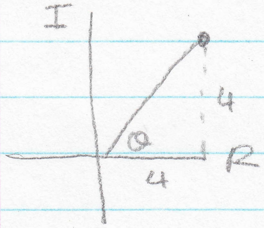
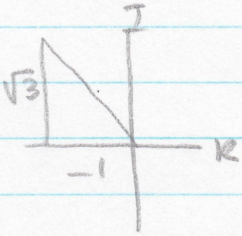
Review 11A

1 

$r = 4\sqrt{2}$

$\theta = \tan^{-1}\left(\frac{4}{4}\right) = 45^\circ$

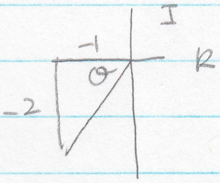
$4+4i = 4\sqrt{2}\ \text{cis}\ 45^\circ$

2\. 

$r = \sqrt{3+1} = 2$

$\theta = 180^\circ - \tan^{-1}\left(\oplus\frac{\sqrt{3}}{1}\right) = 180^\circ - 60^\circ = 120^\circ$

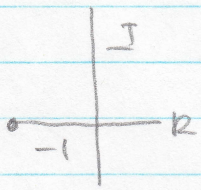
$-1+\sqrt{3}i = 2\,\text{cis}\ 120^\circ$

3 

$r = \sqrt{1+4} = \sqrt{5}$

$\theta = 180^\circ + \tan^{-1}\left(\frac{2}{1}\right) = 243.4^\circ$

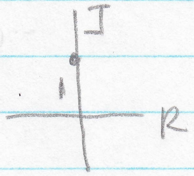
$-1-2i = \sqrt{5}\ \text{cis}\ 243.4^\circ$

4 

$r = 1$

$\theta = 180^\circ$

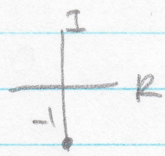
$-1+0i = 1\,\text{cis}\,180^\circ$

5\. 

$r = 1$

$\theta = 90^\circ$

$0 + i = 1\,\text{cis}\ 90^\circ$

6\. 

$r = 1;\ \theta = 270^\circ$

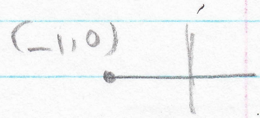
$0 - i = 1\ \text{cis}\ 270^\circ$

7\. $\frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\frac{1}{2}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{\sqrt{3}}{4}i$

8 $8\,\text{cis}\ \pi = 8(\cos\pi + i\sin\pi)$

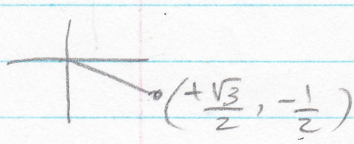
$= 8(-1 + 0i) = -8 + 0i$



9\. $4\,\text{cis}\ \frac{11\pi}{6} = 4\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$

$= 4\left(\frac{\sqrt{3}}{2}\right) + i\,(4)\left(-\frac{1}{2}\right)$

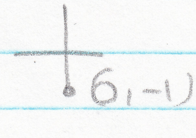
$= 2\sqrt{3} - 2i$

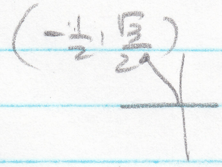


10. $cis\left(-\frac{\pi}{2}\right) = cis\left(-\frac{\pi}{2} + 2\pi\right) = cis\ \frac{3\pi}{2}$

$= \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$

$= 0 + i(-1) = 0 - i$





11. $z_1 \cdot z_2 = (3\ cis\ 80°)\left(\frac{1}{2}\ cis\ 40°\right)$

$= (3)\left(\frac{1}{2}\right) cis\ (80° + 40°)$

$= \frac{3}{2}\ cis\ 120° = \frac{3}{2}(\cos 120° + i\sin 120°)$

$= \frac{3}{2}\left(-\frac{1}{2}\right) + i\left(\frac{3}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{3}{4} + \frac{3\sqrt{3}}{4}i$

$\frac{z_1}{z_2} = \frac{3\ cis\ 80°}{\frac{1}{2}\ cis\ 40°} = \left(3 \div \frac{1}{2}\right) cis\ (80° - 40°)$

$= 6\ cis\ 40° = 6(\cos 40° + i\sin 40°)$

$= 4.6 + 3.9i$

12. $\left(5\ cis\ \frac{2\pi}{3}\right)\left(4\ cis\ \frac{3\pi}{4}\right) = 20\ cis\left(\frac{4}{4}\frac{2\pi}{3} + \frac{3\pi}{4}\frac{3}{3}\right)$

