

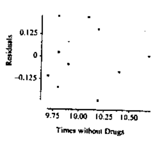


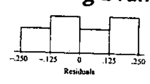
Either use `T-Test` on the TI-84 or calculate $\bar{x}_D = .11$, $s_D = .1912$, $t = \frac{.11 - 0}{.1912/\sqrt{10}} = 1.819$, df $= 10 - 1 = 9$, and $P = .0511$. With this small a P-value, there is some evidence to reject H_0 and conclude that caffeine makes a difference.

(b) The proper test here is the t-test for regression on the matched pairs. Make sure the scatterplot is roughly linear, the residuals have no apparent pattern, and the histogram of the residuals is roughly normal.

On the TI-84, LinRegTTest on H_0: $\beta = 0$, H_a: $\beta \neq 0$ gives $t = 3.82$ and $P = .0051$. With a P-value this small, there is strong evidence to reject H_0 and conclude that knowing a runner's time without drugs does help predict his time using caffeine.







Answer: (a) We use a two-sample t-test, H_0: $\mu_1 - \mu_2 = 0$, H_a: $\mu_1 - \mu_2 > 0$. The given scheme gives independent random samples, and the Central Limit Theorem and large sample sizes give a normal distribution for the differences in sample means. Using `2-SampTTest` on the TI-84, or calculating $t = \frac{357 - 343}{\sqrt{54^2/100 + 62^2/150}} = 1.891$, we find $P = .030$. With a P-value this small, there is evidence to reject H_0 and conclude that evidence indicates students using the innovative approach have a higher average score than students using the traditional approach.

(b) Even with a small P-value, we cannot conclude causation because this is an observational study, not an experiment. For example, it may be that the best schools with the top teachers and students are the ones who choose to try the innovative approach.

(c) $1.96\sqrt{54^2/n + 62^2/n} \leq 10$ gives $\sqrt{n} \geq 16.1$ and $n \geq 259.2$, so choose $n = 260$.

Answer: (a) $1.645\sqrt{\frac{(.76)(.24)}{n}} \leq .03$ gives $\sqrt{n} \geq 23.42$ and $n \geq 548.4$, so 549 people must be surveyed.

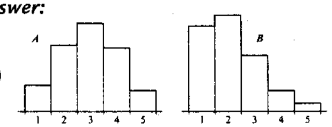
(b) This is a two-sided, one proportion z-test with H_0: $p = .76$ and H_a: $p \neq .76$. The sample is randomly chosen, and both $np = 600(.76) = 456$ and $n(1 - p) = 600(.24) = 144$ are > 10. Either use `1-PropZTest`, or with $\hat{p} = \frac{440}{600} = .7333$, we get $z = \frac{.7333 - .76}{\sqrt{(.76)(.24)/600}} = -1.531$, and thus $P = 2(.0629) = .1258$. With a P-value this large, there is no evidence that the approval rating has changed.

(c) With H_0: $p = .76$ and H_a: $p < .76$, $P = .0629$, and thus there is some evidence (at least at the 10 percent significance level) that the senator's rating has slipped.

(d) There is no contradiction. The tail probability of the test statistic is doubled when finding the P-value in a two-sided test. Thus, what might be a small enough tail probability to reject H_0 for a one-sided test might no longer be small enough when doubled for a two-sided test.

Answer:

(a)



Or use parallel boxplots or double bar charts.

The distribution for detergent A is roughly symmetric and bell-shaped, while that for detergent B is skewed to the right. Both distributions have the same range with lows of 1 and highs of 5; however, both the mean and median scores of detergent A appear greater than those of B.

(b) The proper hypothesis test is the two-sample t-test. Assumption of independent random samples is satisfied by the given scheme. By the Central Limit Theorem and large sample sizes, use of the t-distribution is appropriate. $H_0: \mu_A - \mu_B = 0$ and $H_a: \mu_A - \mu_B > 0$. On the TI–84, we get $\bar{x}_A = 2.944$, $\bar{x}_B = 2.129$, $t = 6.552$, and $P = .0000$. With such a small P-value, there is very strong evidence of greater satisfaction with detergent A as compared to B.

FREE RESPONSE 3

Suppose that jumps by Olympic men high jumpers have a normal distribution with mean 2.12 meters and standard deviation 0.12 meters; women's jumps have a normal distribution with mean 1.80 meters and standard deviation 0.09 meters. A man and woman Olympic high jumper are picked at random.

(a) What is the probability the sum of their jumps is over 4 meters?

(b) What is the probability that the man jumped higher than the woman?

Probability

FREE RESPONSE 7

A company advertises it has a process that can extract 5 kg of protein from 100 kg of seaweed. Using this process, a random sample of fifteen 100-kg clumps of seaweed yields a mean of 4.82 kg of protein with a standard deviation of 0.65 kg. Assume the sample distribution is symmetric and unimodal with no outliers.

(a) Is there sufficient evidence to dispute the advertisement? Justify your answer.

A large-scale test of a second company's process shows yields of protein that are normally distributed with a mean of 4.75 kg and a standard deviation of 0.83 kg.

(b) What is the probability that using this second process a 100-kg clump of seaweed will yield at least 5 kg of protein?

