

11.3 Geometric Sequences

std. 22.0

ex. 1 Given the sequence

1, 3, 9, 27, ...

$$\begin{aligned} a_1 &= 1 \cdot 3^0 \\ a_2 &= 1 \cdot 3^1 \\ a_3 &= 1 \cdot 3^2 \\ a_4 &= 1 \cdot 3^3 \end{aligned}$$

$$a_n = a_1 \cdot r^{n-1}$$

Common ratio
 $r = 3$
 $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \dots$
 $\frac{3}{1} = \frac{9}{3}$

n th term of a geometric sequence: $a_n = a_1 \cdot r^{n-1}$

ex. 2 Given the sequence 5, 2, 0.8, 0.32, ...

a) write a rule for the n th term

b) find a_8

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} \\ a_n &= 5(.4)^{n-1} \end{aligned}$$

$$\begin{aligned} r &= \frac{2}{5} \\ r &= .4 \end{aligned}$$

$$a_8 = 5(.4)^7$$

$$a_8 = .008192$$

ex. 3 In a geometric sequence, $a_4 = 3$ and $r = 3$.

Write a rule for the n th term.

$$a_1 = \frac{1}{9}$$

$$a_n = \frac{1}{9} \cdot 3^{n-1}$$

$$a_n = a_1 \cdot r^{n-1}$$

$$3 = a_1 \cdot 3^3$$

$$3 = a_1 \cdot 3$$

ex. 4

Two terms of a geometric sequence are $a_2 = -4$ and $a_6 = -1024$. Write a rule for the n th term.

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} \\ -4 &= a_1 \cdot r^{2-1} & -4 &= a_1 \cdot r \rightarrow a_1 = \frac{-4}{r} \\ -1024 &= a_1 \cdot r^{6-1} & -1024 &= a_1 \cdot r^5 \\ & & -1024 &= \frac{-4}{r} \cdot r^5 \\ & & & \pm \sqrt[4]{256} = \sqrt[4]{r^4} \end{aligned}$$

$r = 4, a_1 = -1$
 $r = -4, a_1 = 1$

$$\begin{aligned} a_n &= (-1) \cdot 4^{n-1} \\ a_n &= \cancel{(-4)}^{n-1} \end{aligned}$$

ex. 5

Grandma gave Ryker \$1 on his 1st birthday. On his 2nd birthday, she gave him \$2, on his 3rd birthday she gave him \$4, and so on. On which birthday should Grandma give Ryker \$1024 (if she has enough money)?

11th
Birthday

$1, 2, 4, \dots, 1024$
 $a_1 \qquad \qquad \qquad a_n$

$r = 2$

$$\begin{aligned} a_n &= a_1 \cdot r^{n-1} \\ 1024 &= 1 \cdot 2^{n-1} \\ 1024 &= 2^{n-1} \end{aligned}$$
$$\begin{aligned} \log 1024 &= (n-1) \log 2 \\ \frac{\log 1024}{\log 2} &= n-1 \\ 10 &= n-1 \\ 11 &= n \end{aligned}$$