$= 20\ cis\ \frac{17\pi}{12} = 20\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right)$

$= -5.2 - 19.3i$

$\frac{5\ cis\ \frac{2\pi}{3}}{4\ cis\ \frac{3\pi}{4}} = \frac{5}{4}\ cis\left(\frac{4}{4}\frac{2\pi}{3} - \frac{3\pi}{4}\frac{3}{3}\right)$

$= \frac{5}{4}\ cis\left(\frac{-\pi}{12}\right) = \frac{5}{4}\ cos\left(\frac{-\pi}{12} + \frac{2\pi}{1}\frac{12}{12}\right)$

$= \frac{5}{4}\ cis\ \frac{23\pi}{12} = \frac{5}{4}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right)$

$= 1.207 - 0.324i$

13. square roots of i

$i = 0 + i$

$r = 1$; $\theta = 90° + n \cdot 360°$ $\rightarrow i = 1\ cis(90° + n \cdot 360°)$

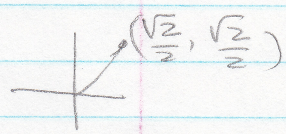
square roots of $i = [1\ cis\ (90° + n \cdot 360°)]^{\frac{1}{2}}$

$= 1^{\frac{1}{2}}\ cis\ \frac{1}{2}(90° + n \cdot 360°)$

$= 1\ cis(45° + n \cdot 180°)$

$n=0$ root #1 $= 1 \text{ cis } 45^\circ = 1 \left(\cos 45^\circ + i \sin 45^\circ\right)$

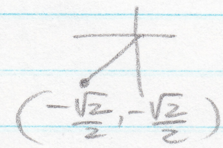
$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$



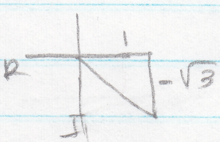
$n=1$ root #2 $= 1 \text{ cis } (45^\circ + 180^\circ) = \text{cis } 225^\circ$

$= \cos 225^\circ + i \sin 225^\circ$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$



14. $1-\sqrt{3}\, i$



$r = \sqrt{1+3} = 2$

$\theta = 360^\circ - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$360^\circ - 60^\circ = 300^\circ + n \cdot 360^\circ$

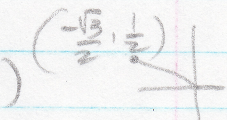
$1-\sqrt{3}\, i = 2 \text{ cis}(300^\circ + n \cdot 360^\circ)$

Square roots of $1-\sqrt{3}\, i =$

$\left[2 \text{ cis}(300^\circ + n \cdot 360^\circ)\right]^{\frac{1}{2}}$

$= 2^{\frac{1}{2}} \text{ cis } \frac{1}{2}(300^\circ + n \cdot 360^\circ)$

$= \sqrt{2} \text{ cis } 150^\circ + n \cdot 180^\circ$



$n=0$ root #1 $\to \sqrt{2} \text{ cis } 150^\circ = \sqrt{2}\left(\cos 150^\circ + i \sin 150^\circ\right)$

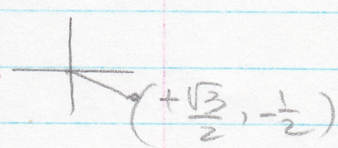
$= \sqrt{2}\left(-\frac{\sqrt{3}}{2}\right) + i\, (\sqrt{2})\left(\frac{1}{2}\right) = -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} i$

$n=1$ root #2 $= \sqrt{2} \text{ cis}(150^\circ + 180^\circ) = \sqrt{2} \text{ cis } 330^\circ$

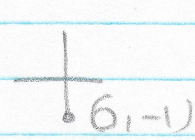
$= \sqrt{2}\left(\cos 330^\circ + i \sin 330^\circ\right)$

$= \sqrt{2}\left(\frac{\sqrt{3}}{2}\right) + i\, (\sqrt{2})\left(-\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} i$

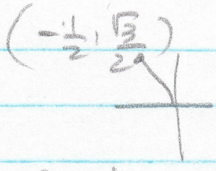


10. $cis(-\frac{\pi}{2}) = cis(-\frac{\pi}{2}+2\pi) = cis\,\frac{3\pi}{2}$



$= \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}$

$= 0 + i(-1) = 0 - i$



11. $z_1 \cdot z_2 = (3\,cis\,80°)(\frac{1}{2}\,cis\,40°)$

$= (3)(\frac{1}{2})\,cis\,(80° + 40°)$

$= \frac{3}{2}\,cis\,120° = \frac{3}{2}(\cos 120° + i\sin 120°)$

$= \frac{3}{2}(-\frac{1}{2}) + i(\frac{3}{2})(\frac{\sqrt{3}}{2}) = -\frac{3}{4} + \frac{3\sqrt{3}}{4}i$

