

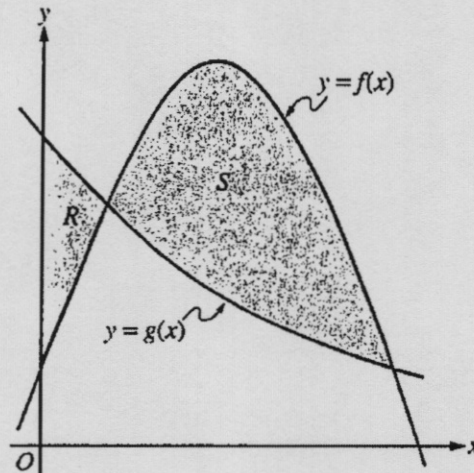
2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of f and g , and let S be the shaded region in the first quadrant enclosed by the graphs of f and g , as shown in the figure above.
- Find the area of R .
 - Find the area of S .
 - Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

3pts a) $\int_0^{.178} (g(x) - f(x)) dx = 0.065$

3pts b) $\int_{.178}^1 (f(x) - g(x)) dx = 0.410$

8pts c) $\pi \int_{.178}^1 [(f(x)+1)^2 - (g(x)+1)^2] dx = 4.559$

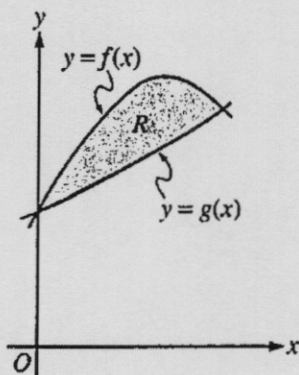
2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



2. Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

- (a) Find the area of R .
 (b) Find the volume of the solid generated when R is revolved about the x -axis.
 (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

1 pt for correct limits in an integral in (a), (b), or (c)

2 pts a) $\int_0^{1.136} (f(x) - g(x)) dx = 0.429$

3 pts b) $\pi \int_0^{1.136} ([f(x)]^2 - [g(x)]^2) dx = 4.267$

3 pts c) $\frac{\pi}{2} \int_0^{1.136} \left(\frac{f(x) - g(x)}{2} \right)^2 dx$

$\frac{\pi}{8} \int_0^{1.136} [f(x) - g(x)]^2 dx = .078$

Please do the following practice free response AP problems. Questions 1-3 allow the use of a graphing calculator and questions 4-7 do NOT.

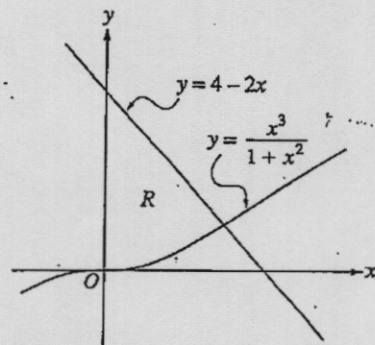
3) Let R be the region bounded by the y -axis and the graphs of

$$y = \frac{x^3}{1+x^2} \text{ and } y = 4 - 2x, \text{ as shown in the figure above.}$$

(a) Find the area of R .

(b) Find the volume of the solid generated when R is revolved about the x -axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.



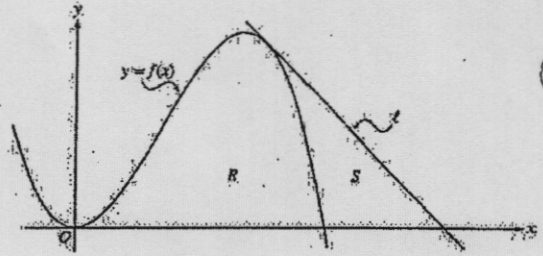
1 pt for correct limits

2 pts a)
$$\int_0^{1.488} \left[4 - 2x - \frac{x^3}{1+x^2} \right] dx = 3.215$$

3 pts b)
$$\pi \int_0^{1.488} \left[(4 - 2x)^2 - \left(\frac{x^3}{1+x^2} \right)^2 \right] dx = 31.885 \text{ or } 10.149\pi$$

3 pts c)
$$\int_0^{1.488} \left[4 - 2x - \frac{x^3}{1+x^2} \right]^2 dx = 8.997$$

9) Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f . Let R be the region bounded by the graph of f and the x -axis, and let S be the region bounded by the graph of f , the line ℓ , and the x -axis, as shown above.



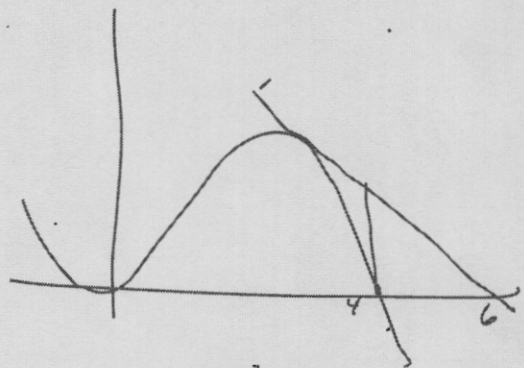
- (a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.
 (b) Find the area of S .
 (c) Find the volume of the solid generated when R is revolved about the x -axis.

2pts a) $f'(x) = 8x - 3x^2$
 $f'(3) = 24 - 27 = -3$
 pt $(3, 9)$
 $y - 9 = -3(x - 3)$
 $y = -3x + 18$

4pts b) $f(x) = 0$ at $x = 4$
 The line goes through $(6, 0)$

$$\text{Area} = \int_0^4 [(18 - 3x) - (4x^2 - x^3)] dx$$

$$+ \int_4^6 (18 - 3x) dx = 1.916 + 6 = 7.916$$



2pts c) $\pi \int_0^4 (4x^2 - x^3)^2 dx = 156.038\pi$
 $= 490.208$