

Order-of-Magnitude Estimating

Name: _____

Date: _____ Period: _____

I. RAPID ESTIMATING

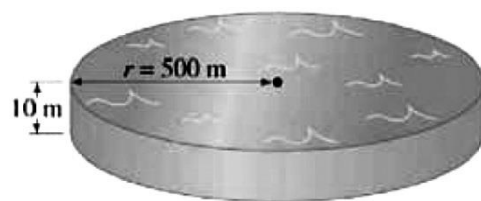
We are sometimes only interested in an approximate value for a quantity. This might be because an accurate calculation would take more time than it is worth or would require additional data that are not available. In other cases, we may want to make a rough estimate in order to check an accurate calculation made on a calculator, to make sure that no blunders were made.

A rough estimate is made by rounding off all numbers to one significant figure and its power of 10, and after the calculation is made, again only one significant figure is kept. Such an estimate is called an order-of-magnitude estimate and can be accurate within a factor of 10, and often better. In fact, the phrase “order of magnitude” is sometimes used to refer simply to the power of 10.

II. EXAMPLE PROBLEMS

1. Estimate how much water there is in a particular lake, which is roughly circular, about 1 km across, and you guess it has an average depth of about 10 m.

No lake is a perfect circle, nor can lakes be expected to have a perfectly flat bottom. We are only estimating here. To estimate volume, we can use a simple model of the lake as a cylinder: we multiply the average depth of the lake times its roughly circular surface area, as if the lake were a cylinder.



(a) Estimate the volume in cubic meters.

(b) Estimate the volume in gallons (1 liter = $10^{-3} \text{ m}^3 \approx \frac{1}{4}$ gallon)

2. Estimate the thickness of this piece of paper.

At first you might think that a special measuring device, a micrometer, is needed to measure the thickness of one page since an ordinary ruler clearly won't do. But we can use a trick or, to put it in physics terms, make use of a symmetry: we can make the reasonable assumption that all the pages in ream of paper are equal in thickness and if you measure the thickness of a ream of paper (500 pages), you might get something like 5.2 cm.

(a) Estimate the thickness in mm.

(b) How many microns is this?

III. PRACTICE PROBLEMS

3. Estimate the total number of heartbeats a typical human heart makes in a lifetime.

A typical rest heart rate is 70 beats/min, but during exercise it can be a lot higher. A reasonable average might be 80 beats/min.

6. Estimate the radius of the universe.

Assume a spherical universe expanding at the speed of light for the age of the universe (13.7×10^9 yr).

4. Estimate how many hours it would take a runner to run (a 10 km/h) across the United States from New York to California.

5. Estimate the number of liters of water a human drinks in a lifetime.