$\frac{z_1}{z_2} = \frac{3\,cis\,80°}{\frac{1}{2}\,cis\,40°} = (3 \div \frac{1}{2})\,cis\,(80° - 40°)$

$= 6\,cis\,40° = 6(\cos 40° + i\sin 40°)$

$= 4.6 + 3.9i$

12. $(5\,cis\,\frac{2\pi}{3})(4\,cis\,\frac{3\pi}{4}) = 20\,cis\,(\frac{4}{4}\cdot\frac{2\pi}{3} + \frac{3\pi}{4}\cdot\frac{3}{3})$

$= 20\,cis\,\frac{17\pi}{12} = 20(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12})$

$= -5.2 - 19.3i$

$\frac{5\,cis\,\frac{2\pi}{3}}{4\,cis\,\frac{3\pi}{4}} = \frac{5}{4}\,cis\,(\frac{4}{4}\cdot\frac{2\pi}{3} - \frac{3\pi}{4}\cdot\frac{3}{3})$

$= \frac{5}{4}\,cis\,(\frac{-\pi}{12}) = \frac{5}{4}\cos(\frac{-\pi}{12} + \frac{2\pi}{1}\cdot\frac{12}{12})$

$= \frac{5}{4}\,cis\,\frac{23\pi}{12} = \frac{5}{4}(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12})$

$= 1.207 - 0.324i$

13 square roots of i

$i = 0 + i$

$r = 1$; $\theta = 90°$

$0 + i = 1\,cis\,90°$

root $= r\,cis\,\alpha \rightarrow (r\,cis\,\alpha)^2 = 1\,cis\,90°$

$r^2\,cis\,2\alpha = 1\,cis\,90°$

$r^2 = 1 \rightarrow r = 1$ (r must be pos)

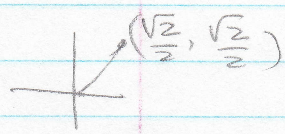
$\text{cis}\, 2\alpha = \text{cis}\, 90^\circ \quad \rightarrow \quad 2\alpha = 90^\circ + n \cdot 360^\circ$

$\alpha = 45^\circ + n \cdot 180^\circ$

Root $= 1 \,\text{cis}\, 45^\circ + n \cdot 180^\circ$

$n=0 \quad r\,\text{cis}\,\alpha = 1\,\text{cis}\,45^\circ = 1\,(\cos 45^\circ + i \sin 45^\circ)$

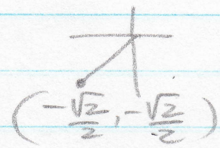
$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$



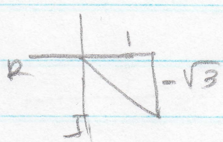
$n=1 \quad r\,\text{cis}\,\alpha = 1\,\text{cis}\,(45^\circ + 180^\circ) = \text{cis}\, 225^\circ$

$= \cos 225^\circ + i \sin 225^\circ$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$



14. $1 - \sqrt{3}\, i$



$r = \sqrt{1+3} = 2$

$\theta = 360^\circ - \tan^{-1}(\frac{\sqrt{3}}{1}) = 360^\circ - 60^\circ = 300^\circ$

$1 - \sqrt{3}\, i = 2\,\text{cis}\, 300^\circ$

root $= r\,\text{cis}\,\alpha \rightarrow (r\,\text{cis}\,\alpha)^2 = 2\,\text{cis}\,300^\circ$

$r^2\,\text{cis}\,2\alpha = 2\,\text{cis}\,300^\circ$

$r^2 = 2 \rightarrow r = \sqrt{2}$

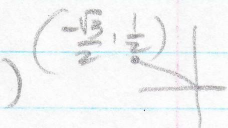
$\text{cis}\,2\alpha = \text{cis}\,300^\circ \rightarrow 2\alpha = 300^\circ + n \cdot 360^\circ$