(c) What is the probability that using this second process on ten randomly selected 100-kg clumps of seaweed, at least two of them yield at least 5 kg of protein?

FREE RESPONSE 9

The probability distribution for the number of magazine subscriptions to which college students subscribe is as follows:

Number of subscriptions	0	1	2	3	4
Relative frequency	.12	.41	.25	.15	.07

(a) Calculate and give a brief interpretation of the mean of this probability distribution.

(b) In a random sample of 10 college students, there are a total of 20 magazine subscriptions. A new random sample of 50 students is planned. How do you expect the average number of subscriptions for this new sample to compare to that of the first sample? Explain.

(c) Find the median of the above distribution, where the median M is defined to be a value such that $P(x \geq M) \geq .5$ and $P(x \leq M) \geq .5$.

PROBABILITY 3

Suppose 80 percent of jurors come to a just decision. In a jury of six people, what is the probability more than half come to a just decision?

(A) .09888

(B) .34464

(C) .80000

(D) .90112

(E) .98304

PROBABILITY 2

Suppose that the probabilities that an answer can be found on Google is .95, on Answers.com is .92, and on both Web sites is .874. Are the possibilities of finding the answer on the two Web sites independent?

(A) Yes, because (.95)(.92) = .874.

(B) No, because (.95)(.92) = .874.

(C) Yes, because .95 > .92 > .874.

(D) No, because .5(.95 + .92) ≠ .874.

(E) There is insufficient information to answer this question.

PROBABILITY 7

A television game show has three payoffs with the following probabilities:

Payoff ($)	0	500	5,000
Probability	.7	.25	.05

What are the mean and standard deviation of the payoff variable?

(A) $\mu = 375, \sigma = 361$

(B) $\mu = 375, \sigma = 1{,}083$

(C) $\mu = 1{,}833, \sigma = 1{,}816$

(D) $\mu = 1{,}833, \sigma = 2{,}248$

(E) None of the above gives a set of correct answers.

Answer: (a) We can assume the two jumps are independent. Then $E(M + W) = E(M) + E(W) = 2.12 + 1.80 = 3.92$ meters, and $\text{var}(M + W) = \text{var}(M) + \text{var}(W) = (0.12)^2 + (0.09)^2 = 0.0225$, so $SD(M + W) = \sqrt{0.0225} = 0.15$ meters. If two independent random variables have normal distributions, so does their sum. Given $N(3.92, 0.15)$, $P(x > 4) = .2969$.

(b) Assuming the jumps are independent, we have $E(M - W) = E(M) - E(W) = 2.12 - 1.80 = 0.32$ meters, and $\text{var}(M - W) = \text{var}(M) + \text{var}(W) = (0.12)^2 + (0.09)^2 = 0.0225$, so the standard deviation of $M - W = \sqrt{0.0225} = 0.15$ meters. If two independent random variables have normal distributions, so does their difference. Given $N(0.32, 0.15)$, $P(x > 0) = .9836$.

Answer: (D) $P(x > 3) = {}_6C_4(.8)^4(.2)^2 + {}_6C_5(.8)^5(.2) + (.8)^6 = .90112$ [or on the TI-84, 1 − binomcdf(6, .8, 3) = .90112].

Probability

Answer:

(a) The correct hypotheses are: $H_0: \mu = 5$, $H_a: \mu < 5$. The correct test should be given by name (one sample *t*-test for a mean) or formula: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$.

The conditions (random sample and roughly normal population) are given.

The test statistic is calculated by $t = \dfrac{4.82 - 5}{0.65/\sqrt{15}} = \dfrac{-0.18}{0.1678} = -1.073$.

With df = 15 − 1 = 14, the *P*-value is calculated to be .151. With this large a *P*-value, there is not sufficient evidence to dispute the advertisement of extraction of 5 kg of protein from 100 kg of seaweed.

(b) With $N(4.75, 0.83)$, $P(x \geq 5) = .3816$.

(c) We have a binomial distribution with $n = 10$ and $p = .3816$.

$P(x \geq 2) = 1 - [P(x = 0) + P(x = 1)] = 1 - [(.6184)^{10} + 10(.6184)^9(.3816)] = 1 - [.00818 + .05047] = .9414$.

Answer: (A) If $P(E \cap F) = P(E)P(F)$, then E and F are independent.

Answer:

(a) $\mu = \sum xP(x) = 0(.12) + 1(.41) + 2(.25) + 3(.15) + 4(.07) = 1.64$

On the average, college students subscribe to 1.64 magazines.

(b) The average for the first sample was 20/10 = 2. It is expected that the average based on 50 students will be closer to 1.64 than was the average based on only 10 students. The variability for sample averages based on 50 students is smaller than for sample averages based on 10 students.

(c) We see that $P(X \leq 1) = .12 + .41 = .53 \geq .5$ and $P(X \geq 1) = .41 + .25 + .15 + .07 = .88 \geq .5$, so the median is 1.

Answer: (B) $\mu = 0(.7) + 500(.25) + 5{,}000(.05) = 375$, $\sigma^2 = (0 - 375)^2(.7) + (500 - 375)^2(.25) + (5{,}000 - 375)^2(.05) = 1{,}171{,}875$, and $\sigma = 1{,}083$.