

2-1 If-then Statements and Converses

Sept 16

An "if-then" statement is a *conditional*.

Example 1: If  $\angle A$  is a right angle, then  $m\angle A = 90$ .

*Handwritten notes:*  $\angle A$  is a right angle is labeled *hypothesis* (with  $p$  above it).  $m\angle A = 90$  is labeled *conclusion* (with  $q$  above it). The whole statement is labeled *if p, then q*.

Other forms:

- $\angle A$  is a right angle *implies*  $m\angle A = 90$ .
- $\angle A$  is a right angle *only if*  $m\angle A = 90$ .
- then*  $m\angle A = 90$  *if*  $\angle A$  is a right angle.

*Handwritten notes:* Arrows point from the words "implies", "only if", and "if" to the notes: "p implies q", "p only if q", and "q, if p".

The *converse* of a conditional statement is formed by switching the hypothesis and conclusion.

Example 2: If  $m\angle B = 110$ , then  $\angle B$  is obtuse.

*Handwritten notes:*  $m\angle B = 110$  is labeled  $H$  (hypothesis) and  $\angle B$  is obtuse is labeled  $C$  (conclusion). The whole statement is labeled  $T$  (true). The converse is labeled  $F$  (false).

Converse: If  $\angle B$  is obtuse, then  $m\angle B = 110$ .

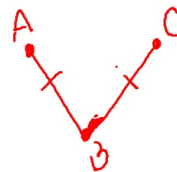
A *counterexample* is an example that shows why a statement is false.

Example 3: Give a counterexample to show why the statement is false.

a) If  $x^2 = 49$ , then  $x = 7$ .

$x = -7$

b) If  $\overline{AB} \cong \overline{BC}$ , then  $B$  is the midpoint of  $\overline{AC}$ .



A **biconditional** is a statement that combines a conditional and its converse.

$p$  if and only if  $q$ .

Conditional If  $\overline{DE} \cong \overline{FG}$ , then  $DE = FG$ . T

Converse

If  $DE = FG$ , then  $\overline{DE} \cong \overline{FG}$ . T

Biconditional

$\overline{DE} \cong \overline{FG}$  if and only if  $DE = FG$ .

**Example 4:** Write this pair of conditionals as a biconditional.

If two angles are congruent, then their measures are equal.

If the measures of two angles are equal, then they are congruent.

Two angles are congruent if and only if their measures are equal.

**Example 5:** Write the biconditional as two conditionals that are converses of each other.

An angle is acute if and only if its measure is less than  $90^\circ$ .

If an angle is acute, then its measure is less than  $90^\circ$ .  
If its measure is less than  $90^\circ$ , then an angle is acute.