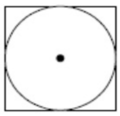
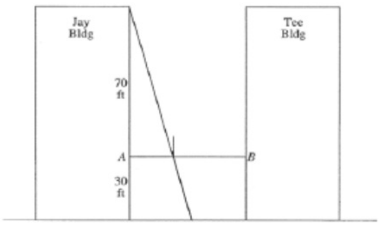


A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$)

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

A tight rope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.



(a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)

(b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)

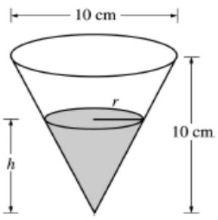
(c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee Building when she is 10 feet from point B? (Indicate units of measure.)

The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second. (Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

Consider the curve defined by $x^2 + xy + y^2 = 27$.

(a) Write an expression for the slope of the curve at any point (x, y).

(b) Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

(c) Find the points on the curve where the lines tangent to the curve are vertical.