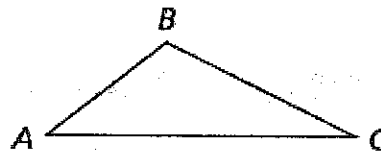


## CHAPTER 4 NOTES: SECTION 2 ANGLE RELATIONSHIPS IN TRIANGLES

Angles in a triangle have various relationships. These relationships can be used in problem solving situations.

1. **TRIANGLE INTERIOR ANGLE SUM THEOREM:** The sum of the angle measures of a triangle is  $180^\circ$ .

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

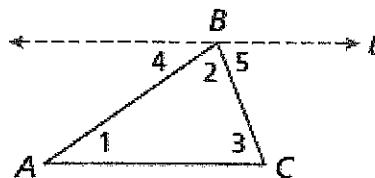


The proof of the Triangle Interior Angle Sum Theorem uses an auxiliary line. An auxiliary line is a line that is added to a figure to aid in a proof.

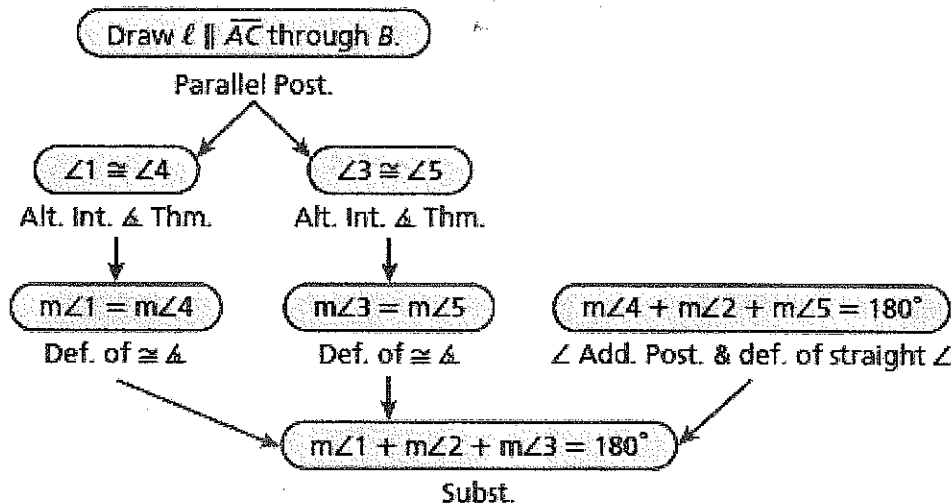
### Triangle Sum Theorem

Given:  $\triangle ABC$

Prove:  $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$



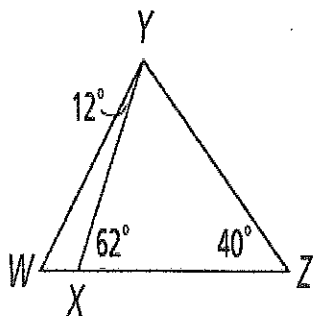
Proof:



### Caution!

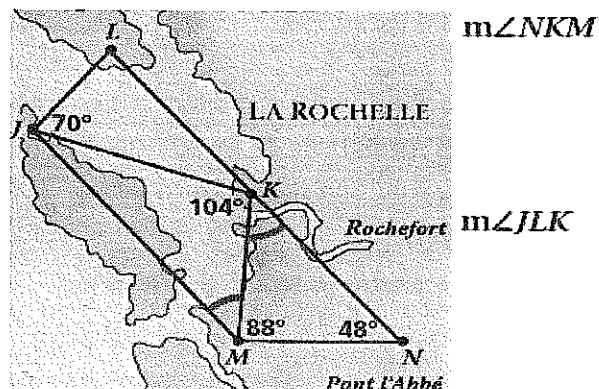
Whenever you draw an auxiliary line, you must be able to justify its existence. Give this as the reason: Through any two points there is exactly one line.

Example: After an accident, the positions of cars are measured by law enforcement to investigate the collision. Use the diagram drawn from the information collected to find the indicated angle measures.



A.  $m\angle XYZ$

B.  $m\angle YWZ$



A **corollary** is a theorem whose proof follows directly from another theorem. Here are two corollaries to the Triangle Interior Angle Sum Theorem.

