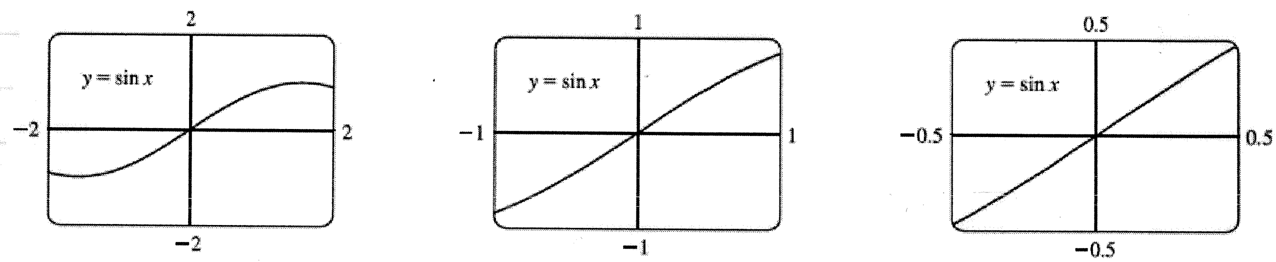
3.1a

1. $y = f(x)$

ⓐ $m_{PQ} = \dfrac{f(x) - f(3)}{x - 3}$

ⓑ $m = \lim\limits_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$

2.



The curve looks more like a line

3. ⓐ $f(x) = 4x - x^2$ point $(1,3)$

ⓘ $m = \lim\limits_{x \to 1} \dfrac{f(x) - f(1)}{x - 1} = \lim\limits_{x \to 1} \dfrac{(4x - x^2) - [4(1) - 1^2]}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{-x^2 + 4x - 3}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{-(x^2 - 4x + 3)}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{-(x-3)(x-1)}{x - 1}$

$= \lim\limits_{x \to 1} -(x - 3)$

$= 2$

3. cont'd

(ii) $m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{[4(1+h) - (1+h)^2] - 3}{h}$

$= \lim_{h \to 0} \frac{[4 + 4h - 1 - 2h - h^2] - 3}{h}$

$= \lim_{h \to 0} \frac{2h - h^2}{h}$

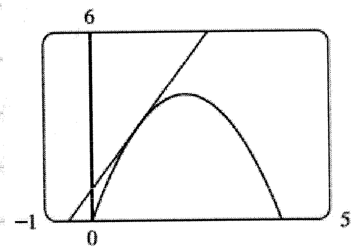
$= \lim_{h \to 0} (2 - h)$

$= 2$

(b) $y - 3 = 2(x-1)$

$y = 2x + 1$

(c)



4. (a) $f(x) = x - x^3$ pt $(1,0)$

(i) $m = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{(x - x^3) - 0}{x - 1}$

$= \lim_{x \to 1} \frac{x(1 - x^2)}{x - 1}$

$= \lim_{x \to 1} \frac{x(1+x)(1-x)}{-(1-x)}$

$= \lim_{x \to 1} -x(1+x)$

$= -1(1+1)$

$= -2$

(ii) $m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{[(1+h) - (1+h)^3] - 0}{h}$

$= \lim_{h \to 0} \frac{1 + h - (1 + 3h + 3h^2 + h^3)}{h}$

$= \lim_{h \to 0} \frac{h(-h^2 - 3h - 2)}{h}$

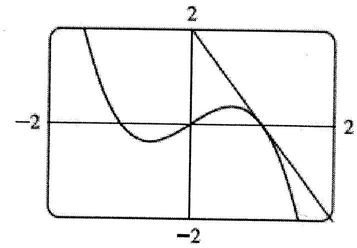
$= \lim_{h \to 0} (-h^2 - 3h - 2)$

$= -2$

4. cont'd (b) $y-0=-2(x-1)$

$y=-2x+2$

(c)



5. $f(x)=\dfrac{x-1}{x-2}$ pt $(3,2)$

$$m=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{\frac{(3+h)-1}{(3+h)-2}-\frac{3-1}{3-2}}{h}$$

$$=\lim_{h\to 0}\left(\frac{h+2}{h+1}-2\right)\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{h+2-2(h+1)}{h+1}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{-h}{h(h+1)}$$

$$=\lim_{h\to 0}\frac{-1}{h+1}$$

$$=-1$$

$\boxed{\text{OR}}$

$$m=\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}=\lim_{x\to 3}\frac{\frac{x-1}{x-2}-2}{x-3}$$

$$=\lim_{x\to 3}\frac{x-1-2(x-2)}{x-2}\cdot\frac{1}{x-3}$$

$$=\lim_{x\to 3}\frac{-x+3}{-(x-2)(-x+3)}$$

$$=\lim_{x\to 3}\frac{1}{-(x-2)}$$

$$=-1$$

tangent line: $y-2=-1(x-3)\Rightarrow y=-x+5$

6. $f(x) = 2x^3 - 5x$ pt $(-1,3)$

$$m = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h \to 0} \frac{[2(-1+h)^3 - 5(-1+h)] - 3}{h}$$

