19. $\tan^{-1} x = 1 \quad \Rightarrow \quad \tan \tan^{-1} x = \tan 1 \quad \Rightarrow \quad x = \tan 1 \;\; (\approx 1.5574)$

20. $\sin x = 0.3 \quad \Rightarrow \quad x = \sin^{-1} 0.3 = \alpha$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. The reference angle for α is $\pi - \alpha$, so all solutions are $x = \alpha + 2n\pi$ and $x = \pi - \alpha + 2n\pi$ [or $(2n+1)\pi - \alpha$]

33. $y = 3^{x \ln x} \quad \Rightarrow \quad y' = 3^{x \ln x}(\ln 3)\dfrac{d}{dx}(x \ln x) = 3^{x \ln x}(\ln 3)\left(x \cdot \dfrac{1}{x} + \ln x \cdot 1\right) = 3^{x \ln x}(\ln 3)(1 + \ln x)$

35. $H(v) = v \tan^{-1} v \quad \Rightarrow \quad H'(v) = v \cdot \dfrac{1}{1+v^2} + \tan^{-1} v \cdot 1 = \dfrac{v}{1+v^2} + \tan^{-1} v$

37. $y = x \sinh(x^2) \quad \Rightarrow \quad y' = x \cosh(x^2) \cdot 2x + \sinh(x^2) \cdot 1 = 2x^2 \cosh(x^2) + \sinh(x^2)$

39. $y = \ln \sin x - \frac{1}{2}\sin^2 x \quad \Rightarrow \quad y' = \dfrac{1}{\sin x} \cdot \cos x - \frac{1}{2} \cdot 2 \sin x \cdot \cos x = \cot x - \sin x \cos x$

41. $y = \ln\left(\dfrac{1}{x}\right) + \dfrac{1}{\ln x} = \ln x^{-1} + (\ln x)^{-1} = -\ln x + (\ln x)^{-1} \quad \Rightarrow \quad y' = -1 \cdot \dfrac{1}{x} + (-1)(\ln x)^{-2} \cdot \dfrac{1}{x} = -\dfrac{1}{x} - \dfrac{1}{x(\ln x)^2}$

43. $y = \ln(\cosh 3x) \quad \Rightarrow \quad y' = (1/\cosh 3x)(\sinh 3x)(3) = 3 \tanh 3x$

45. $y = \cosh^{-1}(\sinh x) \quad \Rightarrow \quad y' = (\cosh x)/\sqrt{\sinh^2 x - 1}$

47. $y = \cos\left(e^{\sqrt{\tan 3x}}\right) \quad \Rightarrow$

$$y' = -\sin\left(e^{\sqrt{\tan 3x}}\right) \cdot \left(e^{\sqrt{\tan 3x}}\right)' = -\sin\left(e^{\sqrt{\tan 3x}}\right) e^{\sqrt{\tan 3x}} \cdot \tfrac{1}{2}(\tan 3x)^{-1/2} \cdot \sec^2(3x) \cdot 3$$

$$= \frac{-3 \sin\left(e^{\sqrt{\tan 3x}}\right) e^{\sqrt{\tan 3x}} \sec^2(3x)}{2\sqrt{\tan 3x}}$$

49. $f(x) = e^{g(x)} \quad \Rightarrow \quad f'(x) = e^{g(x)} g'(x)$

50. $f(x) = g(e^x) \quad \Rightarrow \quad f'(x) = g'(e^x)\, e^x$

51. $f(x) = \ln|g(x)| \quad \Rightarrow \quad f'(x) = \dfrac{1}{g(x)} g'(x) = \dfrac{g'(x)}{g(x)}$

52. $f(x) = g(\ln x) \quad \Rightarrow \quad f'(x) = g'(\ln x) \cdot \dfrac{1}{x} = \dfrac{g'(\ln x)}{x}$

57. $y = (2+x)e^{-x} \quad \Rightarrow \quad y' = (2+x)(-e^{-x}) + e^{-x} \cdot 1 = e^{-x}[-(2+x)+1] = e^{-x}(-x-1)$. At $(0, 2)$, $y' = 1(-1) = -1$, so an equation of the tangent line is $y - 2 = -1(x - 0)$, or $y = -x + 2$.

67. Let $t = \sinh x$. As $x \to 0^+$, $t \to 0^+$. $\displaystyle\lim_{x \to 0^+} \ln(\sinh x) = \lim_{t \to 0^+} \ln t = -\infty$

69. $\lim_{x\to\infty} \dfrac{(1+2^x)/2^x}{(1-2^x)/2^x} = \lim_{x\to\infty} \dfrac{1/2^x+1}{1/2^x-1} = \dfrac{0+1}{0-1} = -1$

71. This limit has the form $\frac{0}{0}$. $\lim_{x\to 0} \dfrac{\tan \pi x}{\ln(1+x)} \overset{\text{H}}{=} \lim_{x\to 0} \dfrac{\pi \sec^2 \pi x}{1/(1+x)} = \dfrac{\pi \cdot 1^2}{1/1} = \pi$

73. This limit has the form $\frac{0}{0}$. $\lim_{x\to 0} \dfrac{e^{4x}-1-4x}{x^2} \overset{\text{H}}{=} \lim_{x\to 0} \dfrac{4e^{4x}-4}{2x} \overset{\text{H}}{=} \lim_{x\to 0} \dfrac{16e^{4x}}{2} = \lim_{x\to 0} 8e^{4x} = 8 \cdot 1 = 8$

