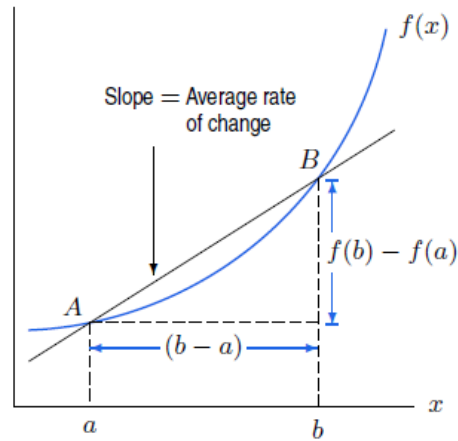


## 2.4 Rates of Change and Tangent Lines

*Average rate of change - speed (mph), growth rates (% per year) monthly rainfall (inches per month)*

The **average rate of change** of a function over an interval is the amount of change divided by the length of the interval



How can we express the average rate of change?

### Example 1:

Find the average rate of change over the interval  $[1, 3]$  of

$$f(x) = x^3 - x$$

**Example 2:**

Suppose a ball is thrown upward so that its height  $f(x)$  in feet is given by the equation, where  $x$  is time, in seconds.

$$f(x) = 96 + 64x - 16x^2$$

Graph  $[0,6] \times [-20,200]$

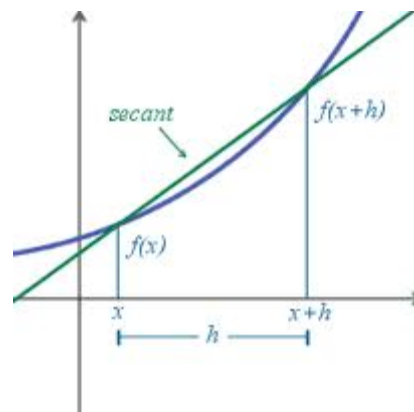
- a) Find the average velocity (average rate of change) of the ball from  $x = 2$  to  $x = 3$  seconds.
  
  - b) Find the average velocity of the ball from  $x = 2$  to  $x = 2.5$  seconds.
  
  - c) Find the average velocity of the ball from  $x = 2$  to  $x = 2.1$  seconds.
  
  - d) Find the slope of the secant line from  $x = 2$  to  $x = 2.1$  seconds.
  
  - e) Find the instantaneous velocity of the ball at  $x = 2$  seconds.
- STOP

What is the problem?

The **average rate of change** of a function from  $x$  to  $x + h$  is the **slope of the secant** line passing through the two points.

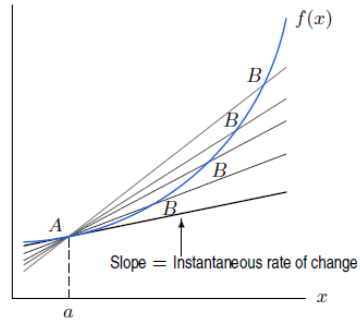
$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

*This is also called the difference quotient of  $f$*



The **instantaneous rate of change** of a function is the **slope of the tangent line or slope of the curve** at a certain point. It is the limit as  $h$  approaches 0 of the average rate of change.

$$m = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$



Back to the problem.....

Find the instantaneous velocity at  $x = 1$  and at  $x = 2$  seconds.

**Example 3:**

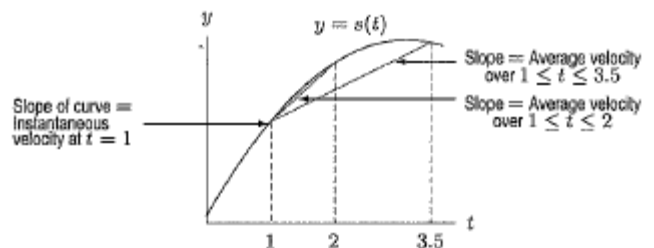
Let  $f(x) = 1/x$

- Find the slope of the curve at  $x = a$ .
- Where does the slope equal  $-1/4$ ?

**Remember, all of these are the same....**

- the slope of  $y = f(x)$  at  $x = a$
- the slope of the tangent to  $y = f(x)$  at  $x = a$
- the instantaneous rate of change of  $f(x)$  with respect to  $x$  at  $x = a$

$$4) \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$



The **normal** line to a curve is the line perpendicular to the tangent at that point.

**Example 4:**

Write an equation for the normal to the curve  $f(x) = 4 - x^2$  at  $x = 1$ .

**0**

**Example 5:**

A rock breaks loose from the top of a cliff,  $y = -4.9t^2$ .

(a) What is the average speed during the first 2 seconds of the fall?

(b) Find the speed of the rock at the instant  $t = 1$  second

**Homework**

Section 2.4 Pg. 87 # 1 – 11 odd, 19, 23, 25, 29, 32

Chapter 2 Review Exercises Pg. 91 #1 - 18