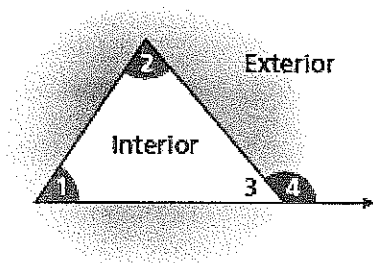
Corollaries		
COROLLARY	HYPOTHESIS	CONCLUSION
<p><b>4-2-2</b> The acute angles of a right triangle are complementary.</p>		<p><math>\angle D</math> and <math>\angle E</math> are complementary.  <math>m\angle D + m\angle E = 90^\circ</math></p>
<p><b>4-2-3</b> The measure of each angle of an equiangular triangle is <math>60^\circ</math>.</p>		<p><math>m\angle A = m\angle B = m\angle C = 60^\circ</math></p>

**Practice: Finding Angle Measures in Right Triangles**

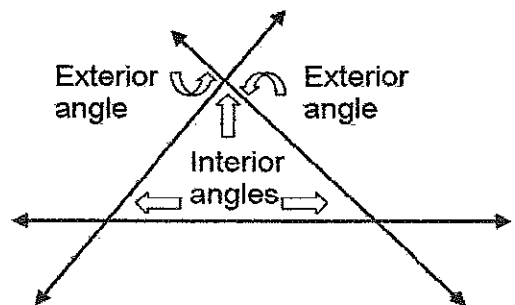
One of the acute angles in a right triangle is given. What is the measure of the other acute angle?

- A.  $22.9^\circ$                       B.  $63.7^\circ$                       C.  $x^\circ$                       D.  $48\frac{2}{5}$

The **interior** is the set of all points inside the figure. The **exterior** is the set of all points outside the figure. An **interior angle** is formed by two sides of the triangle. An **exterior angle** is formed by one side of the triangle and the extension of an adjacent side. Each angle has two **remote interior angles**. A **remote interior angle** is an interior angle that is not adjacent to the exterior angle.

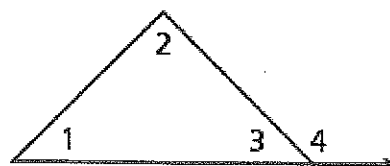


$\angle 4$  is an exterior angle. Its remote interior angles are  $\angle 1$  and  $\angle 2$ .



**2. TRIANGLE EXTERIOR ANGLE THEOREM:** The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

$$m\angle 4 = m\angle 1 + m\angle 2$$

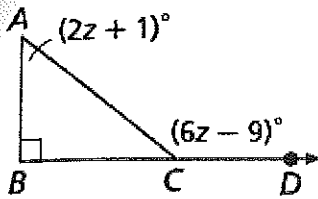
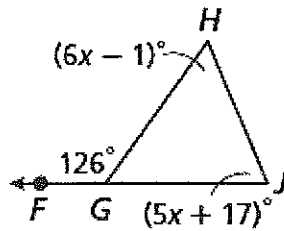
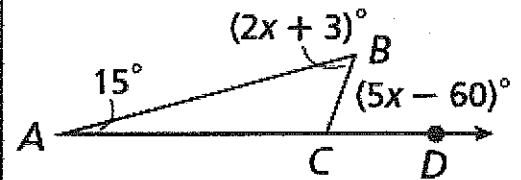


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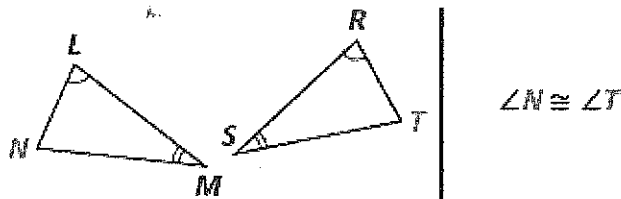
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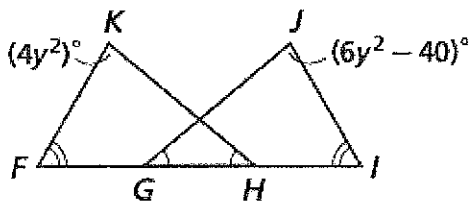
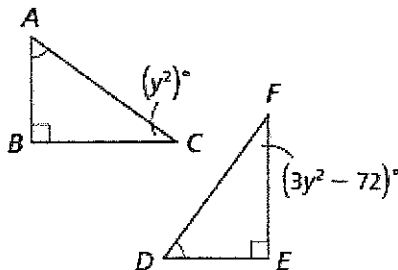
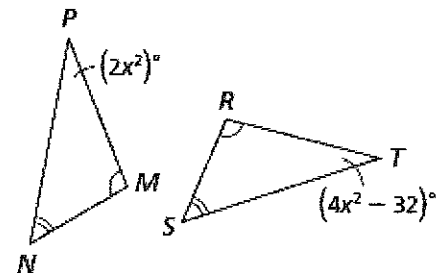
PRACTICE: FIND THE MEASURE OF EACH INDICATED ANGLE.