$\alpha = 150^\circ + n \cdot 180^\circ$

$r\,\text{cis}\,\alpha = \sqrt{2}\,\text{cis}\,150^\circ + n \cdot 180^\circ$

$n=0 \quad r\,\text{cis}\,\alpha = \sqrt{2}\,\text{cis}\,150^\circ = \sqrt{2}\,(\cos 150^\circ + i \sin 150^\circ)$

$= \sqrt{2}\,(-\frac{\sqrt{3}}{2}) + i\,(\sqrt{2})(\frac{1}{2}) = -\frac{\sqrt{6}}{2} + \frac{\sqrt{2}}{2} i$

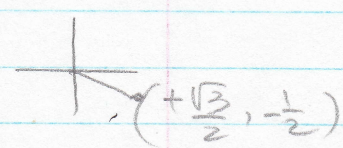


$n=1 \quad r\,\text{cis}\,\alpha = \sqrt{2}\,\text{cis}\,(150^\circ + 180^\circ) = \sqrt{2}\,\text{cis}\,330^\circ$

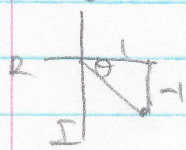
$= \sqrt{2}\,(\cos 330^\circ + i \sin 330^\circ)$

$= \sqrt{2}\,(\frac{\sqrt{3}}{2}) + i\,(\sqrt{2})(-\frac{1}{2})$

$= \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} i$



15. $(1-i)^3$



$r = \sqrt{1+1} = \sqrt{2}$

$\theta = 360° - \tan^{-1}\left(\frac{1}{1}\right) = 360° - 45° = 315°$

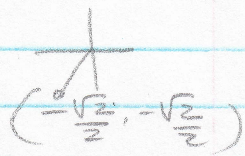
$(1-i) = \sqrt{2} \text{ cis } 315°$

$(\sqrt{2} \text{ cis } 315°)^3 = (\sqrt{2})^3 \text{ cis } 3(315°)$

$= 2\sqrt{2} \text{ cis } 945° = 2\sqrt{2} \sin(945° - 2(360°))$

$= 2\sqrt{2} \text{ cis } 225°$

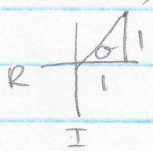
$= 2\sqrt{2}(\cos 225° + i \sin 225°)$



$= 2\sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) + i(2\sqrt{2})\left(-\frac{\sqrt{2}}{2}\right)$

$= -2 - 2i$

16. $(1+i)^{-4}$



$r = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$

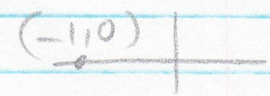
$1+i = \sqrt{2} \text{ cis } 45°$

$(\sqrt{2} \text{ cis } 45°)^{-4} = (\sqrt{2})^{-4} \text{ cis}(-4)(45°)$

$= \frac{1}{(\sqrt{2})^4} \text{ cis } -180°$

$= \frac{1}{4} \text{ cis}(-180° + 360°)$

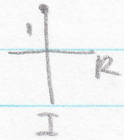
$= \frac{1}{4} \text{ cis } 180°$



$= \frac{1}{4}(\cos 180° + i \sin 180°)$

$= \frac{1}{4}(-1) + \left(\frac{1}{4}\right)(i)(0) = -\frac{1}{4} + 0i$

17 $i^4 =$



$r = 1 \quad \theta = 90°$

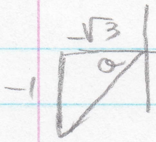
$i^4 = (1 \text{ cis } 90°)^4$

$= 1^4 \text{ cis } 4(90°) = 1 \text{ cis } 360°$

$= (\cos 0° + i \sin 0°) = 1 + i(0)$

$= 1 + 0i$

18 $(-\sqrt{3} - i)^3$



$r = \sqrt{3+1} = 2$

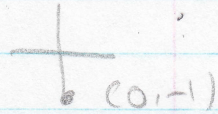
$\theta = 180° + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 180° + 30° = 210°$

$(2 \text{ cis } 210°)^3 = 2^3 \text{ cis } 3(210°)$

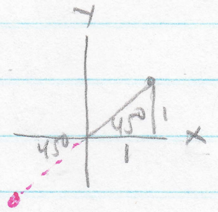
$= 8 \text{ cis } 630° = 8 \text{ cis } (630° - 360°) = 8 \text{ cis } 270°$

$= 8(\cos 270° + i \sin 270°)$

$= 8(0) + 8(i)(-1) = 0 - 8i$



19



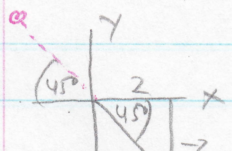
$r = \sqrt{1+1} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$

