

14-6: Sum and Difference Identities

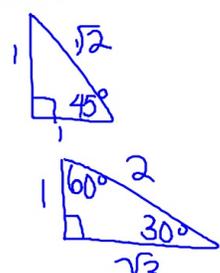
← trig. std. 10.0

★ identities 4-9 on trig identities list

NO CALCULATORS

Examples: find exact values in simplest form.

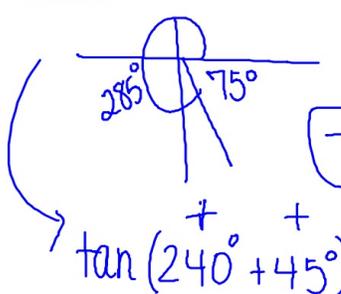
1 $\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$


2 $\cos 105^\circ = \cos(45^\circ + 60^\circ)$

$$= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

Q4 - $\tan 285^\circ = -\tan 75^\circ = -\tan(30^\circ + 45^\circ)$



$\tan(240^\circ + 45^\circ)$

$-\tan(30^\circ + 45^\circ)$

$-\tan(30^\circ + 45^\circ) = -\left(\frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ}\right)$

$= -\left(\frac{\frac{\sqrt{3}}{3} + 1}{1 - \left(\frac{\sqrt{3}}{3} \cdot 1\right)}\right)$

$= -\frac{(\sqrt{3} + 3)}{(3 - \sqrt{3})} \cdot \frac{(\sqrt{3} + 3)}{(3 + \sqrt{3})}$

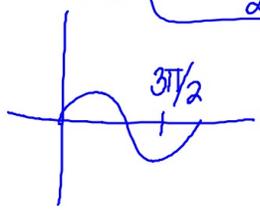
$= \frac{-(3 + 6\sqrt{3} + 9)}{9 - 3} = \frac{-12 - 6\sqrt{3}}{6}$

$-2 - \sqrt{3}$

Examples: simplify to a trig function of a single angle and find the exact value.

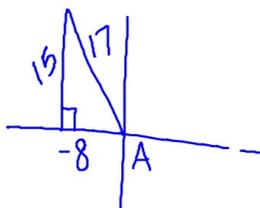
Q4 $\frac{\tan 80^\circ - \tan 20^\circ}{1 + \tan 80^\circ \tan 20^\circ} = \tan(80^\circ - 20^\circ) = \tan 60^\circ = \sqrt{3}$

Q5 $\sin \pi \cos \frac{\pi}{2} + \cos \pi \sin \frac{\pi}{2} = \sin\left(\pi + \frac{\pi}{2}\right) = \sin \frac{3\pi}{2} = -1$



6 Given: $\cos A = \frac{-8}{17}$, $\frac{\pi}{2} < A \leq \pi$ and $\tan B = \frac{4}{3}$, $0 \leq B < \frac{\pi}{2}$

Find exact value of $\tan(A - B)$. $= \frac{77}{36}$



$$= \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\left[\frac{-15}{8} - \frac{4}{3} \right]}{1 + \left(\frac{-15}{8} \cdot \frac{4}{3} \right)}$$

$$= \frac{\left[\frac{-15}{8} - \frac{4}{3} \right] \cdot 24}{\left[1 + \left(\frac{-15}{8} \cdot \frac{4}{3} \right) \right] \cdot 24} = \frac{-45 - 32}{24 - 20} = \frac{-77}{4}$$