Find  $m\angle ACD$ .Find  $m\angle J$ .Find  $m\angle B$ .

3. **THIRD ANGLES THEOREM:** If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.

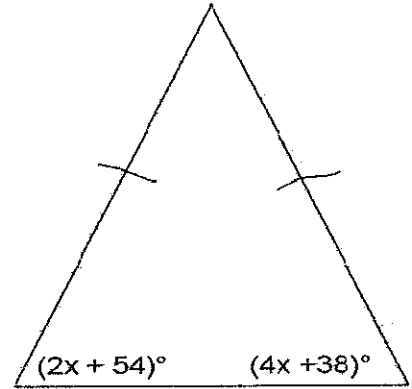


PRACTICE: Find the measure of the indicated angles.

Find  $m\angle K$  and  $m\angle J$ .Find  $m\angle C$  and  $m\angle F$ .Find  $m\angle P$  and  $m\angle T$ .

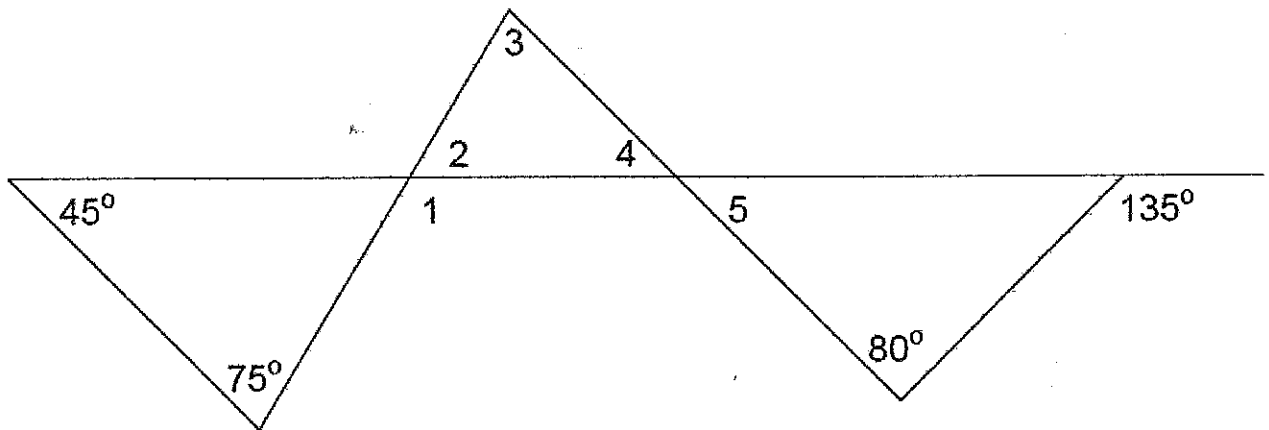
4. **ISOSCELES TRIANGLE THEOREM:** The base angles of an isosceles triangle are congruent.

Practice: Find the value of  $x$  and the measure of each base angle.



**Guided Practice**

Find the measure of each numbered angle and justify how it was found.



$m\angle 1 =$  \_\_\_\_\_

$m\angle 2 =$  \_\_\_\_\_

$m\angle 3 =$  \_\_\_\_\_

$m\angle 4 =$  \_\_\_\_\_

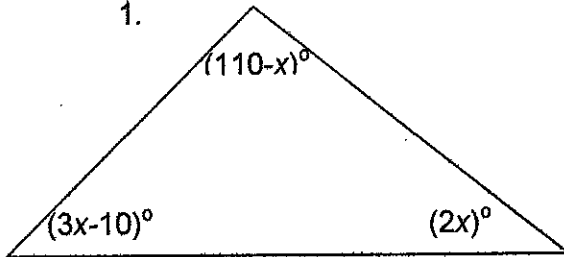
$m\angle 5 =$  \_\_\_\_\_

## Theorems Related to Triangles

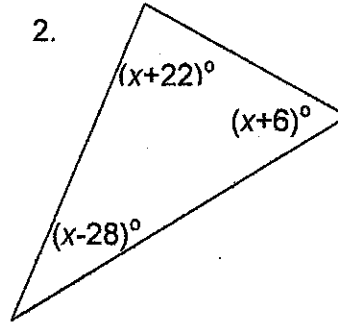
### Practice Problems

Find the measures of the interior angles.

1.