$$= \lim_{h \to 0} \frac{2(-1+3h-3h^2+h^3) + 5 - 5h - 3}{h}$$

$$= \lim_{h \to 0} \frac{h - 6h^2 + 2h^3}{h}$$

$$= \lim_{h \to 0} \frac{h(1-6h+2h^2)}{h}$$

$$= \lim_{h \to 0} (1 - 6h + 2h^2)$$

$$= 1$$

$\boxed{\text{OR}}$

$$m = \lim_{x \to -1} \frac{f(x) - f(-1)}{x-(-1)} = \lim_{x \to -1} \frac{(2x^3-5x) - 3}{x+1}$$

$$= \lim_{x \to -1} \frac{(x+1)(2x^2-2x-3)}{x+1}$$

Note: use synthetic division to factor

$$= \lim_{x \to -1} (2x^2 - 2x - 3)$$

$$= 2(-1)^2 - 2(-1) - 3$$

$$= 1$$

tangent line: $y - 3 = 1(x+1) \Rightarrow y = x + 4$

7. $f(x) = \sqrt{x}$ pt $(1,1)$

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1}$$

$$= \lim_{h \to 0} \frac{(1+h) - 1}{h(\sqrt{1+h} + 1)}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{1+h} + 1)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{1+h} + 1}$$

$$= \frac{1}{2}$$

$\boxed{\text{OR}}$

$$m = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} \cdot \frac{\sqrt{x} + 1}{\sqrt{x} + 1}$$

$$= \lim_{x \to 1} \frac{x - 1}{(x-1)(\sqrt{x} + 1)}$$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x} + 1}$$

$$= \frac{1}{2}$$

tangent line: $y - 1 = \frac{1}{2}(x - 1) \Rightarrow y = \frac{1}{2}x + \frac{1}{2}$

8. $f(x) = \dfrac{2x}{(x+1)^2}$ pt $(0,0)$

$$m = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\frac{2h}{(h+1)^2} - 0}{h}$$

$$= \lim_{h\to 0} \frac{2h}{(h+1)^2} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{2}{(h+1)^2}$$

$$= 2$$

$\boxed{\text{OR}}$

$$m = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{\frac{2x}{(x+1)^2} - 0}{x}$$

$$= \lim_{x\to 0} \frac{2x}{(x+1)^2} \cdot \frac{1}{x}$$

$$= \lim_{x\to 0} \frac{2}{(x+1)^2}$$

$$= 2$$

tangent line: $y-0 = 2(x-0) \Rightarrow y = 2x$

9. (a) $f(x) = 3 + 4x^2 - 2x^3$ at $x = a$

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{[3 + 4(a+h)^2 - 2(a+h)^3] - [3 + 4a^2 - 2a^3]}{h}$$

$$= \lim_{h \to 0} \frac{3 + 4(a^2 + 2ah + h^2) - 2(a^3 + 3a^2h + 3ah^2 + h^3) - 3 - 4a^2 + 2a}{h}$$

$$= \lim_{h \to 0} \frac{8ah + 4h^2 - 6a^2h - 6ah^2 - 2h^3}{h}$$

$$= \lim_{h \to 0} \frac{h(8a + 4h - 6a^2 - 6ah - 2h^2)}{h}$$

$$= \lim_{h \to 0} (8a + 4h - 6a^2 - 6ah - 2h^2)$$

$$= 8a - 6a^2$$

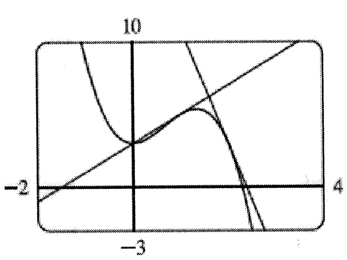
(b) at $(1,5)$ $m = 8(1) - 6(1)^2 = 2$

tangent line: $y - 5 = 2(x-1) \Rightarrow y = 2x + 3$

at $(2,3)$ $m = 8(2) - 6(2)^2 = -8$

tangent line: $y - 3 = -8(x-2) \Rightarrow y = -8x + 19$

(c)