75. This limit has the form $\infty \cdot 0$. $\lim_{x\to\infty} x^3 e^{-x} = \lim_{x\to\infty} \dfrac{x^3}{e^x} \overset{\text{H}}{=} \lim_{x\to\infty} \dfrac{3x^2}{e^x} \overset{\text{H}}{=} \lim_{x\to\infty} \dfrac{6x}{e^x} \overset{\text{H}}{=} \lim_{x\to\infty} \dfrac{6}{e^x} = 0$

77. This limit has the form $\infty - \infty$.

$$\lim_{x\to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right) = \lim_{x\to 1^+} \left(\frac{x \ln x - x + 1}{(x-1)\ln x} \right) \overset{\text{H}}{=} \lim_{x\to 1^+} \frac{x \cdot (1/x) + \ln x - 1}{(x-1)\cdot(1/x) + \ln x} = \lim_{x\to 1^+} \frac{\ln x}{1 - 1/x + \ln x}$$

$$\overset{\text{H}}{=} \lim_{x\to 1^+} \frac{1/x}{1/x^2 + 1/x} = \frac{1}{1+1} = \frac{1}{2}$$

89. (a) $y(t) = y(0)e^{kt} = 200e^{kt} \quad \Rightarrow \quad y(0.5) = 200e^{0.5k} = 360 \quad \Rightarrow \quad e^{0.5k} = 1.8 \quad \Rightarrow \quad 0.5k = \ln 1.8 \quad \Rightarrow$

$k = 2\ln 1.8 = \ln(1.8)^2 = \ln 3.24 \quad \Rightarrow \quad y(t) = 200e^{(\ln 3.24)t} = 200(3.24)^t$

(b) $y(4) = 200(3.24)^4 \approx 22{,}040$ bacteria

(c) $y'(t) = 200(3.24)^t \cdot \ln 3.24$, so $y'(4) = 200(3.24)^4 \cdot \ln 3.24 \approx 25{,}910$ bacteria per hour

(d) $200(3.24)^t = 10{,}000 \quad \Rightarrow \quad (3.24)^t = 50 \quad \Rightarrow \quad t \ln 3.24 = \ln 50 \quad \Rightarrow \quad t = \ln 50 / \ln 3.24 \approx 3.33$ hours

90. (a) If $y(t)$ is the mass remaining after t years, then $y(t) = y(0)e^{kt} = 100e^{kt}$. $y(5.24) = 100e^{5.24k} = \frac{1}{2} \cdot 100 \quad \Rightarrow$

$e^{5.24k} = \frac{1}{2} \quad \Rightarrow \quad 5.24k = -\ln 2 \quad \Rightarrow \quad k = -\frac{1}{5.24}\ln 2 \quad \Rightarrow \quad y(t) = 100e^{-(\ln 2)t/5.24} = 100 \cdot 2^{-t/5.24}$. Thus,

$y(20) = 100 \cdot 2^{-20/5.24} \approx 7.1$ mg.

(b) $100 \cdot 2^{-t/5.24} = 1 \quad \Rightarrow \quad 2^{-t/5.24} = \dfrac{1}{100} \quad \Rightarrow \quad -\dfrac{t}{5.24}\ln 2 = \ln \dfrac{1}{100} \quad \Rightarrow \quad t = 5.24 \dfrac{\ln 100}{\ln 2} \approx 34.8$ years

97. Let $u = \sqrt{x}$. Then $du = \dfrac{dx}{2\sqrt{x}} \quad \Rightarrow \quad \displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C.$

99. Let $u = x^2 + 2x$. Then $du = (2x+2)\,dx = 2(x+1)\,dx$ and

$$\int \frac{x+1}{x^2+2x}\,dx = \int \frac{\frac{1}{2}\,du}{u} = \tfrac{1}{2}\ln|u| + C = \tfrac{1}{2}\ln|x^2+2x| + C.$$

101. Let $u = \ln(\cos x)$. Then $du = \dfrac{-\sin x}{\cos x}\,dx = -\tan x\,dx \quad \Rightarrow$

$\int \tan x \ln(\cos x)\,dx = -\int u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}[\ln(\cos x)]^2 + C.$

103. Let $u = \tan\theta$. Then $du = \sec^2\theta\, d\theta$ and $\int 2^{\tan\theta}\sec^2\theta\, d\theta = \int 2^u du = \frac{2^u}{\ln 2} + C = \frac{2^{\tan\theta}}{\ln 2} + C$.

105. $\int \left(\frac{1-x}{x}\right)^2 dx = \int \left(\frac{1}{x} - 1\right)^2 dx = \int \left(\frac{1}{x^2} - \frac{2}{x} + 1\right) dx = -\frac{1}{x} - 2\ln|x| + x + C$

111. $f_{\text{ave}} = \frac{1}{4-1}\int_1^4 \frac{1}{x}\, dx = \frac{1}{3}\big[\ln|x|\big]_1^4 = \frac{1}{3}[\ln 4 - \ln 1] = \frac{1}{3}\ln 4$

114. $f(x) = x + x^2 + e^x \quad\Rightarrow\quad f'(x) = 1 + 2x + e^x$ and $f(0) = 1 \quad\Rightarrow\quad g(1) = 0$ [where $g = f^{-1}$],

so $g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(0)} = \frac{1}{2}$.