12-2 Pascal's Triangle and Binomial Theorem

Expand $(a+b)^n$

terms = n + 1

$(a+b)^0 = 1$

$(a+b)^1 = 1a^1 + 1b^1$

$(a+b)^2 = 1a^2 + 2a^1b^1 + 1b^2$

$(a+b)^3 = 1a^3 + 3a^2b^1 + 3a^1b^2 + 1b^3$

$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

Pascal's Δ

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 Expand $(x - 2y)^5$

a b

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$= 1x^5 + 5x^4(-2y)^1 + 10x^3(-2y)^2 + 10x^2(-2y)^3 + 5x(-2y)^4 + 1(-2y)^5$

$= x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

BINOMIAL THEOREM

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r\, a^{n-r} b^r$$

Pascal #

$n+1$ terms

$\frac{n!}{r!(n-r)!}$

2 Find the coefficient of x^7 in $(2-3x)^{10}$

a b

$$\frac{10!}{3!7!}(2)^3(-3x)^7 = -2,099,520x^7$$

$$\frac{10 \cdot 9 \cdot 8^4}{3 \cdot 2}\, 120(8)(-2187x^7) =$$

3 Find the 5th term of $(2 - 3x)^{10}$

a

$$\frac{10!}{6!\,4!}(2)^{6}(-3x)^{4}$$

$$\frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2}(64)(81x^{4})$$

$$210(64)(81x^{4})$$