14-3: Simplifying Expressions and Proving Identities

Simplify to a single trig function:

$$\tan y(\tan y + \cot y) = \underbrace{\tan^2 y + 1}$$

$$= \sec^2 y$$

Simplify to a single trig function:

$$\frac{\sec y + \csc y}{1 + \tan y} = \frac{\left[\frac{1}{\cos y} + \frac{1}{\sin y}\right]}{\left[1 + \frac{\sin y}{\cos y}\right]} \frac{\cos y \sin y}{\cos y \sin y}$$

$$= \frac{\sin y + \cos y}{\cos y \sin y + \sin^2 y}$$

$$= \frac{1\ \cancel{\sin y + \cos y}}{\sin y\,\cancel{(\cos y + \sin y)}}$$

$$= \csc y$$

Prove: $\dfrac{\sec^2 y - 1}{\sec^2 y} \equiv \sin^2 y$

$$\frac{\tan^2 y}{\sec^2 y}$$

$$\frac{\sin^2 y}{\cos^2 y} \div \frac{1}{\cos^2 y}$$

$$\frac{\sin^2 y}{\cancel{\cos^2 y}} \cdot \frac{\cancel{\cos^2 y}}{1}$$

$$\sin^2 y \equiv \sin^2 y$$

Prove: $\dfrac{\cos y}{1 + \sin y} + \tan y \equiv \sec y$

$$\frac{\cos y}{\cos y} \cdot \frac{\cos y}{1 + \sin y} + \frac{\sin y\,(1+\sin y)}{\cos y\,(1+\sin y)}$$

$$\frac{\cos^2 y + \sin y + \sin^2 y}{\cos y\,(1 + \sin y)}$$

$$= \frac{\cancel{1 + \sin y}}{\cos y\,\cancel{(1+\sin y)}}$$

$$\frac{1}{\cos y} \equiv \frac{1}{\cos y}$$