10. (a) $f(x) = \frac{1}{\sqrt{x}}$ at $x = a$

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{a+h}} - \frac{1}{\sqrt{a}}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{a} - \sqrt{a+h}}{\sqrt{a} \cdot \sqrt{a+h}} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{a} - \sqrt{a+h}}{h\sqrt{a} \cdot \sqrt{a+h}} \cdot \frac{\sqrt{a} + \sqrt{a+h}}{\sqrt{a} + \sqrt{a+h}}$$

$$= \lim_{h \to 0} \frac{a - (a+h)}{h\sqrt{a} \cdot \sqrt{a+h}\,(\sqrt{a} + \sqrt{a+h})}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{a} \cdot \sqrt{a+h}\,(\sqrt{a} + \sqrt{a+h})}$$

$$= \frac{-1}{\sqrt{a} \cdot \sqrt{a}\,(\sqrt{a} + \sqrt{a})}$$

$$= \frac{-1}{a(2\sqrt{a})}$$

$$= \frac{-1}{2a^{\frac{3}{2}}}$$

(b) at $(1,1)$ $m = \frac{-1}{2(1)^{3/2}} = -\frac{1}{2}$

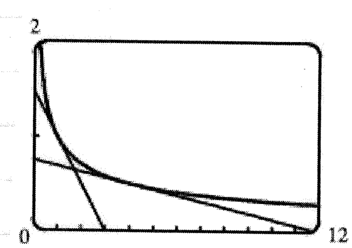
tangent line: $y - 1 = -\frac{1}{2}(x-1) \Rightarrow y = -\frac{1}{2}x + \frac{3}{2}$

at $(4, \frac{1}{2})$ $m = \frac{-1}{2(4)^{3/2}} = \frac{-1}{2(2)^3} = \frac{-1}{16}$

tangent line: $y - \frac{1}{2} = -\frac{1}{16}(x+4)$

$y = -\frac{1}{16}x + \frac{3}{4}$

(c)



13. $f(t) = 40t - 16t^2$

$$v(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{[40(2+h) - 16(2+h)^2] - [40(2) - 16(2)^2]}{h}$$

$$= \lim_{h\to 0} \frac{80 + 40h - 16(4 + 4h + h^2) - 16}{h}$$

$$= \lim_{h\to 0} \frac{-24h - 16h^2}{h}$$

$$= \lim_{h\to 0} (-24 - 16h)$$

$$= -24 \ \text{ft/sec}$$

15. $f(t) = \frac{1}{t^2}$

$$v(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{\frac{1}{(a+h)^2} - \frac{1}{a^2}}{h}$$

$$= \lim_{h\to 0} \frac{a^2 - (a+h)^2}{a^2(a+h)^2} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{a^2 - (a^2 + 2ah + h^2)}{a^2(a+h)^2} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-2ah - h^2}{a^2 h (a+h)^2}$$

$$= \lim_{h\to 0} \frac{h(-2a - h)}{a^2 h (a+h)^2}$$

$$= \lim_{h\to 0} \frac{-2a - h}{a^2 (a+h)^2}$$

$$= \frac{-2a}{a^4}$$

$$= \frac{-2}{a^3}$$

15 Cont'd

$V(1) = \frac{-2}{(1)^3} = -2 \text{ m/sec}$

$V(2) = \frac{-2}{(2)^3} = -\frac{1}{4} \text{ m/sec}$

$V(3) = \frac{-2}{3^3} = \frac{-2}{27} \text{ m/sec}$

37. $f(t) = 100 + 50t - 4.9t^2$

$$\begin{aligned}
v(5) &= \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} \\
&= \lim_{h\to 0} \frac{[100 + 50(5+h) - 4.9(5+h)^2] - [227.5]}{h} \\
&= \lim_{h\to 0} \frac{100 + 250 + 50h - 4.9(25 + 10h + h^2) - 227.5}{h} \\
&= \lim_{h\to 0} \frac{h - 4.9h^2}{h} \\
&= \lim_{h\to 0} (1 - 4.9h) \\
&= 1 \text{ m/sec}
\end{aligned}$$

Speed at 5 sec is $|v(5)| = 1$ m/sec

38. $f(t) = t^{-1} - t = \frac{1}{t} - \frac{t}{1} = \frac{1-t^2}{t}$

$$v(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$$

$$= \lim_{h \to 0} \frac{[(5+h)^{-1} - (5+h)] - [5^{-1} - 5]}{h}$$

$$= \lim_{h \to 0} \frac{(5+h)^{-1} - 5^{-1} - h}{h}$$

$$= \lim_{h \to 0} \left[\frac{1}{5+h} - \frac{1}{5} - \frac{h}{1}\right] \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{5 - (5+h) - h(5)(5+h)}{5(5+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{5 - 5 - h - 25h - 5h^2}{5h(5+h)}$$

$$= \lim_{h \to 0} \frac{-26h - 5h^2}{5h(5+h)}$$

$$= \lim_{h \to 0} \frac{-26 - 5h}{5(5+h)}$$

$$= \frac{-26}{25} \text{ m/sec}$$

speed at 5 sec is $|v(5)| = \frac{26}{25}$ m/sec