

Algebra 2H Extra Notes: Binomial Theorem

The coefficients of the expansion of $(x + y)^n$ are the numbers in Pascal's triangle. The binomial coefficients are also combinations.

| Pascal's Triangle | Combinations | Binomial Expansion |
|-------------------|---|---|
| 1 | ${}_0C_0$ | $(x + y)^0 = 1$ |
| 1 1 | ${}_1C_0$ ${}_1C_1$ | $(x + y)^1 = x + y$ |
| 1 2 1 | ${}_2C_0$ ${}_2C_1$ ${}_2C_2$ | $(x + y)^2 = x^2 + 2xy + y^2$ |
| 1 3 3 1 | ${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$ | $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ |

- The number of terms in the expansion of $(x + y)^n$ is $n + 1$.
- The exponents of x decrease while the exponents of y increase.
- The sum of the exponents of the variables in any term is n .
- The Pascal numbers or combinations are the coefficients of each term.

The **Binomial Theorem** can be used to find a single term of a binomial expansion or instead of Pascal's Triangle to expand $(x + y)^n$.

Binomial Theorem

For any whole number n ,

$$(x + y)^n = \sum_{r=0}^n {}_nC_r x^{n-r} y^r = {}_nC_0 x^n y^0 + {}_nC_1 x^{n-1} y^1 + {}_nC_2 x^{n-2} y^2 + \dots + {}_nC_n x^0 y^n$$

Example 1. Find the term containing x^7 in $(2 - 3x)^{10}$.

Example 2. Find the middle term of $(2 - 3x)^{10}$.

****Show all work on your own paper. Save this handout for reference.****

Use the Binomial Theorem for #1-3.

- Find the term containing x^4 in the expansion of $(x - 3)^7$.
- Find the third term in the expansion $(3x + 2)^9$.
- Find the middle term of $(x^2 + 4)^8$.
- Use Pascal's Triangle or the Binomial Theorem to expand $(2x - 5)^5$.

Give the answers to #5-6 to the nearest thousandth.

- Mrs. Bryant bought 4 boxes of Chocolate Krave cereal. One out of every 5 boxes has a coupon for a free box of Krave.
 - What is the probability that Mrs. Bryant got exactly 3 coupons?
 - What is the probability that Mrs. Bryant got at least 2 coupons?
- In a manufacturing plant, there is a 2% chance that a stamp will be placed on a box upside down. If the plant shipped 30 boxes today, what is the probability that at least 2 of the boxes have an upside-down stamp?
- Suppose that in 2004, 95% of the high school students in a city graduated. A follow-up study showed that at age 25, 81% of the high school graduates held full-time jobs while only 63% of those who did not graduate held full-time jobs. What is the probability that a randomly selected former student did not have a full-time job at age 25? Give the answer as a percent rounded to the nearest whole number.

8. Tomas is trying to decide on the best possible route to drive to work so that he is on time. Over a 40-day period, he randomly selects a route each day and keeps track of whether he is late. The table of joint and marginal relative frequencies for the results of his study are shown.

| | Late | Not late | Total |
|---------|-------|----------|-------|
| Route A | 0.1 | 0.25 | 0.35 |
| Route B | 0.075 | 0.175 | 0.25 |
| Route C | 0.1 | 0.3 | 0.4 |
| Total | 0.275 | 0.725 | 1 |

Use conditional probabilities to determine the best route for Tomas to drive so that he is on time for work.

Answers:

1. $-945x^4$ 2. $314,928x^7$ 3. $17,920x^8$ 4. $32x^5 - 400x^4 + 2000x^3 - 5000x^2 + 6250x - 3125$

5a. 0.026

5b. 0.181

6. 0.121

7. 20%

8. Tomas should choose Route C.