$(\sqrt{2}, 45°)$ no i ; no cis $= (\sqrt{2}, 45° - 360°) = (\sqrt{2}, -315°)$

$(-\sqrt{2}, 225°) = (-\sqrt{2}, 225° - 360°) = (-\sqrt{2}, -135°)$

20



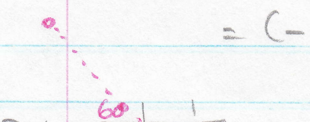
$r = \sqrt{4+4} = 2\sqrt{2}$

$\theta = 360° - \tan^{-1}\left(\frac{2}{2}\right) = 360° - 45° = 315°$

$(2\sqrt{2}, 315°) = (2\sqrt{2}, 315° - 360°) = (2\sqrt{2}, -45°)$

$= (-2\sqrt{2}, 135°) = (-2\sqrt{2}, 135° - 360°) = (-2\sqrt{2}, -225°)$

21



$r = \sqrt{1+3} = 2$

$\theta = 360° - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 360° - 60° = 300°$

$(1, -\sqrt{3}) = (2, 300°) = (2, 300° - 360°) = (2, -60°)$

$= (-2, 120°) = (-2, 120° - 360°) = (-2, 240°)$

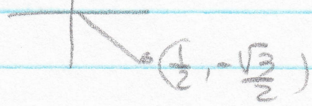
22

$r = 9 ; \theta = 180°$

$(9, 180°) = (9, 180° - 360°) = (9, -180°)$

$(-9, 0°) = (-9, 0° - 360°) = (-9, -360°)$

23 $(-4, 300^\circ)$

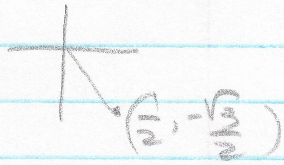


$x = -4\cos 300^\circ = -4(\frac{1}{2}) = -2$

$y = -4\sin 300^\circ = -4(-\frac{\sqrt{3}}{2}) = 2\sqrt{3}$

$(-2, 2\sqrt{3})$

24. $(-6, -\frac{\pi}{3}) = (-6, \frac{-\pi}{3} + 2\pi) = (-6, \frac{5\pi}{3})$

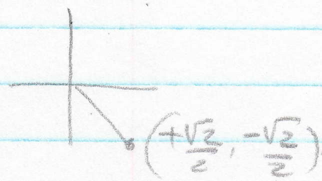


$x = -6\cos\frac{5\pi}{3} = -6(\frac{1}{2}) = -3$

$y = -6\sin\frac{5\pi}{3} = -6(-\frac{\sqrt{3}}{2}) = 3\sqrt{3}$

$(-3, 3\sqrt{3})$

25 $(-\frac{3}{2}, \frac{-\pi}{4}) = (-\frac{3}{2}, \frac{-\pi}{4} + 2\pi) = (-\frac{3}{2}, \frac{7\pi}{4})$

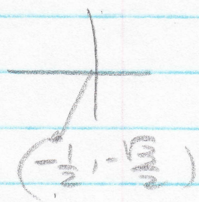


$x = -\frac{3}{2}\cos\frac{7\pi}{4} = -\frac{3}{2}(\frac{\sqrt{2}}{2}) = \frac{-3\sqrt{2}}{4}$

$y = -\frac{3}{2}\sin\frac{7\pi}{4} = -\frac{3}{2}(-\frac{\sqrt{2}}{2}) = \frac{3\sqrt{2}}{4}$

$(-\frac{3\sqrt{2}}{4}, \frac{3\sqrt{2}}{4})$

26 $(+3, -\frac{2\pi}{3}) = (+3, -\frac{2\pi}{3} + 2\pi) = (+3, \frac{4\pi}{3})$



$x = +3\cos\frac{4\pi}{3} = +3(-\frac{1}{2}) = -\frac{3}{2}$

$y = +3\sin\frac{4\pi}{3} = +3(\frac{-\sqrt{3}}{2}) = -\frac{3\sqrt{3}}{2}$

$(\frac{-3}{2}, \frac{-3\sqrt{3}}{2})$

27 $r = 1 - 2\cos\theta$

θ	$\cos\theta$	$1-2\cos\theta$
$0°$	1	−1
$60°$	0.5	0
$90°$	0.	1
$120°$	−0.5	2
$180°$	−1	3
$240°$	−0.5	2
$270°$	0	1
$300°$	0.5	0
$360°$	1	−1