Extra Practice/Review

Classify as arithmetic, geometric, or neither.

1) 4, 7, 10, ... Arith.
2) 6, 12, 24, ... geo
3) 1, 4, 9, 16..... neither $\sum_{k=1}^{\infty} k^2$
4) 27, 9, 3, ... geo
5) 11, 9, 7, ... Arith
6) 1, 3, 7, 15, 31..... neither

+2 +4 +8 +16

Sec 11-4

Infinite Geometric Series

22.0 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\ldots$$

$S_1 = \frac{1}{2} = .5$

$S_2 = \frac{1}{2}+\frac{1}{4} = \frac{3}{4} = .75$

$S_3 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8} = \frac{7}{8} = .88$

$S_4 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16} = \frac{15}{16} = .94$

$S_5 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32} = \frac{31}{32} = .97$

Even though this series has infinitely many terms it has a finite sum

The Sum of a Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and a common ratio r

$$S = \frac{a_1}{1-r}$$

provided $|r| < 1$. If the $|r| \geq 1$, the series has no sum.

S = sum of infinitely many terms
a_1 = first term
r = common ratio

IMPORTANT: this only works for an infinite geometric series where | r | < 1

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\ldots \quad r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{2}{4} = \frac{1}{2}$$

$$S = \frac{a_1}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

(-20) + (-10) + (-5) + (-5/2) + (-5/4) + ...

S_1 = -20 $\quad r = \frac{-10}{-20} = \frac{1}{2}$

S_2 = -20 + (-10) = -30

S_3 = -20 + (-10) + (-5) = -35

S_4 = -20 + (-10) + (-5) + (-5/2) = -37.5

S_5 = -20 + (-10) + (-5) + (-5/2) + (-5/4) = -38.75

$S_{30} = -20\left(\frac{1-\left(\frac{1}{2}\right)^{30}}{1-\frac{1}{2}}\right) = -39.999996$

$S = \frac{-20}{1-\frac{1}{2}} = \frac{-20}{\frac{1}{2}} = -40$

Find the sum of the infinite geometric series.

$\sum_{i=1}^{\infty} 3(2)^{i-1}$

a_1 r

NO SUM

Answer:

$S = \frac{a_1}{1-r}$

Find the sum of the infinite geometric series.

$\sum_{i=1}^{\infty} 3(0.7)^{i-1}$

a_1 r

$\frac{a_1}{1-r} = \frac{3}{1-0.7} = \frac{3}{0.3} = 10$

Answer:

$S = \frac{a_1}{1-r}$

Find the sum of the infinite geometric series.

$3 + 9 + 27 + 81 + \ldots$

$r = \frac{9}{3} = 3$

NO SUM

Answer:

$S = \frac{a_1}{1-r}$

Find the sum of the infinite geometric series.

$1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \ldots\ldots$

$r = \frac{-\frac{1}{4}}{1} = \frac{-1}{4}$

$S = \frac{a_1}{1-r} = \frac{1}{1-\left(\frac{-1}{4}\right)} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$

Answer:

$S = \frac{a_1}{1-r}$

Find the sum of the infinite geometric series.

$\sum_{i=1}^{\infty} 2(0.1)^{i-1} = \frac{a_1}{1-r} = \frac{2}{1-0.1} = \frac{2}{0.9} = \frac{20}{9}$

$r = 0.1$

Answer:

$S = \frac{a_1}{1-r}$

Find the sum of the infinite geometric series.

$12 + 4 + \frac{4}{3} + \frac{4}{9} + \ldots\ldots$

$r = \frac{4}{12} = \frac{1}{3}$

$S = \frac{12}{1-\frac{1}{3}} = \frac{12}{\frac{2}{3}} = 18$

Answer:

$S = \frac{a_1}{1-r}$

An infinite geometric series with first term $a_1 = 4$ and a sum of 10. What is the common ratio?

$S = \frac{a_1}{1-r}$

$\frac{10}{1} = \frac{4}{1-r}$

$4 = 10(1-r)$

$4 = 10 - 10r$

$-10 \quad -10$

$-6 = -10r$

$\frac{-6}{-10} = \frac{-10r}{-10}$

$\boxed{\frac{3}{5} = r}$

Answer:

$S = \frac{a_1}{1-r}$

An infinite geometric series with first term $a_1 = 5$ and a sum of 27/5. What is the common ratio?

$S = \frac{a_1}{1-r}$

$\frac{27}{5} = \frac{5}{1-r}$

$25 = 27(1-r)$

$25 = 27 - 27r$

$-27 \quad -27$

$-2 = -27r$

$\frac{-2}{-27} = \frac{-27r}{-27}$

$\boxed{r = \frac{2}{27}}$

Answer:

$S = \frac{a_1}{1-r}$