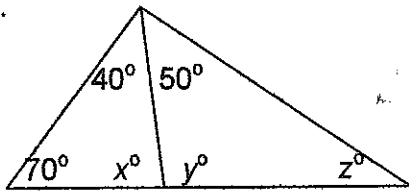


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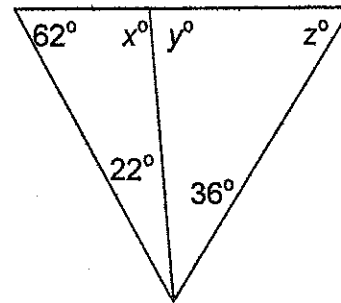


Find the value of the variables in each problem.

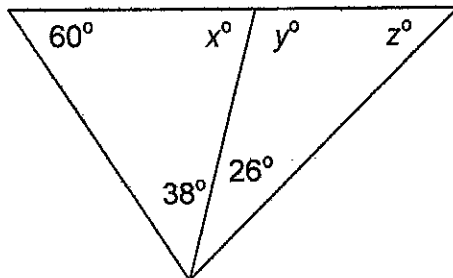
3.



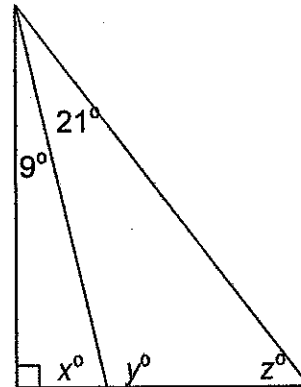
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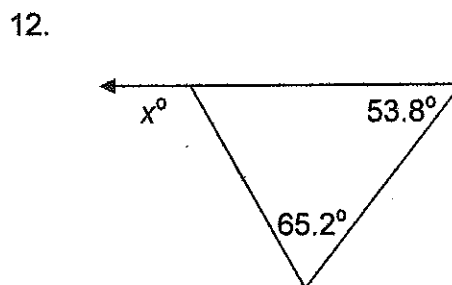
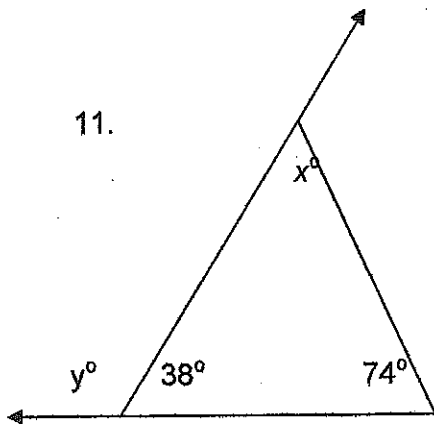
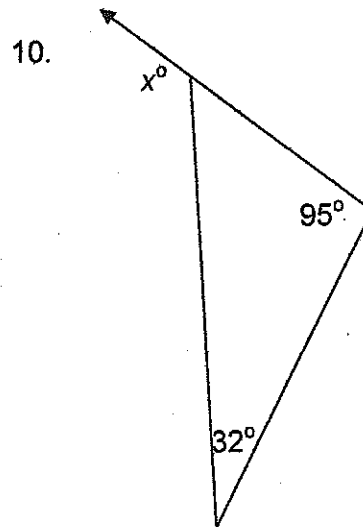
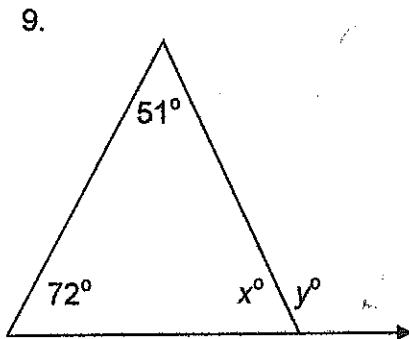
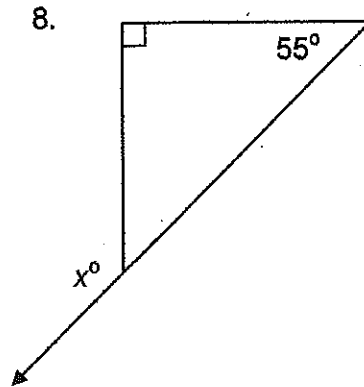
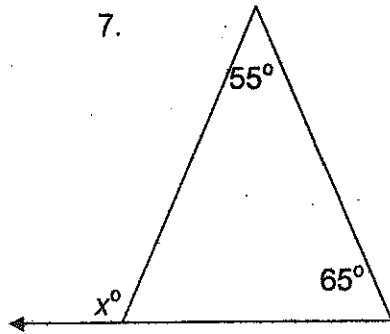
5.



6.



### Theorems Related to Triangles



13. Compare and contrast the sum of the angles in a triangle for Euclidean, spherical, and hyperbolic geometry.